
What To Do When the Trisector Comes

Underwood Dudley

A *trisector* is a person who has, he thinks, succeeded in dividing any angle into three equal parts using straightedge and compass alone. He *comes* when he sends you his trisection in the mail and asks your opinion, or (worse) calls you to discuss his work, or (worse still) shows up in person. You may think that the problem of how to deal with trisectors is not an important one; I intend to show that it is.

Trisectors form a subset of mathematical cranks, which in turn are a subset of all cranks. By "crank" I mean what Augustus DeMorgan meant by "paradoxxer" in his wonderful book *A Budget of Paradoxers*:

A great many individuals, ever since the rise of the mathematical method, have, each for himself, attacked its direct and indirect consequences. I shall call each of these persons a *paradoxxer*, and his system a *paradox*. I use the word in the old sense: a paradox is something which is apart from general opinion, either in subject-matter, method, or conclusion.

I pulled that book off a library shelf nearly thirty years ago not knowing what was in it, and it started my interest in cranks in general and mathematical cranks in particular. You might think that it would be easier for mathematicians to handle cranks than, say, economists. An advocate of Henry George's single tax could argue endlessly and inconclusively, but the only way to see if the tax would work would be to get a nation or collection of nations to give the idea a try. In mathematics, there ought to be less endless argument: given a "proof" of Fermat's Conjecture, something like "there is an error on page 4, line 12" should settle the matter, and given a trisection, there is no need even to read as far as page 4 since there is no chance whatsoever that it is correct. But unfortunately it is not that way. Trisectors can and do argue just as endlessly as any other cranks.

By the way, if you have any trisections or any works of mathematical cranks I would be most grateful to have copies. I have sought such things out for years, and my collection of analyzed trisections—standardized diagrams with neat letters, and computer printouts of errors—is in its second hundred. But I have the collector's lust: I want them *all*. Quadratures of the circle (the most popular target of cranks in the last century but now not common), duplications of the cube, proofs of Fermat's Conjecture: all are welcome.

One obvious characteristic of trisectors is that they are old. The typical trisector heard of the trisection in his geometry class, but did not succeed with his construction until many years later, usually after retirement. "His" in the last sentence is not sexist because almost all trisectors are male. From the two female trisectors I know of, it follows by an invalid statistical calculation that we can be 95% sure that the proportion of female trisectors is less than .04. Women have too much sense to waste time on such things. Trisectors are old men. An Illinois lawyer wrote in 1953:

I became interested in this problem in my junior high school year (1913–14) at Philo, Illinois. My geometry teacher suggested it. I turned it over in my mind and thought of it many times.

An engineer in 1973:

It all started in 1936. From that date on, more or less of my spare time was devoted in the field of trisection.

From the *California Freemason*, 1972:

My teacher at that time said that it was the opinion of mathematicians that a solution was impossible. For over 55 years that puzzle has bugged me.

Underwood Dudley



From Düsseldorf, 1973:

In more than 12,000 working hours I have in the course of 40 years found this solution. I am not a mathematician but a retired civil servant, now 69 years of age.

Twelve thousand working hours! Full-time is forty hours a week for fifty weeks, so the poor deluded man had devoted the equivalent of six years of his life to something as useless as trying to find two even integers whose sum is odd. What wouldn't you or I give for an extra six years of time? What couldn't we accomplish? What a deplorable waste!

Another obvious characteristic of trisectors is that they fail to understand what "impossible" means in mathematics. It is one of the great failures of mathematics education that the essential nature of mathematics is not made clear to students. Typical is the trisector who wrote:

I received through the mail an advertising brochure, from a science magazine, that had in it a simple statement—and it went something like this—the FORMULA for TRISECTING AN ANGLE has never been worked-out.

This really intrigued me.

I couldn't believe, after hundreds of years of math, that this could be true.

So he went to the library and found that all the books said that it was impossible:

How could men of science be so stupid?

Any scientist or mathematician who declares that a thing is impossible is *showing his limitations* before he even starts on the problem at hand.

A trisector in New Orleans wrote in 1933:

Moreover, we find our modern authorities of mathematics not attempting to solve these involved problems, but writing treatises showing the impossibility of proving them. Instead of offering inducements to the solution of these problems, they discourage others and dub them as "cranks."

This trisector had a testimonial from a Professor of Mathematics whom I will not name at a real University which I will not name:

I have carefully checked your work on "the trisection of the angle" and was not able to detect any fallacy in your work.

Do not do as the Professor did when the trisector comes. You will, true enough, get rid of him in the short run, but in the long run it is bad for the trisector and bad for you. I could give many other examples, including a large number of variations on the "tell me a thing is impossible and I will immediately set to work on it" theme, but those are enough.

A third characteristic of trisectors is that they do not know much mathematics. High school geometry is about as far as they have gone. Some have not even gone that far. One trisector wrote in 1902:

It was necessary to get outside of the problem to solve it, and it was not solved by a study of geometry and trigonometry, as the author has never made a study of these branches of learning.

You might think that anyone who knows higher mathematics could not be a trisector, but that is not always so. One trisector applied Desargues' Theorem in his proof, and another gave a trigonometric proof that was full of partial derivatives.

Trisectors think that the trisection is important. No one has told them that a protractor will do the job quickly and well and, if caught without one, that Archimedes' straightedge with two scratches on it will also work. An 1892 trisector had picked up an odd idea:

... the author of the present work has devoted careful study to the solving of the problem so useful and necessary to every branch of science and art.

Another said:

The study of technical magazines and data shows that a solution is being sought whereby a standard construction permits the thrice division of any given angle by the simple use of compass and ruler.

But he did not mention the magazines. The most unusual application of the trisection I have seen was suggested in 1934:

... the TRISECTION OF THE ANGLE may conceivably prove to be the key to the discovery of the modern philosopher's stone, by means of which it would be possible to change one element into another—that is to say, practical alchemy.

One trisector in Ohio refused to send me his construction because he thought that the trisection was worth money and he was afraid that I would steal it. For revenge, I put him in touch with a trisector in Texas, but as revenge it was not successful: they corresponded and each concluded that the other was not making sense. Trisectors commonly copyright, register, and have their constructions witnessed:

When the time arrived for me to submit this project to a publisher, I was very much concerned about the copyright. I was fearful that if I submitted to a publisher, they might steal the entire trisection and I would have to go to court and try to establish my right to the trisection.

Another trisector wanted to know where to apply for the prize money he had heard had been offered for the

solution. (His trisection was a new one to me: it consisted of cutting the angle out, rolling it up, cutting off the bottom so as to make a right circular cone, trisecting its circular base and unrolling it. I told him that this was a construction using straightedge, compass, and scissors and that there was no prize money, and I have not heard from him since.) A Massachusetts M.D. wrote in 1890:

Trusting that this effort of mine will prove a benefit to science, I will forgive the years of toil for the glory of the achievement.

"I will forgive." What is more infuriating than the condescension of the ignorant?

Trisectors draw complicated diagrams. Figure 1 is typical. It was too much work for me to reproduce the most extreme example in my collection, but on it the

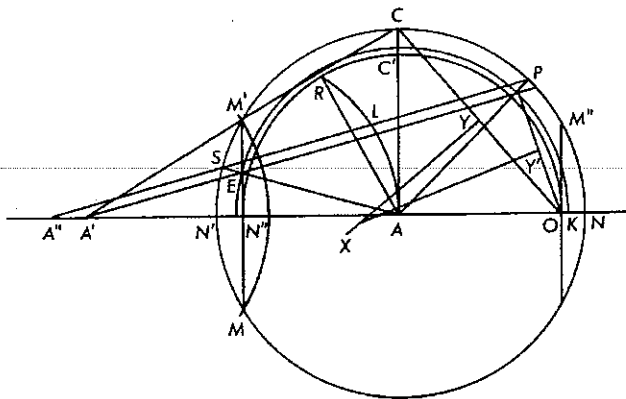
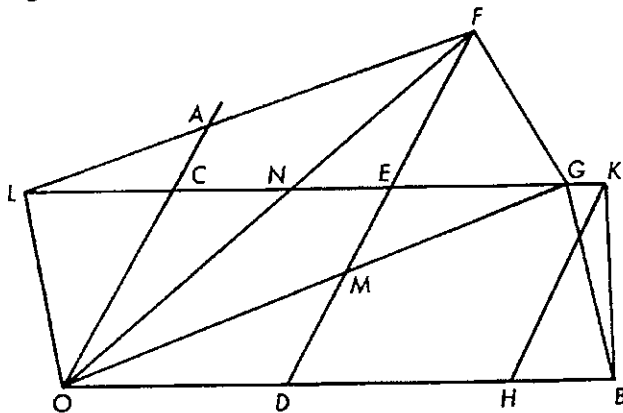


Figure 1

lines and arcs cover what seems to be all of the available space, and it is utterly impenetrable. Without exception, the constructions can be simplified, sometimes drastically. The reason for the complexity is not clear. Perhaps the trisectors become so involved with their work that they are not able to step back and look at it with a fresh eye. Perhaps they think that complexity is impressive or, consciously or not, perhaps

Figure 2



they are a bit unsure of their work and think that errors will be harder to find in a complicated diagram with many letters. Figure 2 gives what is a relatively simple trisector's diagram, and Figure 3 its simplifica-

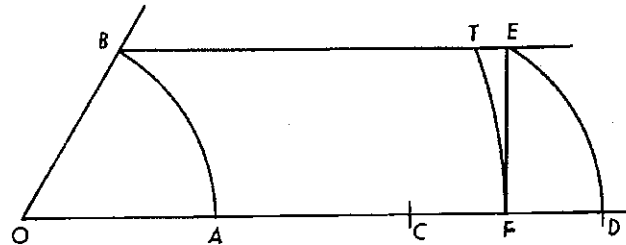


Figure 3

tion. I will not go through the details of how to produce the first diagram, but for the second, draw a line through B parallel to OA, mark off AC and CD each equal to OA, draw the arc DE with center C and radius CD, drop a perpendicular from E to OD, and draw the arc FT with center O and radius OF to get the trisection point T. As trisections go, this one is not terribly distinguished and in fact has been rediscovered several times, but as far as I know it is the only one ever perpetrated by a President of a University. The author, writing in 1933, was at the time President of a third-rank but respectable university, and in addition to the trisection he duplicated the cube and proved Euclid's Fifth Postulate. You would have thought that he would have spoken to his Department of Mathematics about his work, but many trisectors do not seek advice for reasons of fear or pride. Or perhaps he did consult and no one objected—teaching jobs were hard to come by in 1933. His construction is equivalent to asserting that

$$\sin \frac{A}{3} = \frac{\sin A}{2 + \cos A}$$

and verifying this is a good trigonometry exercise. In one of Martin Gardner's *Scientific American* columns, he wrote mistakenly that the President "had in effect merely taken an angle, tripled it, and then found the original angle again." He got that idea, I think, from one of Irving Adler's books, but I would be derelict in my duty to trisection scholarship not to point out the error. The President was better than that: he really had an approximate trisection.

Trisectors are great letter-writers. They correspond eagerly with mathematicians and are hard to get rid of. My most extreme example is a crank, not a trisector, who discovered the following fact. Take the six permutations of 1, 2, 3, arrange them in increasing order, and take first differences.

123	132	213	231	312	321
9	81	18	81	9	

The sums of the two lines are 1332 and 198. Now, take

the digits of pi in groups of three and add that magic constant, 198.

314	159	265
<u>198</u>	<u>198</u>	<u>198</u>
512	357	463

The sum of the three totals is, of all things, 1332. That is perfectly astonishing even setting aside the coincidences of $512 = 2^9$ and 3, 5, 7 in order, and I wrote the crank to tell him so. In the next three weeks I got *twelve* multi-page letters, even though I replied to none of them, and not long ago he gave it one more try.

It is almost always a mistake to correspond with trisectors because it is virtually impossible to convince them that they have made an error. One trisector sent me copies of correspondence he had had with a mathematician with the RAND Corporation. No doubt the mathematician derived satisfaction from writing:

I thought you might be interested in *my* method of angle trisection. Since the construction is rather simple (much more so than yours) and involves no change in the compass setting, I thought it would have mystic appeal to you.

He continued in a heavily ironic vein. He deserved the satisfaction, because the correspondence endured for at least seven years. It is best not to start such things.

One trisector's letters to me became increasingly abusive over time (and I later discovered that he had followed the same pattern with others). Eventually there came a letter written with a blue marker in letters two inches high; the writer was bursting with rage and frustration. Shortly after that I was sent a copy of a letter he had written to the President of my university, accusing me of various crimes and misdemeanors. I don't think my President paid much attention (he was an exceptionally vague President), but I sent the trisector a letter mentioning the laws of libel and slander and the correspondence ceased, the trisector no doubt going on to some other victim.

A corollary to the love of letter-writing is that trisectors as a group can take a great deal of time from the mathematical community, not to mention money for postage. Many mathematics departments do not bother with crank work, throwing it out or putting it in a file labeled "nuts" or "crackpots." But some of us are so filled with the teacher's urge to educate that we try to reason with the trisector. This is almost always futile. You may come close, but you will seldom succeed. A 1975 trisector in Guyana wrote:

One great scholar stated, and I quote: "I feel I should tell you it was proved a long time ago that it is impossible to trisect an angle using ruler and compass alone; consequently, any attempt to perform this construction is bound to be a waste of time."

He then prepared for me a simplified thesis proving the impossibility; and this *nearly* [italics mine] shattered my faith in the Supreme Mathematician, to whom I had made intercession for revelation. However, intuition and grace kept me constant in the face of many reverses till revelation came, pointing out to me that the requirement for the trisection of angles was the construction of a fourth root and not a cube root as the scholar had laboured to prove.

Labor in vain—proof is helpless against revelation!

Some trisectors waste vast amounts of mathematical time. A 1951 trisector in Detroit, aged 82 at the time, broadcast his constructions (he had two, one with straightedge and compass and the other with straightedge alone) to the leading state university of each state, to high-reputation private institutions, to Albert Einstein, to over 100 places in all. He had more than 60 replies! Think how many mathematician-hours went into producing those answers, some of which the trisector excerpted in his next widely distributed letter.

From the National Academy of Sciences: "There is no issue. This question has been settled once and for all." From *Mathematics Magazine*: "This has long ago been proved impossible with ungraduated ruler and compass alone." From Chicago: "Chicago will examine solutions for a fee, to cover the time cost."

There were other answers from M.I.T., Columbia, Cornell, Illinois . . . , but the best was Einstein's, which used the delightful formula:

I am so overwhelmed with correspondence that despite every desire to do so I have no time to reply to all my letters.

Some trisectors have the means to publish and distribute their works widely. One trisector had his book, *Trisection of the 120 Degree Angle* (his construction was in fact applicable only to that angle), published by a vanity press in 1973, complete with a picture of its author, a pleasant-looking old gentleman, on the back cover. I must have been in a harsh mood when I concluded a letter to him with:

It is a shame that you have spent so much time, energy, and money trying to do the same as trying to prove that the final score of a football game could be 7 to 1; that is impossible, and it can be proved. It is also a shame that you have seen fit to publish a book which can only mislead its readers and spread error.

I was properly shamed by his reply, which said:

I am most grateful to you for the interest you are showing in my trisection, and the time you have taken to correspond with me.

Pictures do not lie: he *was* a nice old gentleman. Nevertheless, the spread of error and waste of time and money are evils to be put down whenever possible.

One summer I went to visit three trisectors, and I want to describe them briefly because I think that they exemplify the most common types. My first trisector was in his middle fifties, living with his wife in the South. He was enormously pleased to see me and couldn't stop talking. He was bursting with energy and couldn't keep still, in spite of his wife's efforts to slow him down. He had no job, but had previously been in the Army; I inferred, perhaps wrongly, that he had been discharged for psychiatric reasons and was living on a pension. He had been reading mathematics books in the time left from his other activities: he painted, wrote (he had had an unsuccessful novel published by a publisher I had never heard of), he was widely read in many areas of the occult and was up on Edgar Cayce, Atlantis, and Rosicrucianism. We talked about his trisection and I tried (I was younger then, and not as wise) to show him the error of his ways. It was no good. He *seemed* to be listening to what I was saying, but none of it was making any impression. I made the mistake of mentioning the quadrature of the circle as another impossible problem with straight-edge and compass, and on returning home after a cordial stay, full of good feeling on the trisector's part, I got a quadrature from him. He later moved to California, where he belonged, I thought, but now lives in Arizona. The trisection has not affected his life greatly one way or another, I think, and he has probably now moved beyond it to other things.

My second trisector lived in a Midwestern state university town. When I turned on my tape recorder, he turned on *his* and proceeded to a long, long monologue. He had been a high-school teacher of chemistry, he said, and had thought of the trisection one day while substituting in a mathematics class. After a while he succeeded in the trisection and started to try to get his construction accepted as being correct. He tried several people at the local university, and made several attempts to address mathematical meetings, once succeeding in getting a place on the program of the state's Academy of Science. He wrote to anyone who would write back, and he said that two hundred and fifty people had examined his trisection and none had been able to find anything wrong with it. (I did not interrupt to say that I had found something wrong with it. He would not have heard.) Towards the end of the monologue he gave off guarded hints about big things he had done on other large unsolved problems. I tried to reason with him, but it was in vain. We had already had a voluminous correspondence, and after a while I noticed that all of his letters were essentially the same. They repeated each other, sometimes in the same words, and he was say-

ing in person the same things he wrote. The patterns had become fixed in his brain and could not be altered. He was a man obsessed; his life was his trisection, and its purpose was to get his achievement acknowledged. What a life! A life with no joy, only endless disappointment and continual frustration; a life blighted by the trisection.

My third trisector was in a small Midwestern city. He was a short man in his 70's, and he lived in an old house on the decaying fringe of downtown. He would not let me in the house: it was too dirty and he was ashamed for anyone to see it. The first thing we had to do was to go to the local newspaper office and see one of the editors about getting something about this meeting in the paper. The editor was noncommittal and didn't respond to the trisector's broad hints that a photographer should be fetched to take a picture. But he was prepared for that: the next stop was a photographer's, for a picture of us shaking hands. We talked for about four hours in the lobby of a hospital in whose heavily-subsidized cafeteria he ate all his meals. Besides learning about his trisection, I found out about his dropping out of college, his failed ambition for the ministry, his succession of jobs, and his aloneness. He was a gentle man, one who had probably never done anyone any harm, but still with a flame of ambition burning inside. The reason he attempted the trisection, he said, was so that he might get a little respect; respect from people as a man who had accomplished something out of the ordinary. That's all he wanted, he said—a little respect. A couple of weeks after I left, I got in the mail a clipping from the local newspaper, with the picture the photographer had taken, under the headline "LOCAL MATHEMATICIAN SCORES NEAR MISS." The story was accurate, and my trisector was pleased with it. He wrote that he had, in fact, gotten some respect. I hope that for him the trisection had a happy ending.

The happy ending is an exception, I am sure. More often, the trisection produces frustration, unhappiness, and obsession.

It is time, at last, to say what to do when the trisector comes. But first, let me say what *not* to do. One temporary way to get rid of the trisector is to say, "Well, yes, that looks pretty close, but you know you have to have a *proof* that it's right. You know, a series of statements and reasons, like in your old geometry book?" The trisector will leave, but he will come back, with proof.

At that point you might say, "Well, let me look at it," find the error and point it out to the trisector. The trisector will leave, but he will be back with a revised proof, longer, more complicated, and with the error harder to find. Enough repetitions of the revision process result in a proof that you are unable or unwilling to find the error in.

The next step, also a mistake, is to say, "I really

don't have time to go over this proof, but you know that a man named Wantzel proved in 1837 that you can't trisect the angle with straightedge and compass. There's his proof, and here's your proof; they can't both be right, so what you have to do is find the error in Wantzel." That will once again get rid of the trisector, but sooner or later he will be back with a refutation of Wantzel phrased in such a way that it is impossible to find the meaning in it. Nothing can stop the dedicated trisector.

Then what is the right thing to do when the trisector comes? To the first letter from a trisector respond politely, being sure to congratulate him for the goodness of his approximation, or its simplicity, or his cleverness in finding a new approximation. Include a computer printout giving the errors in the construction for angles of various sizes—I go from 0 to 180 degrees in steps of three. This is important because the computer still has the power to inspire respect and awe. Also, enclose some other approximate trisections with some remark like, "I thought you might be interested in seeing how other people have gotten approximate trisections."

I have greatly improved my success rate in recent years with this technique. I still remember my gratification at my first success. An engineer in New Jersey had produced a large hard-bound book, more than 250 pages long, with the title *Adventures in Geometry* stamped on the cover in gold. I thought that anyone who had invested so much in the trisection was beyond salvation, but in response to my letter he wrote in part:

I am satisfied that I have achieved only an approximation, and I will now put it aside.

A soul snatched from damnation! I have had some other recent successes and perhaps some of the now silent trisectors are convinced too.

If this technique does not suffice, then be brutal. Write a letter that is harsh, scathing, and designed to make the writer hate you. He will in any event never bother *you* again, and some of the hate may be transformed into a dislike of mathematicians and a disinclination to pursue the trisection further, because we do not do things that cause us pain if we can help it. If everyone followed this course, the race of trisectors would wither and die. Then all those people who are cranks because it is part of their nature to be cranks would go and bother the economists, physicists, or theologians, and we could live in serenity and security, knowing that never again would the trisector come.

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