FINAL EXAM

Math 204 Name _____

December 16, 1997

1. (20 pts) Solve the DE $y' + (\tan t)y = \sin 2t$.

2. (20 pts) Find the general solution of $4y'' + y = 2t + 3$.

3. Consider the DE $y' - 2y = 0$.

(a) (5 pts) Solve using separation of variables.

(b) (5 pts) Solve using the characteristic equation.

(c) (5 pts) Solve using the Laplace transform.

(d) (5 pts) This is a _____, _____, _____,

_____DE, and the solutions form a vector space of dimension _____.

4. Consider the DE $y^{vi} - 3y^{iv} + 3y'' - y = 0$.

(a) (4 pts) This DE has _____linearly independent solutions.

(b) (16 pts) Find the general solution.

5. (20 pts) Solve the DE $y'' + y = g(t), \quad y(0) = 0, \quad y'(0) = 0, \quad where \quad g(t) = \begin{cases} t & 0 \le t < 3 \\ 3 & t \ge 3 \end{cases}$

6. Let $A = \begin{pmatrix} 0 & -1 & -1 \\ 1 & 0 & -2 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 3 \\ 3 & -4 \\ -2 & 1 \end{pmatrix}$.

(a) (12 pts) Find AB and BA.

(b) (8 pts) A theorem from Linear Algebra says "If A and B are matrices such that AB and BA both exist, then the nonzero eigenvalues of AB are the same as the nonzero eigenvalues of BA." Given that the eigenvalues of AB are 4 and -4, find the eigenvalues of BA. What is the determinant of BA? (You should be able to do this without calculation.)

7. (20 pts) If $\vec{x}(t) = \vec{c}e^{\lambda t}$ is a nonzero solution of the system $\vec{x}'(t) = A\,\vec{x}(t)$, prove that λ must be an eigenvalue of A with corresponding eigenvector $\vec{c}$. Hint: Substitute the solution into the system.

8. (20 pts) Find the general solution of $\vec{x}'(t) = A\,\vec{x}(t)$, when $A = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix}$.

9. (20 pts) Find the general solution of $\vec{x}'(t) = A\,\vec{x}(t)$, when $A = \begin{pmatrix} 1 & -\frac{9}{16} \\ 4 & -2 \end{pmatrix}$.

10. (20 pts) Find the general solution of $\vec{x}'(t) = A\,\vec{x}(t)$, when $A = \begin{pmatrix} 1 & -\frac{25}{16} \\ 4 & -2 \end{pmatrix}$.