1. (20 pts) Solve the DE \( y' + (\tan t)y = \sin 2t \).

2. (20 pts) Find the general solution of \( 4y'' + y = 2t + 3 \).

3. Consider the DE \( y' - 2y = 0 \).
   (a) (5 pts) Solve using separation of variables.
   (b) (5 pts) Solve using the characteristic equation.
   (c) (5 pts) Solve using the Laplace transform.
   (d) (5 pts) This is a \( \) \( \) \( \) \( \) \( \) DE, and the solutions form a vector space of dimension \( \) \( \) \( \) \( \) \( \).

4. Consider the DE \( y^{vi} - 3y^{iv} + 3y'' - y = 0 \).
   (a) (4 pts) This DE has \( \) \( \) \( \) \( \) \( \) linearly independent solutions.
   (b) (16 pts) Find the general solution.

5. (20 pts) Solve the DE \( y'' + y = g(t), \ y(0) = 0, \ y'(0) = 0, \) \( \) where \( \) \( \) \( t \) \( \) \( 0 \leq t < 3 \) \( t \geq 3 \) \( \) \( \).

6. Let \( A = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & -2 \end{pmatrix} \) and \( B = \begin{pmatrix} 1 & 3 \\ 3 & -4 \\ -2 & 1 \end{pmatrix} \).
   (a) (12 pts) Find \( AB \) and \( BA \).
   (b) (8 pts) A theorem from Linear Algebra says "If \( A \) and \( B \) are matrices such that \( AB \) and \( BA \) both exist, then the nonzero eigenvalues of \( AB \) are the same as the nonzero eigenvalues of \( BA. \)" Given that the eigenvalues of \( AB \) are \( 4 \) and \( -4 \), find the eigenvalues of \( BA \). What is the determinant of \( BA \)? (You should be able to do this without calculation.)

7. (20 pts) If \( \vec{x}(t) = \vec{c}e^{\lambda t} \) is a nonzero solution of the system \( \vec{x}'(t) = A \vec{x}(t) \), prove that \( \lambda \) must be an eigenvalue of \( A \) with corresponding eigenvector \( \vec{c} \). Hint: Substitute the solution into the system.

8. (20 pts) Find the general solution of \( \vec{x}'(t) = A \vec{x}(t) \), when \( A = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \).

9. (20 pts) Find the general solution of \( \vec{x}'(t) = A \vec{x}(t) \), when \( A = \begin{pmatrix} 1 & -\frac{9}{16} \\ 4 & -2 \end{pmatrix} \).

10. (20 pts) Find the general solution of \( \vec{x}'(t) = A \vec{x}(t) \), when \( A = \begin{pmatrix} 1 & -\frac{25}{16} \\ 4 & -2 \end{pmatrix} \).