Exam 4

Math 8 BB Name_____

December 11, 1996 Recitation time 7:30 8:30 9:30

1. (15 pts) True or False. If False, you must give a reason or a counterexample for full credit.

T F If f and g are integrable on $[a,b]$, then $\displaystyle\int_a^b f(x)\,g(x)\,dx = \left(\int_a^b f(x)\,dx\right)\left(\int_a^b g(x)\,dx\right)$.

T F If f and g are integrable on $[a,b]$, then $\displaystyle\int_a^b (f(x)+g(x))\;dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$.

T F If f and g are continuous and $f(x) \geq g(x)$ on $[a,b]$, then $\displaystyle\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$.

T F For the problem of finding the volume of the solid obtained by rotating the region bounded by $y = x(x-1)^2$ and $y = 0$ about the y-axis, the washer method is simpler than the method of cylindrical shells.

T F If f and g are continuous on $[a,b]$ and the average value of f on $[a,b]$ is A, then $\int_a^b f(x)\,g(x)\,dx = A\int_a^b g(x)\,dx$.

2. (3 pts each) Short answer. Use the appropriate properties of and theorems about integrals.

(a) $\displaystyle\int_0^{\pi/2} \frac{d}{dx}\left(\sin\frac{x}{2}\cos\frac{x}{3}\right)\,dx =$ _____

(b) $\displaystyle\frac{d}{dx}\int_0^{\pi/2} \sin\frac{x}{2}\cos\frac{x}{3}\,dx =$ _____

(c) $\displaystyle\frac{d}{dx}\int_x^{\pi/2} \sin\frac{t}{2}\cos\frac{t}{3}\,dt =$ _____

(d) The average value of $f(x) = x^n$ on $[0,1]$ is _____

3. (5 pts each) Evaluate the following integrals.

(a) $\displaystyle\int_{-1}^{1} |x|\,dx$

(b) $\displaystyle\int \frac{2x^3}{\sqrt[3]{x^2+1}}\,dx$

(c) $\displaystyle\int_{-4}^{4} \sqrt{16-x^2}\,dx$

4. Consider the function defined by $f(x) = 2 + (x-2)^2$ on the interval $[0, 2]$.

(a) (6 pts) Find the Riemann sum for f over $[0, 2]$ using a regular partition with $n = 4$ and choosing x_i^* to be the right endpoint of the ith subinterval.

(b) (4 pts) Sketch the graph of f and the approximating rectangles for $0 \le x \le 2$.

(c) (2 pts) Is the Riemann sum you found in (a) larger or smaller than $\displaystyle\int_0^2 f(x)\,dx$?

5. (13 pts) Let $\mathcal{R}$ be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$. Find the area of $\mathcal{R}$.

6. (10 pts) Set up, but do not evaluate, a definite integral for the volume of the solid obtained by rotating the region bounded by $y = 0$ and the portion of $y = \sin x$ between 0 and π about the x-axis.

7. (13 pts) Find the volume of the solid obtained by rotating the region bounded by $y = 1$ and $y = x^2$ about the line $x = 1$.

8. (10 pts) A spherical tank of radius 5 ft is half full of water. (Water weighs 62.5 lb/ft^3.) Set up, but do not evaluate, a definite integral for the work required to pump the water out of an outlet 1 ft above the top of the tank.