Questions 1-4 deal with one or more of the following four matrices:

\[ A = \begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 4 & 1 \\ 2 & 0 & 1 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & -2 \\ -3 & 4 \\ 5 & -6 \end{pmatrix}, \quad D = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \]

1. (10 pts) For each product, either give the dimensions of the product matrix, or say the product does not exist.

\[ CB \quad DC \quad AD \quad BA \quad BC \]

2. (10 pts) Compute \( B^T A - CD \).

3. (25 pts) Find the general solution of \( \vec{x}' = A\vec{x} \).

4. (25 pts) Find the general solution of \( \vec{x}' = D\vec{x} \).

In questions 5-8, the matrix \( A \) is a \( 2 \times 2 \) matrix with eigenvalues 1 and \(-2\), and corresponding eigenvectors \( \begin{pmatrix} -3 \\ 2 \end{pmatrix} \) and \( \begin{pmatrix} -2 \\ 1 \end{pmatrix} \).

5. (10 pts) Find the solution of \( \vec{x}' = A\vec{x} \), \( \vec{x}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \).

6. (5 pts) A solution matrix for \( \vec{x}' = A\vec{x} \) is a matrix, having the same dimensions as \( A \), whose columns are each a solution vector. If the columns of a solution matrix are also linearly independent, the solution matrix is called a fundamental matrix. The matrices

\[ \Phi(t) = \begin{pmatrix} -3e^t & 6e^t \\ 2e^t & -4e^t \end{pmatrix} \quad \text{and} \quad \Psi(t) = \begin{pmatrix} -3e^t & -2e^{-2t} \\ 2e^t & e^{-2t} \end{pmatrix} \]

are both solution matrices, but only one is a fundamental matrix. Which one?

7. (10 pts) For the matrix \( \Psi(t) \) in problem 6, find the inverse of \( \Psi(0) \).

8. (5 pts) For the matrix \( \Psi(t) \) in problem 6, find \( \Psi(t)\Psi^{-1}(0) \).