Exam 4

Math 8 BB
December 11, 1996

Name__________________________
Recitation time  7:30  8:30  9:30

1. (15 pts) True or False. If False, you must give a reason or a counterexample for full credit.

T F If $f$ and $g$ are integrable on $[a, b]$, then $\int_a^b f(x) g(x) \, dx = \left( \int_a^b f(x) \, dx \right) \left( \int_a^b g(x) \, dx \right)$.

T F If $f$ and $g$ are integrable on $[a, b]$, then $\int_a^b (f(x) + g(x)) \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx$.

T F If $f$ and $g$ are continuous and $f(x) \geq g(x)$ on $[a, b]$, then $\int_a^b f(x) \, dx \geq \int_a^b g(x) \, dx$.

T F For the problem of finding the volume of the solid obtained by rotating the region bounded by $y = x(x - 1)^2$ and $y = 0$ about the $y$-axis, the washer method is simpler than the method of cylindrical shells.

T F If $f$ and $g$ are continuous on $[a, b]$ and the average value of $f$ on $[a, b]$ is $A$, then $\int_a^b f(x) g(x) \, dx = A \int_a^b g(x) \, dx$.

2. (3 pts each) Short answer. Use the appropriate properties of and theorems about integrals.

(a) $\int_0^{\pi/2} \frac{d}{dx} \left( \sin \frac{x}{2} \cos \frac{x}{3} \right) \, dx =$

(b) $\frac{d}{dx} \int_0^{\pi/2} \sin \frac{x}{2} \cos \frac{x}{3} \, dx =$

(c) $\frac{d}{dx} \int_0^{\pi/2} \sin \frac{t}{2} \cos \frac{t}{3} \, dt =$

(d) The average value of $f(x) = x^n$ on $[0, 1]$ is ________________________
3. (5 pts each) Evaluate the following integrals.

(a) \( \int_{-1}^{1} |x| \, dx \)

(b) \( \int \frac{2x^3}{\sqrt{x^2 + 1}} \, dx \)

(c) \( \int_{-4}^{4} \sqrt{16 - x^2} \, dx \)

4. Consider the function defined by \( f(x) = 2 + (x - 2)^2 \) on the interval \([0, 2]\).

(a) (6 pts) Find the Riemann sum for \( f \) over \([0, 2]\) using a regular partition with \( n = 4 \) and choosing \( x_i^* \) to be the right endpoint of the \( i \)th subinterval.

(b) (4 pts) Sketch the graph of \( f \) and the approximating rectangles for \( 0 \leq x \leq 2 \).

(c) (2 pts) Is the Riemann sum you found in (a) larger or smaller than \( \int_{0}^{2} f(x) \, dx \)?
5. (13 pts) Let $\mathcal{R}$ be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$. Find the area of $\mathcal{R}$.

6. (10 pts) Set up, but do not evaluate, a definite integral for the volume of the solid obtained by rotating the region bounded by $y = 0$ and the portion of $y = \sin x$ between 0 and $\pi$ about the $x$-axis.
7. (13 pts) Find the volume of the solid obtained by rotating the region bounded by \( y = 1 \) and \( y = x^2 \) about the line \( x = 1 \).

8. (10 pts) A spherical tank of radius 5 ft is half full of water. (Water weighs 62.5 lb/ft\(^3\).) Set up, but do not evaluate, a definite integral for the work required to pump the water out of an outlet 1 ft above the top of the tank.