Numerical Modeling of Periodic Composite Media for Electromagnetic Shielding Application

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Abstract—This paper describes a methodology to extract effective electrical properties for periodic composite medium. The extraction algorithm is based on a periodic finite-difference time-domain (FDTD) method. The results are compared with conventional mixing theories and 3D Fourier series expansion methods. Two results show satisfactory agreement. With the extracted effective permittivity and conductivity, one can readily use these parameters to study electrical properties of composite materials with arbitrary micro-geometry and the shielding effects of using composite materials.

Keywords—composite medium, FDTD method, shielding, mixture theory, periodic structures.

I. INTRODUCTION

Recently, composite materials are being explored as shielding material for electromagnetic compatibility (EMC) and electromagnetic interference (EMI) applications [1]-[7]. Various composite materials and associated structures have been proposed and tested. To understand the EMC shielding performance of these materials, it is imperative to understand the electrical properties of these medium in terms of effective permittivity, conductivity, and permeability. To accurately extract these parameters, methods that can homogenize the composite materials using these macroscopic parameters are required.

Many analytical and numerical approaches have been proposed to evaluate the effective dielectric constant and conductivity of composite material with inclusions. Among them, Maxwell-Garnett [8] and Bruggeman [9] formulations, based on a conventional mixing theory, have served as the fundamentals for these techniques. In these formulations, the macroscopic parameters are related to the volume fraction of different inclusions. To investigate the effect of inclusion shapes on the macroscopic parameters, some analytical techniques, such as the Fourier series expansion method [10]-[11] and simplified Fourier series expansion methods [12][13], were developed. These techniques allowed us to study the effect of several typical shapes of inclusion, but only limited to a small class of shapes. To analyze structures with arbitrary shapes of inclusions, several numerical techniques were introduced. The FDTD method was used to analyze electric behavior of two-dimensional dielectric mixtures bounded within transverse electromagnetic perfect electric conductor waveguide [14][15]. The effective parameters were then obtained based on reflection and transmission coefficients. In addition, Toumelin et al [16] applied the method of moment to analyze two-dimensional cylindrical composite mixtures. The calculated current density and electric fields were then used to extract the effective properties of mixtures. To evaluate the electrical properties of three-dimensional dielectric mixtures, three-dimensional FDTD method [17] and finite element method [18] with simplified periodic boundary conditions were utilized.

However, previous approaches used electric-static approximation. With the electric-static approximation, simplification in the periodic boundary condition was also used. Such approximations may not meet the needs for EMC shielding applications where shielding materials need to have effective shielding performance over a frequency range. In order to thoroughly characterize and understand the frequency-dependent behavior of these composite media, it is imperative to develop more rigorous modeling tool that can extract macroscopic parameters as function of frequency.

In this paper, a full-wave three-dimensional periodic FDTD method is developed to extract effective dielectric properties of periodic composite material [19]. Following a brief description of the methodologies, we show that this technique can be used to accurately extract the dielectric properties of composite media with arbitrary inclusions. The shielding effectiveness of several single layer composite materials was also shown.

II. METHODOLOGY

The 3D composite mixtures considered here can be approximated by an array of periodic sub-cells in three directions shown as Fig. 1(a). Since this structure is geometrically periodic and infinite large, with plane wave incidence, the original problem can be formulated into the
modeling a single unit element with appropriate periodic boundary conditions in all three directions [10][13][17], shown as Fig. 1(b).

When an external field propagates through the medium, electric and magnetic fields within the medium are closely related to its internal structure. If internal electric field and the electric flux density are known, the effective electrical property of such single periodic cell can be calculated by:

\[ e'(\omega) = e_r(\omega) - j \frac{\sigma(\omega)}{\omega \varepsilon_0} = \frac{D(\omega)}{E(\omega)} = \frac{\text{FFT}(D^{\text{avg}})}{\text{FFT}(E^{\text{avg}})} \]

where \( D^{\text{avg}} \) and \( E^{\text{avg}} \) are the averaged electric flux density and the electric field across a unit cell. To evaluate the induced electric and magnetic field distribution within the composite medium, the FDTD method with appropriate periodic boundary conditions should be applied.

\[ (\mathbf{E}(0,x,y), \mathbf{H}(0,x,y)) = (\mathbf{E}(p_x,x,y), \mathbf{H}(p_x,x,y)) \]

where \( p_x = k_x \sin \theta = \omega \sin \theta / c_0 \), \( p_x \) is the periodicity along the \( x \) direction, and \( c_0 \) is the speed of light in the vacuum.

A cross sectional view of a unit periodic element is shown in Fig. 2. In this figure, the incident signal is propagating along the \(-y\) direction with an incident angle \( \theta \). At the top and bottom of this structure, the perfect matched layer (PML) boundary condition is used to truncate the simulation domain. Periodic boundary conditions are implemented at other interfaces of a unit element. The relationship between electromagnetic fields at two boundaries satisfies the periodic boundary conditions as:

\[ \mathbf{E}(x=0,y,z) = \mathbf{E}(x=p_x,y,z) \exp(jk_xp_x) \]

\[ \mathbf{H}(x=0,y,z) = \mathbf{H}(x=p_x,y,z) \exp(jk_xp_x) \]

where \( k_x = k_y \sin \theta = \omega c_0 / c_0 \), \( p_x \) is the periodicity along the \( x \) direction, and \( c_0 \) is the speed of light in the vacuum.

III. NUMERICAL EXAMPLES AND DISCUSSION

A. Validation Example

Using the numerical method developed above, a numerical example is used to validate this approach. In this example, as shown in Fig. 1(b), the effective dielectric constant of a simple unit cubic cell (\( \varepsilon_r = 1.0 \)) with a spherical inclusion (\( \varepsilon_r = 2.0 \)) is calculated using the proposed method, the Maxwell-Garnett formula and the Fourier series expansion method [13]. Figure 3 illustrates the effective dielectric constant versus permittivity ratio between the two media, where the ratio between the sphere radius and the unit cell size is at 0.4. It is noticed that the FDTD simulation results agree very well with those two analytical solutions when the contrast between two medium is not very high. However, as the contrast increases, more sampling points maybe required for the FDTD simulations.

B. A periodic composite material with lossy spherical inclusions.

With this validated code, we consider the second example to extract the frequency-dependent effective electrical properties of a lossy sphere embedded in a lossy cubic cell. The geometrical dimension of this structure is same as that of the previous example as shown in Fig. 1(b). The relative dielectric constant and conductivity of host medium is 10.0 and 0.001 s/m, and the relative dielectric constant and conductivity of
inclusion is 40.0 and 0.1 s/m, respectively. The side length of the host cubic cell is assumed to be 1 cm. The extracted effective permittivity and conductivity as a function of frequency for different ratios, defined as the spherical inclusions radius over the side length of the host cell, are shown in Figures 4 and 5. As expected, the structure with larger inclusion will have higher effective permittivity and conductivity. These results have similar trends as those obtained using the Bergman-Milton formula [11]. In general, the effective permittivity decreases as frequency increases while the effective conductivity increases as frequency increases. Both of them tend to saturate when frequency is above 1 GHz.

Reflection and transmission coefficients of composite media at normal incidence are calculated and illustrated in Figures 7 and 8. In the figure, the thickness of this structure is only 1 cm. Therefore, we refer to this structure as a single layer composite structure. Based on the transmission coefficients, the shielding effectiveness can be calculated. As expected, the higher volume fraction of inclusion typically leads to higher shielding effectiveness over the whole frequency range. In addition, it is observed that the peak value of shielding occurs at different frequencies for different size of inclusions. Based on this characteristic, it is possible for optimize the inclusion sizes for a specific frequency band of interests. The thickness of composite material should also be considered in the design of EMC shielding designs. The normalized electric fields at 1 GHz for different volume-fractions are shown in Fig. 8 for three center-cut planes as well. The first two subfigures correspond to the planes that are perpendicular to the wave propagation direction while the last subfigure illustrated the electric field distribution along the plane of incident wave. From these figures, we can see that the material with larger inclusions has a better shielding effect than those with smaller inclusions at 1 GHz.
C. A periodic composite medium with complex inclusions.

The third example considered in this paper is the extraction of effective electrical properties for a complex mixture as illustrated in Fig. 9. The detailed geometrical parameters can be found in Fig. 9. For this example, the volume fraction of inclusion is fixed. Two simulations are carried out for this example. For the first simulation, the relative permittivity and conductivity of the host material is set to be 4.0 and 0.0001 s/m, and the relative permittivity and conductivity of inclusion is set as 16.0 and 0.1 s/m. For the second simulation, the relative permittivity and conductivity of host material is unchanged while the relative permittivity and conductivity of inclusion is changed to 40.0 and 0.2 s/m. The calculated effective permittivity and conductivity values are shown in Figures 10 and 11. The corresponding reflection and transmission coefficients for normal incidence are depicted in Figures 12 and 13. As expected, similar phenomena as the second example can be observed. Different inclusion permittivity will cause different frequency-dependent behaviors in terms of effective properties and corresponding shielding effectiveness. The normalized electric fields at 1GHz for the first simulation are plotted for three center-cut planes in Fig. 14. For this example, two different simulations have very similar patterns of field distributions.

Figure 8. Normalize electric fields along three center-cut planes.

Figure 9. A unit cell for the third example.

Figure 10. Effective relative permittivity versus frequency for various inclusions.

Figure 11. Effective conductivity versus frequency for various inclusions.

Figure 12. Reflection coefficient versus frequency for various inclusions (normal incidence).
Three-dimensional finite-difference time-domain method with periodic boundary condition is employed in this paper to calculate the effective electrical properties of periodic composite material used for electromagnetic shielding applications. It has also been shown that this method is capable of characterizing the dispersion behaviors of composite mixture with arbitrary shapes, orientation, and filling factors. Consequently, this method can be utilized to analyze and design appropriate composite media for shielding at microwave frequencies.

IV. CONCLUSION

REFERENCES