Introduction

Most of the physical and chemical properties of matter which we are familiar with are a result of the number and configuration of atomic electrons.

That's why we have spent most of the course to date concentrating topics related to atomic electrons.

Nevertheless, atomic nuclei are vitally important for a number of reasons, including:

- The number of electrons an atom can have depends on how many protons the nuclei has. Thus, the nucleus plays a large, if indirect, role in determining atomic structure.
- Most of the energies liberated in everyday processes involve nuclear reactions.

The first couple of sections of this chapter describe several problems the nucleus presents us.
For example, consider this problem.

Take two protons at their approximate separation in the nucleus. Calculate the repulsive Coulomb energy between them.

\[ V = \frac{e^2}{4\pi\varepsilon_0 R} = \frac{(1.6\times10^{-19})^2}{4\pi\varepsilon_0 (10^{-15})} = 1.4 \text{ MeV}. \]

That’s a large repulsive potential! Far more than electronic bonding energies.

Now imagine the repulsive Coulomb energy for several dozen protons packed tightly into a nucleus.

We have a major problem here. How can a nucleus stick together?

A logical guess is that the nucleus contains electrons, which reduce the Coulomb repulsion.

**11.1 Nuclear Composition**

A good guess might be that half of an atom’s electrons are contained within the nucleus, and reduce the electrostatic repulsive forces between protons.

Other facts which suggest the nucleus might contain electrons are nuclide masses, which are nearly multiples of the hydrogen mass (which contains an electron).

In addition, some nuclei undergo beta decay, in which an electron is spontaneously emitted from the nucleus.

But other experiments demonstrate that the nucleus cannot contain electrons...

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**Reason 1 -- nuclear size.**

The Heisenberg uncertainty principle places a lower bound on the energies of particles confined to a nucleus.

Take a typical nucleus of radius $5\times10^{-15}$ m. Suppose an electron exists inside the nucleus. In the example on page 114, we estimated the minimum momentum such an electron must have. The estimated momentum corresponds to a kinetic energy of **at least** 20 MeV.

Electrons emitted during nuclear decay are found to have only 2 or 3 MeV of energy—not nearly enough to correspond to an electron escaping from a nucleus.

Protons, with their much larger masses, would only need to have a few tenths of an MeV of energy to be confined to a nucleus. This is possible.

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**Reason 2 -- nuclear spin.**

Electrons and protons both have spins of 1/2. A deuteron (an isotope of hydrogen) has a mass roughly equal to two protons.

If the deuterium nucleus contains two protons and one electron (whose mass is small enough to not worry about here), then deuterium should have a nuclear spin of $\pm \frac{1}{2}$ or $\pm \frac{3}{2}$ (from $\pm \frac{1}{2} \pm \frac{1}{2} \pm \frac{1}{2}$).

The deuterium nuclear spin is measured to be 1. Its nucleus cannot contain an electron. (If it did, angular momentum would not be conserved.)
Reason 3 -- nuclear magnetic moments.
Electrons have magnetic moments about 6 times larger than protons.

If nuclei contain electrons, their magnetic moments should be comparable to electron magnetic moments.

Observed nuclear magnetic moments are comparable to proton magnetic moments. Nuclei cannot contain electrons.

Reason 4 -- electron-nuclear interactions.

The energies binding nuclear particles together are observed to be very large, on the order of 8 MeV per particle.

Remember that atomic electronic binding energies are of the order eV to a few keV.

Why, then, can some atomic electrons "escape" from being bound inside the nucleus?

In other words, if you allow any electrons to be bound inside the nucleus, you really must require all of them to.

Of course, it is obvious to us that nuclei don't contain electrons.

But that's mainly because we've been taught that way for so long.

If we were starting from scratch 70 years ago, we would probably try to "put" electrons inside nuclei.

Some definitions and facts.

The most abundant type of carbon atom is defined to have a mass of exactly 12 u, where u is one atomic mass unit:

\[ 1 \text{ u} = 1.6604 \times 10^{-27} \text{ kg} = 931.48 \text{ MeV}. \]

Atomic masses always refer to neutral atoms. In other words, atomic masses include the masses of all of the electrons in the neutral atom.

Atomic masses always refer to neutral atoms. In other words, atomic masses include the masses of all of the electrons in the neutral atom. Yes, I meant to write that twice. I wanted to make sure you remember that statement!

Not all atoms of an element have the same mass. Isotopes are atoms of the same element having different masses.

A nuclide is simply any particular nuclear species. Hydrogen and deuterium are isotopes. They are also nuclides. Carbon-12 is a nuclide, but it is not an isotope of hydrogen.
Rutherford—who we met earlier in this course—predicted the existence of the neutron in 1920.

The neutron, being a neutral particle, proved difficult to detect. “Many” tried and failed.

In 1930, German Physicists Bothe and Becker were experimenting with alpha particles and beryllium. When they bombarded beryllium with alpha particles, the beryllium emitted a mysterious radiation.

The radiation was neutral (they could test for that with magnetic fields) and passed through a whopping 200 millimeters of lead (one millimeter stops a proton).

The only uncharged particle known at the time was the photon.

Irene Curie and her husband Frederic tried putting a block of paraffin wax in front of the mysterious beam coming out of the beryllium.

Huh? Paraffin?

Sure! Paraffin was the duct tape of the old days. No lab could get by without it. (Although the labs I did my early research in had only white and red paraffin.)

Paraffin is made of light hydrocarbons. It contains lots of protons. It is a good “test block” for studying collisions.

The “mystery rays” knocked protons out of the paraffin.

The protons come out with energies up to 5.7 MeV (big!).

The gamma ray energy needed to produce such energetic protons is about 55 MeV.

Gamma rays of this much energy were not observed.

About 20% of the observed 5.7 MeV energy is the most that can be produced by gamma rays.

Chadwick in 1932 proposed that the unknown radiation could be neutral particles having about the mass of protons.

Charge neutrality is necessary for the radiation to easily penetrate matter.

Because a collision between particles of equal mass can transfer all of the kinetic energy from the projectile to the target, the neutrons needed to have only 5.7 MeV of energy, which was a much more reasonable value.
Chadwick devised an experiment to test his hypothesis.

http://hyperphysics.phy-astr.gsu.edu/hbase/particles/neutrons.html

The discovery of the neutron won Chadwick the 1935 Nobel prize.

The neutron:
- mass = 1.00867u, just a little more than the proton
- charge = 0  
- spin = 1/2
- is unstable outside of nuclei (lifetime is about 15 minutes), and decays into a proton, an electron, and an antineutrino
- neutrons produce attractive forces which help hold nuclei together.

The Nucleus.
- The number of protons in a nucleus (and electrons in the atom, if the atom is not ionized) is represented by Z.
- N is the number of neutrons in the nucleus.
- The atomic mass number A is given by A = Z + N.
- Neutrons and protons are called nucleons, so A is the number of nucleons in a nucleus.
- We identify nuclides by writing $^A_Z X$. For example, the most abundant isotope of iron has 26 protons and electrons, and a mass number of 56, so we write $^{56}_{26}$Fe.

Isotopes.
- Isotopes of an element have the same number of protons and electrons, but different numbers of neutrons.
- Because most physical and chemical properties are determined by the number and arrangement of atomic electrons, isotopes of an element are very similar in behavior.
- As Beiser states, all isotopes of chlorine make good bleach and are poisonous.
- Some properties, such as density and freezing points, are different for different isotopes of the same element, but the differences are usually so slight that isotopes are difficult to separate.

\[
R = R_0 A^{1/3},
\]

where $R_0$ is a constant and $R_0 = 1.2 \times 10^{-15}$ m.
The nucleus does not have a sharp boundary, so the "constant" \(R_0\) is only approximate; also, nuclear matter and nuclear charge do not seem to be identically distributed.

The unit of length \(10^{-15}\) m is called a femtometer, abbreviated \(\text{fm}\), and also often called a fermi, so \(R = 1.2A^{1/3}\) in units of \(\text{fm}\).

Example: the radius of the \(^{107}_{47}\)Ag nucleus is \(R = 1.2x(107)^{1/3} \approx 5.7\) fm.

If a nucleus is not spherically symmetric, it will produce an electric field that will perturb atomic electronic energy levels.

Such an effect is, in fact, observed, but it is small -- "hyperfine." The departures from spherical symmetry are small.

A plot of \(N\) versus \(Z\) for the stable nuclides looks like this:

(I see two "typos," not mine!)

Neutrons produce attractive forces within nuclei, and help hold the protons together.

For small numbers of protons, about an equal number of neutrons is enough to provide stability, hence \(N = Z\) for small \(Z\).

As the number of protons gets larger, an excess of neutrons is needed to overcome the proton-proton repulsion.

The stability of nuclei follows a definite pattern.

- The majority of stable nuclei have both even \(Z\) and even \(N\) ("even-even" nuclides).
- Most of the rest have either even \(Z\) and odd \(N\) ("even-odd") or odd \(Z\) and even \(N\) ("odd-even").
- Very few stable nuclei have both \(Z\) and \(N\) odd.

The reasons for this pattern are the Pauli exclusion principle and the existence of nuclear energy levels.

Each nuclear energy level can contain two nucleons of opposite spin.

Skip the subsection on nuclear spin and magnetic moment.

11.3 Stable Nuclei

We can begin to understand why certain nuclei are stable and others unstable by realizing that nucleons have spins of \(1/2\) and obey the Pauli exclusion principle.

Nucleons, like electrons and their electronic energy levels, occupy discrete nuclear energy levels.

Minimum energy configurations (i.e., nucleons in the lowest possible energy levels) give the most stable nuclei.
The neutrons and protons occupy separate sets of energy levels.

When both \( Z \) and \( N \) are even, the energy levels can be filled. The nucleus doesn’t “want” to gain or lose nucleons by participating in nuclear reactions. The nucleus is stable.

When both \( Z \) and \( N \) are odd, the nucleus is much more likely to “want” to participate in nuclear reactions or nuclear decay, because it has unfilled nuclear energy levels.

Attractive nuclear forces are limited in range and primarily operate between nearest neighbors (“saturation”), so there is a nuclear size beyond which neutrons are unable to overcome the proton-proton repulsion.

The heaviest stable nuclide is \(^{209}\text{Bi}\). Heavier ones decay into lighter nuclides through alpha decay (the emission of a \(^{4}\text{He}\) nucleus):

\[
{\text{^A\text{X}}} \rightarrow {\text{^A-4\text{Y}}} + {\text{^{4}\text{He}}}.
\]

\( X \) is called the parent nucleus and \( Y \) is called the daughter nucleus.

Example: \(^{12}\text{C}\)

All of its neutrons and protons in filled energy levels—very stable.

Energy level diagrams illustrative only; not quantitatively accurate!

Example: \(^{12}\text{B}\)

“Extra” neutron in higher energy level; therefore unstable.

Decays via \( \beta \) decay into \(^{12}\text{C}\).

It may be that a nucleus produced by alpha decay has too many neutrons to be stable. In this case, it may decay via beta decay:

\[
n^0 \rightarrow p^+ + e^- .
\]

If the nucleus has too few neutrons, it may decay via positron emission

\[
p^+ \rightarrow n^0 + e^+ .
\]

or by electron capture

\[
p^+ + e^- \rightarrow n^0 .
\]

Note that \( Z \) decreases by 1 as a result of positron emission or electron capture, but \( \text{increases} \) by 1 as a result of beta decay.
11.4 Binding Energy

The binding energy that holds nuclei together “shows up” as “missing” mass.

Deuterium is an isotope of hydrogen which contains a neutron, a proton, and an “orbiting” electron.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass of hydrogen</td>
<td>1.0078 u</td>
</tr>
<tr>
<td>mass of neutron</td>
<td>1.0087 u</td>
</tr>
<tr>
<td>sum</td>
<td>2.0165 u</td>
</tr>
<tr>
<td>mass of deuterium</td>
<td>2.0141 u</td>
</tr>
<tr>
<td>difference</td>
<td>0.0024 u</td>
</tr>
</tbody>
</table>

Since 1 u of mass has an energy equivalent of 931 MeV, the missing mass is equal to 931×0.0024 MeV = 2.2 MeV.

The fact that this mass deficit is the binding energy is demonstrated by experiments which show that it takes 2.2 MeV of energy to split a deuterium into a neutron and a proton.

Nuclear binding energies range from 2.2 MeV for deuterium to 1640 MeV for bismuth-209.

These binding energies are enormous; millions of times greater than even the energies given off in highly energetic chemical reactions.

We usually talk in terms of binding energy per nucleon, which is 2.2/2=1.1 MeV per nucleon for deuterium, or 1640/209=7.8 MeV per nucleon for bismuth-209.

The figure below shows a plot of binding energy per nucleon as a function of nucleon number.

![Graph showing binding energy per nucleon](image)

Keep in mind that energies are reduced on binding. The binding energy is negative, but when we say the words “binding energy” we associate them with the magnitude of the binding energy.

In other words, this plot is upside down. Let’s fix it.

More difficult to read the lettering, but makes more physical sense!

Notice the local minimum at $^4$He, which is a very stable nucleus.

Notice the absolute minimum at $^{56}$Fe, which is the most stable nucleus of all.
If $^{56}_{26}$Fe is so stable, how come heavier elements exist?

Heavier elements are less stable, but stable enough to exist. It takes enormous energies to make elements heavier than iron-56. The only place in the universe where those energies are available are supernovae.

Do you have gold in a ring (or silver in the fillings in your teeth)? If you do, you are carrying with you debris from a supernova.

Let's go back and consider some implications of the binding energy per nucleon plot.

I'll display the plot “right (??)” side up again, because that's they way you'll usually see it. Remember, higher on the plot means lower in energy and more stable.

The positive energy is the energy released in the fission reaction.

Consider a nucleus with a large A. If we could split it into two smaller nuclei, with A's closer to iron, the two nuclei would have more binding energy per nucleon.

But remember, binding energies are negative. If the resulting nuclei have more negative energy than the starting element, some positive energy must have been released in splitting the starting element.

Sample binding energy problem

Homework problem 11.16 Find the binding energy per nucleon in $^{197}_{79}$Au.

There is no equation in your text, so I’ll make one up.

$$E_b(M,A,Z) = [M - Zm_\text{H} - (A - Z)m_n] 931.5$$

Note that all masses must be in units of u, and all electrons are automatically counted in this calculation.
\[ E_b(M, A, Z) = \left[ M - Zm_n - (A - Z)m_n \right] 931.5 \]

This gives the total nuclear binding energy for the atom. We usually want the binding energy per nucleon, so I'll make another OSE:

\[ E_{b, \text{per nucleon}}(M, A, Z) = \frac{E_b(M, A, Z)}{A}. \]

Because \( E_b(M, A, Z) \) is in units of MeV, the binding energy per nucleon is in units of MeV/nucleon.

Now, back to our problem.

The mass of gold-197 is 196.966560 u. You could look that up, or I would give it to you on an exam or quiz (unless it were the quantity I wanted you to calculate).

For gold-197, \( A=197 \) and \( Z=79 \).

\[ E_b = \left[ (196.966560) - (79)(1.007825) - ((197) - (79))(1.008665) \right] 931.5 \]

\[ E_b = -1559 \text{ MeV} \]

\[ E_{b, \text{per nucleon}} = \frac{E_b(M, A, Z)}{197} = -7.916 \text{ MeV}. \]

The binding energy is negative, as it must be.

Two interesting sections. Only once have I had time to teach them. You won't be tested on them.

11.5 Liquid-Drop Model

11.6 Shell Model

The forces between nucleons involves exchange of particles called \( \pi \) mesons.

The \( \pi \) meson is a short-lived, relatively heavy particle (about 250 times the mass of an electron). In fact, it is so short-lived that we never have time to "catch" a proton or neutron lacking a meson.

Mesons were predicted as the basis of nuclear forces by Yukawa in 1934. Experimental verification came in 1937.

Yukawa was awarded the Nobel prize in 1949 for his theory.
OK, smarty. If nucleons are “held together” by the exchange of pi mesons, explain the other forces in nature. You can’t have one force due to particle exchange but not the others.

Sure! Gravity—the attractive force between any two masses—is due to exchange of gravitons.

We haven’t found any gravitons yet (gravity is an incredibly weak force). That’s OK. I believe the theory. You do too, don’t you?

The weak force—another nuclear force—is due to the exchange of vector bosons.

Really!

Finally, the electromagnetic force is mediated by the exchange of virtual photons.

That’s why electromagnetic waves propagate at the speed of light!

The photons are “virtual” because we cannot detect them. But theory says they exist, so therefore they must exist.

A good reference for forces, including nuclear forces:
http://hyperphysics.phy-astr.gsu.edu/hbase/forces/funfor.html#c2