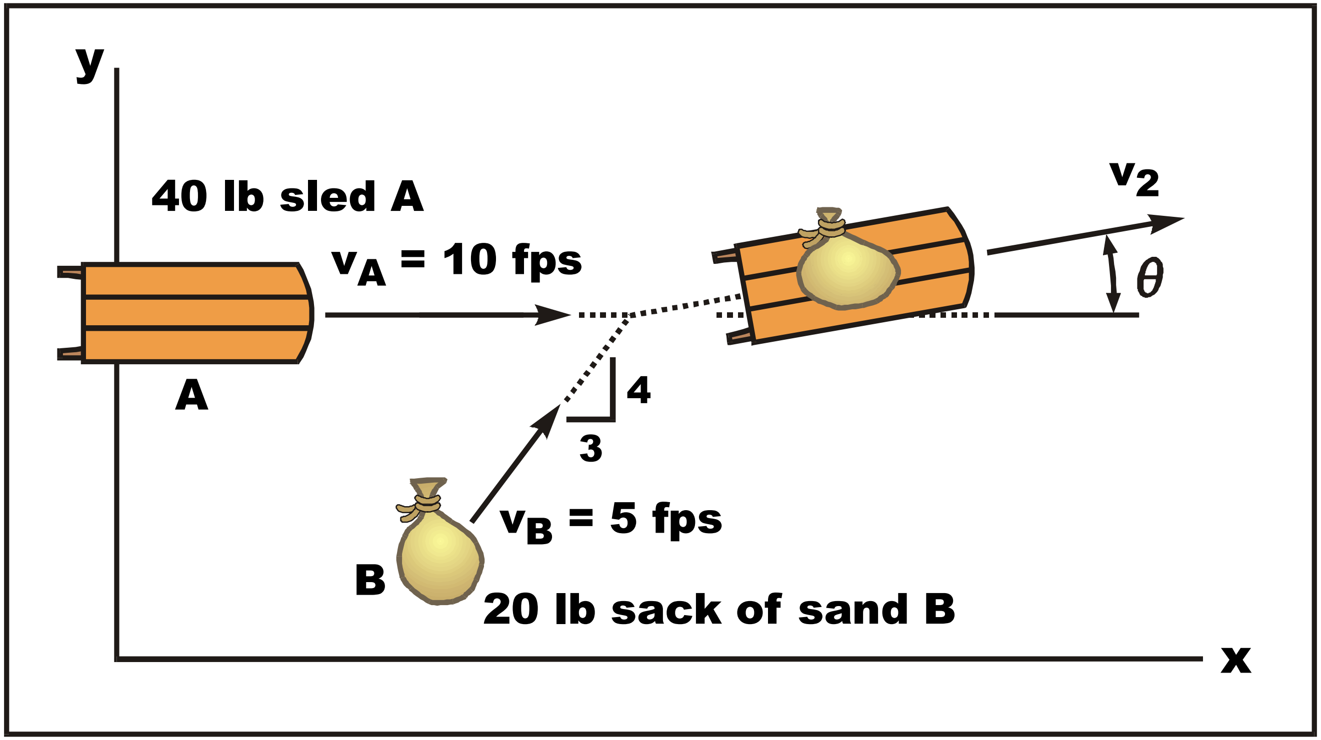
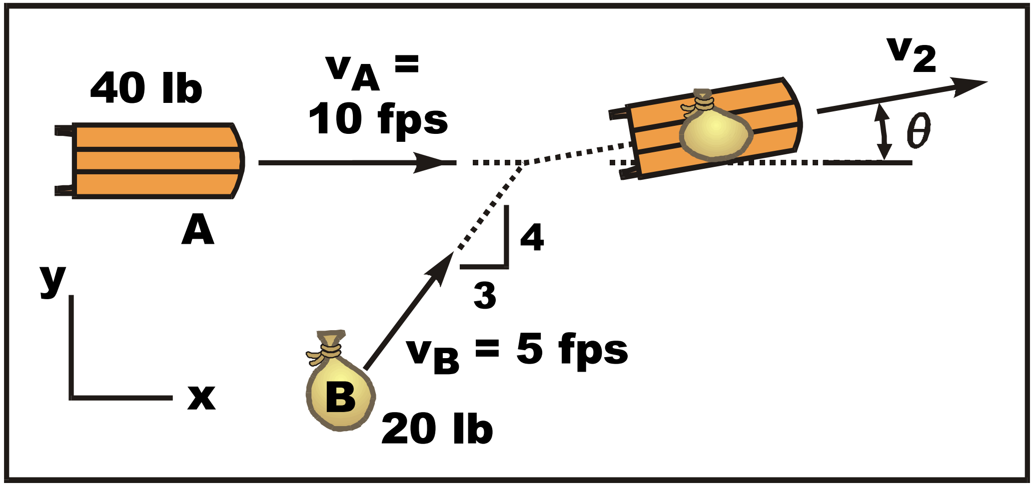
Conservation of Momentum: Example Problem 1

A 20 lb sack of sand B is thrown with a velocity of 5 fps onto a 40 lb sled A moving to the right at 10 fps. If the sack remains on the sled and if friction between the sled and the ground is negligible, please determine:

(a) The final velocity [v_2 @ θ] of the sled and sack.
(b) The impulse of the sack onto the sled.





Write the x and y scalar conservation of momentum equations:

Momentum in the x-direction:

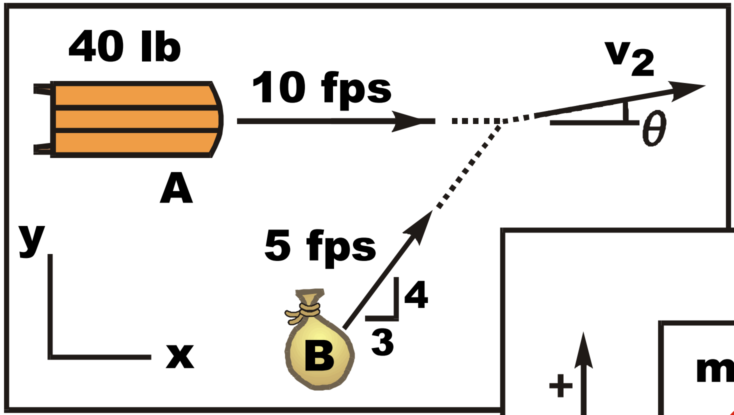
$$\overset{+}{\rightarrow} \quad \boxed{m_A v_{Ax_1} + m_B v_{Bx_1} = (m_A + m_B) v_{2x}}$$

$$\overset{+}{\rightarrow} \quad \overbrace{\left[\frac{40}{32.2}\right]}^{m_A} \overbrace{(10)}^{v_{Ax_1}} + \overbrace{\left[\frac{20}{32.2}\right]}^{m_B} \overbrace{\left(\frac{3}{5}\right)(5)}^{v_{Bx_1}} = \overbrace{\left[\frac{40 + 20}{32.2}\right]}^{(m_A + m_B)} \overbrace{v_{2x}}^{v_{2x}}$$

Elim the 32.2's

$$400 + 60 = 60\, v_{2x}$$

$$\boxed{v_{2x} = 7.67 \text{ fps}} \quad \longrightarrow$$



Momentum in the y-direction:

$$+\uparrow \quad \boxed{\cancelto{0}{m_A v_{Ay_1}} + m_B v_{By_1} = (m_A + m_B) v_{2y}}$$

$$+\uparrow \quad \overbrace{0}^{m_A v_{Ay_1}} + \overbrace{\left[\frac{20}{32.2}\right]}^{m_B} \overbrace{\left(\frac{4}{5}\right)(5)}^{v_{By_1}} = \overbrace{\left[\frac{40+20}{32.2}\right]}^{(m_A + m_B)} \overbrace{v_{2y}}^{v_{2y}}$$

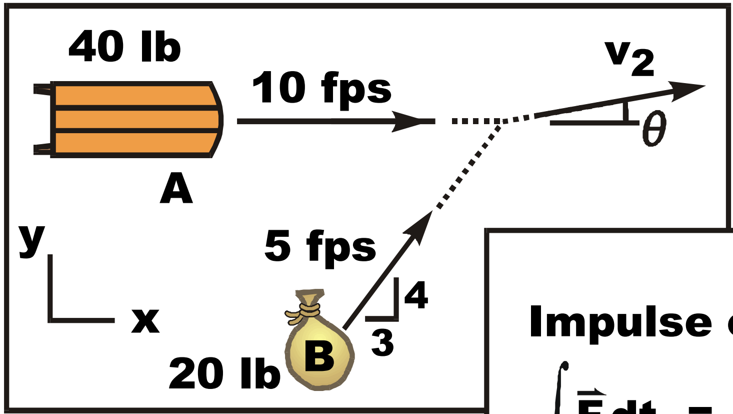
Elim the 32.2's

$$80 = 60 v_{2y}$$

$$\boxed{v_{2y} = 1.33 \text{ fps}} \quad \uparrow$$

Final Vector, v_2:

$$\boxed{\begin{aligned} \vec{v}_2 &= [\, 7.67\,\hat{i} + 1.33\,\hat{j} \,] \text{ fps} \\ &= [\, 7.78 \text{ fps @ } 9.84^\circ] \end{aligned}}$$



$$\vec{v}_2 = [\,7.67\,\hat{i} + 1.33\,\hat{j}\,]\ \text{fps}$$
$$= [\,7.78\ \text{fps} @ 9.84°]$$

To find the impulse of the sack onto the sled, write the Impulse-Momentum Equation for the sled A:

Write the I-M Eqn for the SLED

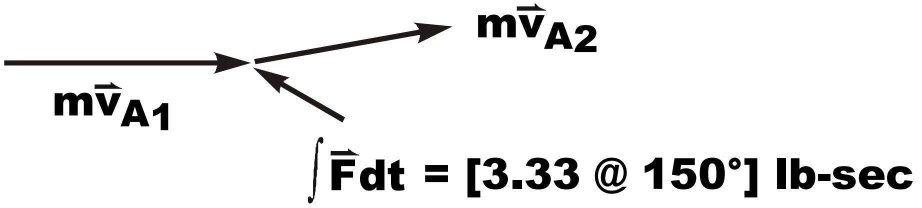
Impulse of the sack onto the sled:

$$\int \vec{F}\,dt = m\vec{v}_{A2} - m\vec{v}_{A1} = m(\,\vec{v}_{A2} - \vec{v}_{A1}\,)$$
$$= \left[\frac{40}{32.2}\,\text{slug}\right]\left[\left[7.62\,\hat{i} + 1.33\,\hat{j}\right] - \left[10\,\hat{i} + 0\,\hat{j}\right]\right]$$

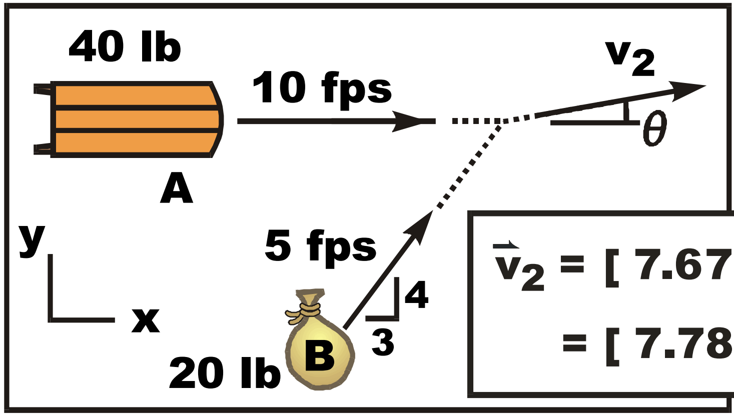
$$\boxed{\int_{\text{Sack onto sled}} \vec{F}\,dt = [\,-2.89\,\hat{i} + 1.65\,\hat{j}\,] = [\,3.33 @ 150.3°\,]\ \text{lb-sec}}$$

Picture of the initial and final momentum of the sled and the effect of the impulse.

Impulse of the Sack onto the Sled



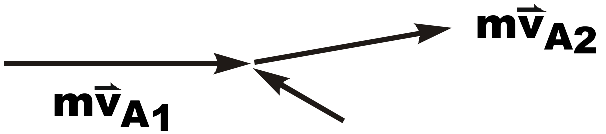
The impulse of the sack onto the sled acts to slow the sled in the x-direction and to impart a small y-direction velocity. The overall speed v_2 of the sled-sack combo is smaller (7.78 vs. 10 fps) than the original sled.



$$\vec{v}_2 = [\ 7.67\ \hat{\imath} + \ 1.33\ \hat{\jmath}\ ]\ \text{fps}$$
$$= [\ 7.78\ \text{fps} @ 9.84°]$$

Picture of the initial and final momentum of the sled and the effect of the impulse.

Impulse of the Sack onto the Sled



$$\int \vec{F}dt = [3.33 @ 150°]\ \text{lb-sec}$$

When the sack lands on the sled, the sack slides momentarily until the sack and sled speeds match. A momentary friction force acts between the sack and the sled. This friction force (the x-component) briefly accelerates the sack and decelerates the sled until their x speeds match. The y-compoent of friction accels the sled and imparts a y-velocity to the sled….