Particle F=ma: Introduction and Straight Line Motion

**Kinematics vs. Kinetics:**

**Kinematics:** In the first half of this course, we investigated kinematics, which is the description of motion without regard to forces. We calculated accelerations, but never asked what forces are needed to produce these accelerations.

**Demonstration:** Grab with your hand and shake back and forth as rapidly as possible:

1. A pen or pencil. Easy, right? Small mass = small force.
3. A large textbook, like a calculus book. Very difficult to shake this rapidly, especially with one hand. Even with two hands it is hard, because you cannot develop enough grip force.

**Analysis:** Shaking the objects—the rapid changes in direction—involves large accelerations. If the object has much mass at all, you discover that you cannot develop enough force to produce these accelerations.
The shaking book demonstration shows that accelerations require forces, and large accelerations can require large forces, esp. if the mass of the body is large.

**Ink Jet Printer**: Consider the print head on an ink jet printer. Recall how it *zips, zips, zips* back and forth. Do you realize that a key design constraint on such a printer is to make the print head as light as possible? The print head switching direction involves large accelerations. Making this as light as possible enables all of the mechanism to be correspondingly lightweight.

**Formula 1 Race Car Engine**: Pistons, connecting rods, crankshafts, cams—every moving element—are all made as light as possible. They experience high accelerations, but the forces are more manageable, and less energy is devoted to moving engine components.

**Can you think of other things** that change direction rapidly? Olympic gymnasts? Scanner heads? A ping pong ball vs. a bowling ball? Punt returners vs. Offensive Linemen?
To determine the forces associated with accelerations, we will use the particle equation of motion:

\[ F = ma \]

Definitions:  
- \( F \) = force, a vector  
- \( m \) = mass, a scalar  
- \( a \) = acceleration, a vector.

Units:  
- Newtons (N) or pounds (lb).  
- Kilograms (kg) or slugs.  
- \( \text{m/sec}^2 \) or \( \text{ft/sec}^2 \) or other.

The force necessary to produce a given acceleration is proportional to the mass of the object being accelerated.

As it is a vector equation, the acceleration \( a \) occurs in exactly the direction of the net force \( F \) acting on the body.

\( F \) is a net force, the resultant of the several forces that act on the body.
### “Base” vs. “Derived” Units:

**SI System:**

**Base Units:** mass (kg), length (m), and time (seconds).

**Derived Unit:** *Force* (the Newton) is a derived unit  
(derived directly from $F = ma$):

From $F=ma$: A 1 N force accelerates a 1 kg mass at 1 m/s².

**U.S. System:**

**Base Units:** force (lb), length (ft), and time (seconds).

**Derived Unit:** *Mass* (the slug) is a derived unit,  
(derived directly from $F = ma$).

From $F=ma$: A 1 lb force accelerates a 1 slug mass at 1 ft/s².
Concept of Weight:

Weight: \[ W = mg \]

The weight \( W \) of a body is its mass times the local acceleration of gravity, \( g \).

Typical accelerations of gravity:

- SI: 9.81 m/s\(^2\)
- US: 32.2 ft/s\(^2\)

(The difference between these is the conversion: 1 ft = 0.3048 m)

Example 1: If a student weighs 180 lb, what is his mass in slugs?

\[ m = \frac{W}{g} = \frac{180}{32.2} = 5.59 \text{ slug} \]

Example 2: If a body has a mass of 50 kg, what is its weight in Newtons?

\[ W = mg = (50 \text{ kg})(9.81) = 490.5 \text{ N} \]

What is your mass in slugs?

What is your mass in kg? (There are 2.2 lb/kg).
Procedure for solving $F = ma$ particle problems:

1. Draw a complete Free Body Diagram (FBD) showing all forces acting on the body.

2. Draw a complete Kinetic Diagram (KD) which shows the $ma$ (kinetic) terms and their assumed directions.

3. Write the equations of motion, using coordinates appropriate to the problem:

\[
\begin{align*}
\sum F_x &= ma_x \\
\sum F_y &= ma_y
\end{align*}
\]

or

\[
\begin{align*}
\sum F_n &= ma_n \\
\sum F_t &= ma_t \\
\sum F_r &= ma_r \\
\sum F_\theta &= ma_\theta
\end{align*}
\]

4. You may need to write an additional equation, such as a pulley kinematics relationship, a friction equation ($F = \mu N$), and/or others.

5. Solve the equations.