Conservation of Momentum: Example Problem 1

A 20 lb sack of sand B is thrown with a velocity of 5 fps onto a 40 lb sled A moving to the right at 10 fps. If the sack remains on the sled and if friction between the sled and the ground is negligible, please determine:

(a) The final velocity \( v_2 @ \theta \) of the sled and sack.
(b) The impulse of the sack onto the sled.
Write the x and y scalar conservation of momentum equations:

\[ m_A v_{Ax1} + m_B v_{Bx1} = (m_A + m_B)v_{2x} \]

Momentum in the x-direction:

\[ \begin{align*}
+ & \\
& \left[ \frac{40}{32.2} \right] (10) + \left[ \frac{20}{32.2} \right] \left( \frac{3}{5} \right) (5) = \left[ \frac{40 + 20}{32.2} \right] v_{2x} \\
& 400 + 60 = 60 v_{2x} \\
& v_{2x} = 7.67 \text{ fps}
\]
Momentum in the y-direction:

$$v_{2y} = 1.33 \text{ fps}$$

Final Vector, $\vec{v}_2$:

$$\vec{v}_2 = [7.67\hat{i} + 1.33\hat{j}] \text{ fps}$$

$$= [7.78 \text{ fps @ 9.84°}]$$
To find the impulse of the sack onto the sled, write the Impulse-Momentum Equation for the sled A:

\[
\vec{v}_2 = [7.67 \hat{i} + 1.33 \hat{j}] \text{ fps} = [7.78 \text{ fps} @ 9.84^\circ]
\]

Write the I-M Eqn for the SLED

**Impulse of the sack onto the sled:**

\[
\int \vec{F} \, dt = m\vec{v}_{A2} - m\vec{v}_{A1} = m(\vec{v}_{A2} - \vec{v}_{A1})
\]

\[
= \left[ \frac{40}{32.2} \text{ slug} \right] \left[ [7.62 \hat{i} + 1.33 \hat{j}] - [10 \hat{i} + 0 \hat{j}] \right]
\]

\[
\int \vec{F} \, dt = [-2.89 \hat{i} + 1.65 \hat{j}] = [3.33 @ 150.3^\circ] \text{ lb-sec}
\]

Picture of the initial and final momentum of the sled and the effect of the impulse.

**Impulse of the Sack onto the Sled**

\[
\int \vec{F} \, dt = [3.33 @ 150^\circ] \text{ lb-sec}
\]
The impulse of the sack onto the sled acts to slow the sled in the x-direction and to impart a small y-direction velocity. The overall speed $v_2$ of the sled-sack combo is smaller (7.78 vs. 10 fps) than the original sled.

When the sack lands on the sled, the sack slides momentarily until the sack and sled speeds match. A momentary friction force acts between the sack and the sled. This friction force (the x-component) briefly accelerates the sack and decelerates the sled until their x speeds match. The y-component of friction accels the sled and imparts a y-velocity to the sled.