Measurement of Electromagnetic Parameters and FDTD Modeling of Ferrite Cores

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Abstract—The paper describes a methodology for an efficient design of novel products based on magneto-dielectric (ferrite) materials with desirable frequency responses that satisfy EMC and SI requirements. The methodology starts from estimating complex permittivity and permeability of these materials. This requires measurement techniques, approximation resultant frequency characteristics for permittivity and permeability using a curve-fitting procedure, and development of a full-wave numerical simulation tool that could deal with frequency-dispersive materials. An example of a ferrite material measurement, constitutive parameters extraction using a genetic algorithm, and corresponding FDTD modeling over the frequency range from 10 to 500 MHz is provided.

I. I N T R O D U C T I O N

The objective of this paper is to develop a methodology for analyzing magneto-dielectric materials and components that can be used for electromagnetic compatibility (EMC) and signal integrity (SI) purposes. This includes characterization of constitutive electromagnetic properties of magneto-dielectric materials, i.e., their complex permittivity and permeability. The methodology includes measurement techniques, parameter extraction procedure, and numerical tool that would allow for simulating magnetic and dielectric materials with complex frequency-dispersive characteristics.

The methodology presented in this paper is tested on a sample made of a soft spinel ferrite, such as Ni-Zn, Mn-Zn, or other. Measurements of complex permittivity and permeability are based on obtaining input impedance of the sample using either a Vector Network Analyzer (VNA), or an Impedance Analyzer (IA).

Frequency-dependent complex permittivity and permeability are calculated from the input impedance measurements using the coaxial transmission line theory. A curve-fitting procedure, based on a robust optimization genetic algorithm (GA) technique [1], is adopted to extract the parameters of the ferrite as a double-Debye material (DDM) over a wideband frequency range. DDM is defined as a material whose permittivity and permeability can be represented as sums of Debye-like terms [2, 3]. The DDM is comparatively easy to implement in the finite-difference time-domain (FDTD) numerical code. In this particular study, the method based on discretized auxiliary equations (ADE) [4] is used in the FDTD codes. This code is then used to compare the simulation results with the original measurements.

II. M E T H O D O L O G Y

A. Complex Permittivity and Permeability Measurement

Although there are known many methods of measuring permittivity and permeability [5]-[7], in this study, a very simple, but effective method based on the transmission line theory, has been employed. There are two kinds of transmission lines which are suitable for measuring parameters of ferrite materials. They are coaxial line and stripline, and the key point is that the magnetic field lines form a close loop inside the material under test so that the demagnetization effect is minimized, and the material fully responds to the incident magnetic field. In the present study, the sample, or a “device under test” (DUT), is a ferrite cylindrical core with internal and external surfaces coated by silver (see Fig. 1).

Fig 1. Ferrite core coated with silver.
an IA is preferable, since it is cheaper and simpler than a VNA.

The DUT is connected to a coaxial cable with a specially designed precision adaptor, as is shown in Fig. 2 (a, b). The operating frequency in this setup ranges from 10 to 500 MHz. Permeability and permittivity are estimated by impedance measurements in short-circuit and open-circuit regimes. In the short-circuit regime, the rear end of the ferrite core is completely coated with silver, while in the open-circuit regime silver coating is removed.

As is known from the transmission-line theory, for a one-end shorted transmission line with \( l \) length, the input impedance is [8]

\[
Z_{\text{short}} = jZ_0 \tan(\gamma l),
\]

where \( Z_0 \) is the characteristic impedance of the transmission line, and \( \gamma \) is the complex propagation constant. For an open end of transmission line, the input impedance is [8]

\[
Z_{\text{open}} = Z_0 (j \tan(\gamma l)).
\]

After two separate measurements of \( Z_{\text{short}} \) and \( Z_{\text{open}} \), the characteristic impedance \( Z_0 \) and the time delay (TD) can be calculated as

\[
Z_0 = \sqrt{Z_{\text{open}} \cdot Z_{\text{short}}},
\]

and

\[
TD = \frac{1}{\omega} \cdot \arctan \left( \frac{1}{j \sqrt{Z_{\text{open}} \cdot Z_{\text{short}}}} \right).
\]

It should be underlined that both \( Z_0 \) and \( TD \) are complex numbers, since the material under study is lossy.

Important conditions are

\[
\text{Re}(Z_0) \geq 0; \text{Im}(TD) \leq 0.
\]

From the wave impedance of the coaxial structure and propagation velocity, it is easy to get

\[
\sqrt{\frac{\mu}{\varepsilon}} = \frac{2\pi \cdot Z_0}{\ln(OD/ID)},
\]

and

\[
\sqrt{\mu} \varepsilon = \frac{TD}{l},
\]

where \( \mu = \mu_e \cdot \mu_0 \), \( \varepsilon = \varepsilon_r \cdot \varepsilon_0 \), and \( \mu_0 = 4\pi \times 10^{-7} \) H/m, \( \varepsilon_0 = 8.854 \cdot 10^{-12} = \frac{1}{36\pi} \cdot 10^{-9} \) F/m. Herein, \( OD \) and \( ID \) are the outer and inner diameters of the core, respectively.

Thus the relative permeability can be calculated as

\[
\mu_r = \frac{Z_0 \cdot TD(ns)}{2 \ln(OD/ID) \cdot l(cm)},
\]

and the relative permittivity is

\[
\varepsilon_r = \frac{1800 \ln(OD/ID) \cdot TD(ns)}{Z_0 \cdot l(cm)}.
\]

B. A Genetic Algorithm for Material Properties Extraction

Constitutive parameters \( \varepsilon_r \) and \( \mu_r \) of a ferrite material are complex and frequency-dependent. They can be approximated as series of Debye-like terms (with poles of the first order) as [3]

\[
\varepsilon_r(\omega) = \varepsilon_{r_0} + \sum_{p=1}^{\infty} \frac{\varepsilon_{r_p} - \varepsilon_{r_0}}{1 + j\omega\tau_{r_p}} + \frac{\sigma_r}{j\omega\varepsilon_0},
\]

and

\[
\mu_r(\omega) = 1 + \sum_{p=1}^{\infty} \frac{\chi_{\mu_p}}{1 + j\omega\tau_{\mu_p}}.
\]

The parameters of these Debye-like terms \( \varepsilon_{r_p}, \varepsilon_{r_0}, \chi_{\mu_p}, \tau_{r_p}, \sigma_r \) and \( \tau_{\mu_p} \) can be extracted using a curve-fitting algorithm. In this work, a genetic algorithm has been adopted due to its robustness and global-search optimization nature. GAs are based on the mechanics of natural selection and genetics [1]. The GA only needs to evaluate the objective function to guide its search, and there is no requirement for derivatives or other auxiliary knowledge [9]. To implement a GA for solving an optimization problem, it is necessary to formulate the problem mathematically by defining an objective function, building up an analytic model, and choosing GA operators, such as selection, recombination, and mutation. Figures 3 (a-d) show the measured complex permittivity and permeability frequency dependencies and the corresponding curves fitted using the GA. There is some deviation, since a limited number of Debye terms have been used (two or three), and the measured results might not always

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Fig 2. (a) Measurement setup, and (b) a precision adaptor.
be causal and accurate, especially at frequencies above 100 MHz.

The permeability was approximated using three Debye terms, and the permittivity was fitted using two Debye terms and a low-frequency ohmic conductivity term that depends on $\sigma_c$.

The extracted permittivity data is $\varepsilon_{\text{r}1} = 15.63$ ; $\varepsilon_{\text{r}2} = 14.20$; $\varepsilon_{\infty} = 13.40$; $\tau_{\varepsilon} = 6.025 \times 10^{-8}$ s; $\tau_{\varepsilon}^* = 4.505 \times 10^{-9}$; and $\sigma_c = 4.312 \times 10^{-4}$ S/m.

The extracted permeability data is $\mu_{\text{i}1} = \chi_{\text{i}1} + 1 = 551.9$; $\mu_{\text{i}2} = \chi_{\text{i}2} + 1 = 105.6$; $\mu_{\text{i}3} = \chi_{\text{i}3} + 1 = 156.2$; $\tau_{\mu} = 2.085 \times 10^{-8}$ s; $\tau_{\mu}^* = 2.934 \times 10^{-9}$; and $\tau_{\mu}^* = 5.211 \times 10^{-8}$ s.

It should be mentioned that some of the terms might appear to be negative for curve-fitting purposes. This is a mathematical result rather than physical, and for this reason we call them “Debye-like terms” rather than “Debye terms”.

\[ \begin{align*}
\varepsilon_r'(\omega) &= \varepsilon_{\infty} + \sum_{p=1}^{P} \frac{\varepsilon_p}{\varepsilon_{\infty}} + \frac{\sigma_c}{j\omega \varepsilon_{\infty}} \\
\mu_r'(\omega) &= \mu_{\infty} + \sum_{p=1}^{P} \frac{\mu_p}{\mu_{\infty}} + \frac{\sigma_c}{j\omega \mu_{\infty}} \\
\varepsilon_r''(\omega) &= \sum_{p=1}^{P} \varepsilon_p \\
\mu_r''(\omega) &= \sum_{p=1}^{P} \mu_p
\end{align*} \]

C. Numerical Modeling of a Double-Debye Material Using FDTD Technique

Representation of frequency-dispersive permeability and permittivity of a magneto-dielectric (ferrite) material as a sum of Debye-like terms allows for implementing this material in numerical electromagnetic codes. There are almost no commercially available codes that can deal with simultaneously dispersive dielectric and magnetic properties of materials. This is the only CST Microwave Studio code where two-term Debye permittivity and permeability have been recently implemented. The EZ-FDTD code developed in the MS&T (former UMR) together with IBM can model up to five-term permeability and permittivity. The EZ-FDTD code uses so-called auxiliary differential equations (ADE) approach [3, 4] to treat complex frequency dependencies of constitutive parameters of materials. We have found this method to be the simplest and the most efficient compared to the other possible FDTD algorithms of treating dispersive materials, such as the recursive convolution (RC), the piecewise linear recursive convolution (PLRC), and the Z-transform method [10]-[12].

The application of the ADE method is identical for permittivity and permeability, and this makes this method comparatively simple for implementation in codes.

The permittivity function (10) and permeability function (11) can be represented as

\[ \begin{align*}
\varepsilon_r'(\omega) &= \varepsilon_{\infty} + \sum_{p=1}^{P} \frac{\varepsilon_p}{\varepsilon_{\infty}} + \frac{\sigma_c}{j\omega \varepsilon_{\infty}} \\
\mu_r'(\omega) &= \mu_{\infty} + \sum_{p=1}^{P} \frac{\mu_p}{\mu_{\infty}} + \frac{\sigma_c}{j\omega \mu_{\infty}} \\
\varepsilon_r''(\omega) &= \sum_{p=1}^{P} \varepsilon_p \\
\mu_r''(\omega) &= \sum_{p=1}^{P} \mu_p
\end{align*} \]

Magnetic Debye terms are analogous to the dielectric Debye terms associated with polarization dynamics. But in soft ferrites they describe the domain-wall dynamics at high-frequency exposure. Ohmic conductivity in dielectrics is associated with impurities causing some free charge, and, hence, some very low conductivity current density. However,
physically, there cannot be any magnetic “conductivity” in ferrites or other magnetic material. The magnetic “conductivity” term has been added in (13) only for total analogy of equations. This magnetic “conductivity” can be introduced in the equations, if it is associated with some magnetic loss mechanism, different from the domain wall rotation or resonances. Also, it is convenient to have this magnetic “conductivity” for modeling materials with constant values of \( \mu' \) and \( \mu'' \) over comparatively narrow frequency bands. In particular case, this magnetic “conductivity” may be analogous to each other [9].

The H-field loop iteration and E-filed loop iteration are analogous to each other [9].

\[
H^{\ddagger} = \left( \frac{2\mu_s[\mu_e + \sum_{n=1}^{P} \beta_n^e] - \sigma_s \Delta t}{2\mu_s[\mu_e + \sum_{n=1}^{P} \beta_n^e] + \sigma_s \Delta t} \right) H^* + \frac{2\Delta t}{2\mu_s[\mu_e + \sum_{n=1}^{P} \beta_n^e] + \sigma_s \Delta t} \left[ \nabla \times E^* - \frac{1}{2} \sum_{n=1}^{P} (1 + k_n^e M_n^e) \right] \tag{14}
\]

and

\[
E^{\ddagger} = \left( \frac{2\epsilon_i[\epsilon_s + \sum_{n=1}^{P} \beta_n^s] - \sigma_i \Delta t}{2\epsilon_i[\epsilon_s + \sum_{n=1}^{P} \beta_n^s] + \sigma_i \Delta t} \right) E^* + \frac{2\Delta t}{2\epsilon_i[\epsilon_s + \sum_{n=1}^{P} \beta_n^s] + \sigma_i \Delta t} \left[ \nabla \times H^{\ddagger} - \frac{1}{2} \sum_{n=1}^{P} (1 + k_n^s J_n^s) \right]. \tag{15}
\]

In (14) and (15), there are auxiliary magnetic \( M \) and electric \( J \) sources introduced at each time-step \( n \):

\[
M_n^{\ddagger} = k_n^e M_n^e + 2\mu_e \beta_n^e \left( \frac{H^{\ddagger} - H^*}{\Delta t} \right) \tag{16}
\]

and

\[
J_n^{\ddagger} = k_n^s J_n^s + 2\epsilon_e \beta_n^s \left( \frac{E^{\ddagger} - E^*}{\Delta t} \right), \tag{17}
\]

where \( \Delta t \) is the time step for the differential operators in Maxwell equations. The auxiliary parameters in (14) and (15) are

\[
k_n^e = \frac{2r_n^e - \Delta t}{2r_n^e + \Delta t}; \quad \beta_n^e = \frac{(\mu_s - \mu_e) \Delta t}{2r_n^e + \Delta t};
\]

\[
k_n^s = \frac{2r_n^s - \Delta t}{2r_n^s + \Delta t}; \quad \beta_n^s = \frac{(\epsilon_s - \epsilon_e) \Delta t}{2r_n^s + \Delta t}. \tag{18}
\]

D. Comparison between Simulations and Measurements

The input impedances in short-circuit and open-circuit cases have been measured using an IA, and the complex permittivity and permeability were extracted as described above. The extracted \( \mu(\omega) \) and \( \epsilon_i(\omega) \) then were approximated using a GA with the Debye-like terms, and these data were used in the EZ-FDTD modeling of the same structure as was available in the measurements. The expected corresponding impedances obtained from experiment and numerical modeling should agree, and this agreement would be a verification of the EZ-FDTD model with DDM implementation.

Table I gives the extracted parameters for DDM model using the GA technique for the ferrite material of the magnetic core that was studied in the measurements. The permittivity frequency characteristics for this particular material were approximated with three Debye-like terms, and permeability was approximated with four Debye-like terms, one of which is negative. However, the presence of the negative term does not prevent the EZ-FDTD modeling.

<table>
<thead>
<tr>
<th>TABLE I</th>
<th>PARAMETERS OF DDM MODEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_e )</td>
<td>13.2</td>
</tr>
<tr>
<td>( \epsilon_{e_1} )</td>
<td>14.9</td>
</tr>
<tr>
<td>( \epsilon_{e_2} )</td>
<td>14.3</td>
</tr>
<tr>
<td>( \epsilon_{e_3} )</td>
<td>14.2</td>
</tr>
<tr>
<td>( \epsilon_{e_4} )</td>
<td>-</td>
</tr>
<tr>
<td>( r_{i_1}(s) )</td>
<td>4.9 \times 10^{-8}</td>
</tr>
<tr>
<td>( r_{i_2}(s) )</td>
<td>5.0 \times 10^{-9}</td>
</tr>
<tr>
<td>( r_{i_3}(s) )</td>
<td>7.7 \times 10^{-9}</td>
</tr>
<tr>
<td>( r_{i_4}(s) )</td>
<td>-</td>
</tr>
<tr>
<td>( \sigma_{ij} ) (S/m)</td>
<td>0.4 \times 10^{-3}</td>
</tr>
</tbody>
</table>

Fig. 4 shows the setup of the structure with a ferrite core for EZ-FDTD model. The conductor inner \( r_0 \) and outer \( r_1 \) radii are 3.6 mm and 7 mm, respectively. The actual core has a length of 28 mm. In this model, to simulate infinitely long conductors and provide the TEM mode propagation in the modeled structure, it was proposed to extend the conductors to touch the FDTD absorbing boundary at the left (cross-section A). The source of excitation (cross-section B) was not a plane-wave source, but a modulated Gaussian 50-Ohm voltage source. It was placed away from the actual input end of the core (which is cross-section D), at the distance equal to the doubled length of the core. This was done to support the only TEM mode waves propagating through the core. And the ferrite core was also extended, as Fig. 4 shows, from the actual cross-section D to the cross-section C, which became
an interface between the ferrite and air. This was done to have reflections only from the shorted or open end of the core, and to reduce, due to the substantial loss in ferrite, unwanted reflections from the cross-section D.

Since EZ-FDTD code can have only a rectangular discretization mesh, the circular cross-sectional geometry was approximated by a staircase grid with a cell size 0.4 mm in all three directions (x, y, and z). This cell size was chosen as a compromise between an accuracy and stability of the modeling results at the comparatively low frequencies (~10-500 MHz). The smaller cell size (0.2 mm) led to numerical instability when using the DDM model of ferrite. However, the bigger cell size (0.6 mm) does not capture the geometrical features well enough, and the accuracy suffers.

![Fig. 4. Geometry of a cylindrical magnetic core: (a) Side view, (b) Cross section view](image)

**Fig. 4. Geometry of a cylindrical magnetic core: (a) Side view, (b) Cross section view**

![Fig. 5. EZ-FDTD modeled and measured input impedance of the ferrite core in the open-circuit case](image)

**Fig. 5. EZ-FDTD modeled and measured input impedance of the ferrite core in the open-circuit case**

![Fig. 6. EZ-FDTD modeled and measured input impedance of the ferrite core in the short-circuit case](image)

**Fig. 6. EZ-FDTD modeled and measured input impedance of the ferrite core in the short-circuit case**

Figs. 5 and 6 show the EZ-FDTD simulated results together with the measured data for both “open” and “short” cases. As is seen from the figures, the agreement between these two sets of results in both open-circuit and short-circuit cases is satisfactory. The discrepancy might be caused by the approximation of the rectangular cells to cylindrical geometry. Besides, there might be some inaccuracy in approximating Debye data for permittivity and permeability with the GA. The other reason might be some effect of reflections that interfere with a pure TEM mode in the modeled structure.

### III. CONCLUSION

A practical design for estimating both the permittivity and permeability of a magneto-dielectric material using a hollow cylindrical sample has been investigated in this paper. The measurements have been performed with an Impedance Analyzer for the frequency range from 10 MHz to 500 MHz. The extraction of dispersive dielectric and magnetic properties was done based on the transmission line theory and application of a genetic algorithm.

A satisfactory agreement between the measured and the full-wave FDTD-modeled was achieved for both open-circuit and short-circuit regimes. Using the proposed method with cylindrical (and also rectangular stripline) geometries, dielectric and magnetic properties of dispersive materials can be extracted effectively under the assumption of only TEM/quasi-TEM wave propagation.

### REFERENCES


