8 Rough Footing on a Mohr-Coulomb Material

8.1 Problem Statement

The prediction of collapse loads under steady plastic flow conditions is one that can be difficult for a numerical model to simulate accurately (Sloan and Randolph 1982). A simple example of a problem involving steady flow is the determination of the bearing capacity of a footing on an elastic-plastic soil. The bearing capacity is dependent on the steady plastic flow beneath the footing, thereby providing a measure of the ability of UDEC to model this condition.

A strip footing is evaluated to demonstrate the capability of UDEC to predict collapse loads and model plastic flow of intact material. The strip footing has a rough base with a width of 6.0 m, and is located on a frictionless, cohesive soil that has the following properties.

- density ($\rho$) 1000 kg/m$^3$
- shear modulus ($G$) 100 MPa
- bulk modulus ($K$) 200 MPa
- cohesion ($c$) 10 kPa
- friction angle ($\phi$) 0
- dilation angle ($\psi$) 0

8.2 Analytical Solution

The bearing capacity for a strip footing is from the solution to “Prandtl’s wedge,” as given by Terzaghi and Peck (1967):

$$q = (2 + \pi) c$$

or

$$q = 5.14 c$$  (8.1)

where $c$ is the cohesion of the material, and $q$ is the bearing capacity stress at failure. The solution is based on the mode of failure, as shown in Figure 8.1:
8.3 UDEC Model

A plane-strain analysis is performed for the strip-footing problem. Half-symmetry is used, and boundary conditions are applied, as shown in Figure 8.2:

Two model grids are created for this problem. The first is a single-block model, and the second is a two-block model created with a diagonal construction joint.

Figure 8.1 Prandtl’s wedge problem of a strip footing on a frictionless soil

Figure 8.2 UDEC model boundary conditions
The first model grid, shown in Figure 8.3, is composed of 2048 triangular zones in a diametrically opposed triangular pattern. As discussed in Section 1.2.5 in Theory and Background, this zone pattern has been demonstrated to provide reasonable accuracy for calculations involving plastic collapse. This pattern is created with either the command

\texttt{gen edge 0.625}

or the command

\texttt{gen quad 0.64}

The second model grid, shown in Figure 8.4, is created by first dividing the model region into two blocks with the command

\texttt{crack (0,0) (20,10) join}

In this case, only the \texttt{GENERATE edge} command can be applied because each block only contains three corners. The command

\texttt{gen edge 0.625}

creates zoning with boundary gridpoints at the same locations as those in the single-block model. The two-block model contains 3104 zones.

The test is velocity-controlled with a downward velocity of $1.0 \times 10^{-3}$ m/sec applied to the gridpoints located along the boundary corresponding to the area representing the footing. A zero velocity is applied in the $x$-direction to represent the rough footing condition. The gridpoint locations of the fixed-velocity boundary condition are indicated on the plots in Figures 8.3 and 8.4.

The footing load is calculated in FISH function \texttt{stripload} by summing the $y$-direction forces at the footing gridpoints and dividing by the representative footing area. The footing load is monitored as a history for comparison with the bearing capacity calculated from Eq. (8.1).
Figure 8.3  UDEC zone geometry for strip footing – single-block model

Figure 8.4  UDEC zone geometry for strip footing – two-block model with diagonal construction joint
8.4 Results and Discussion

Figure 8.5 shows the model conditions at the end of the analysis for the single-block model. The behavior shown is very close to that expected from Figure 8.1. Figure 8.6 shows a history of the bearing capacity versus vertical displacement of the footing for the model using GENERATE quad zoning. The final value of the bearing capacity for the strip footing is 50.6 kPa, giving an error of 1.66% when compared to the expected value of 51.4 kPa. The results using the GENERATE edge zoning are essentially identical to those using GENERATE quad zoning for the single-block model.

In the two-block model, the triangular zoning has an irregular pattern (as shown in Figure 8.4). This introduces kinematic constraints in the plastic flow calculation and results in an excessively stiff response, as indicated by the bearing capacity history plot in Figure 8.7. The error after 2 cm of vertical settlement of the footing is over 10%, and is increasing.

This problem is discussed in Section 1.2.5 in Theory and Background, and can be corrected by applying “nodal mixed discretization” (also described in this section). By adding the command

```plaintext
set nodal on
```

after the GENERATE edge command, nodal mixed discretization is applied to the triangular zoning. The improved result is shown in Figure 8.8. Now the error in the calculated bearing capacity is reduced to 0.8%.

This exercise illustrates that whenever plastic failure and collapse of deformable blocks is to be simulated, the GENERATE quad command (or the GENERATE edge command with the SET nodal on command) should be applied in order to obtain an accurate solution.
Figure 8.5  Steady state x-velocity contours and velocity vectors at collapse load for strip footing

Figure 8.6  History of strip footing load; exact solution also shown – single-block model
Figure 8.7  History of strip footing load; exact solution also shown
– two block model (zoning by GENERATE edge)

Figure 8.8  History of strip footing load; exact solution also shown
– two-block model (zoning by GENERATE edge with SET nodal on)
8.5 References


8.6 Listing of Data File

Example 8.1 PRAN.DAT

; File: pran.dat
; Title: Prandtl's Wedge Test
; rough footing on cohesive material
new
round 0.01
edge 0.02
block 0,0 0,10 20,10 20,0
;
; GEN quad zoning for single-block model
gen quad 0.64
;
; GEN edge zoning for single-block model and for two-block model
; gen edge 0.625
;
; nodal mixed discretization for two-block model
; set nodat on
;
; material properties
group zone 'clay'
zone model mohr density 1E3 bulk 2E8 shear 1E8 cohesion 1E4 tension 1E10 &
range group 'clay'
;
; boundary conditions
boundary xvelocity 0 range -0.1,0.1 -0.1,10.1
boundary xvelocity 0 range 19.9,20.1 -0.1,10.1
boundary xvelocity 0 range -0.1,20.1 -0.1,0.1
boundary yvelocity 0 range -0.1,20.1 -0.1,0.1
boundary yvelocity 0 range -0.001 range -0.1,3 9.9,10.1
boundary xvelocity 0 range -0.1,3 9.9,10.1
;
; comparison to analytical solution
call 'boucnr.fin'
def p_cons
  p_xp = gp_near(3.12,10.0)
p_xm = gp_near(2.50,10.0)
p_y0 = gp_near(0.0,10.0)
solution=(2.0 + pi)*1e4
end
def p_cons
;
def stripload
  sum =0.0
\begin{verbatim}
ib = block_head
loop while ib # 0
  ig = b_gp(ib)
  loop while ig # 0
    if gp_y(ig) > 9.8 then
      if gp_x(ig) < 3.0 then
        ibou=gp_bou(ig) ; index of boundary corner
        if(ibou) > 0 then ; exterior boundary
          if (imem(ibou+$KBDY)) = 4 then
            forcey = fmem(ibou+$KBDFY) ; total y-force
            sum = sum - forcey
          endif
        endif
      endif
    endif
  endif
  ig = gp_next(ig)
endloop
ib = ib_next(ib)
endloop
x_p = gp_x(p_xp)
x_m = gp_x(p_xm)
p_load = 2.0 * sum / (x_p + x_m)
y Disp = -gp_ydis(p_y0)
stripload = p_load
err = (p_load-solution)/solution
end
stripload
history err
history stripload
history solution
history y Disp
set small
save strip1.sav
;
cycle 35000
save strip2.sav
\end{verbatim}