Particle Impact: Ex Prob 4 (Ball Oblique on Surface)

A ball moving at speed v_1 = 10 m/s strikes the ground at an angle of θ_1 = 60º and rebounds with speed v_2 at angle θ_2. Please determine v_2 and θ_2. Assume no friction between the ball and the ground, and treat the ball as a particle.

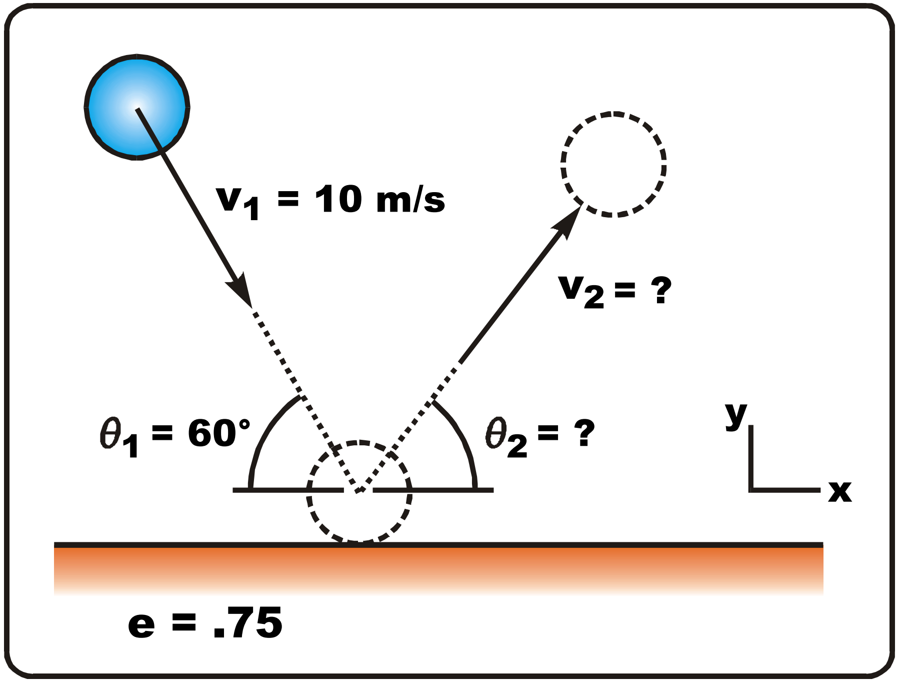
Can you guess the rebound angle? Rebound speed v_2?

Students usually think that the ball rebounds at the same angle at which it strikes the surface. In other words, they think that $\theta_2 = \theta_1 = 60º$.

This is only true, however, if e = 1.

If e < 1 (which is always, really!) then $\theta_2 < \theta_1$, and $v_2 < v_1$.

Let's calculate these and see!



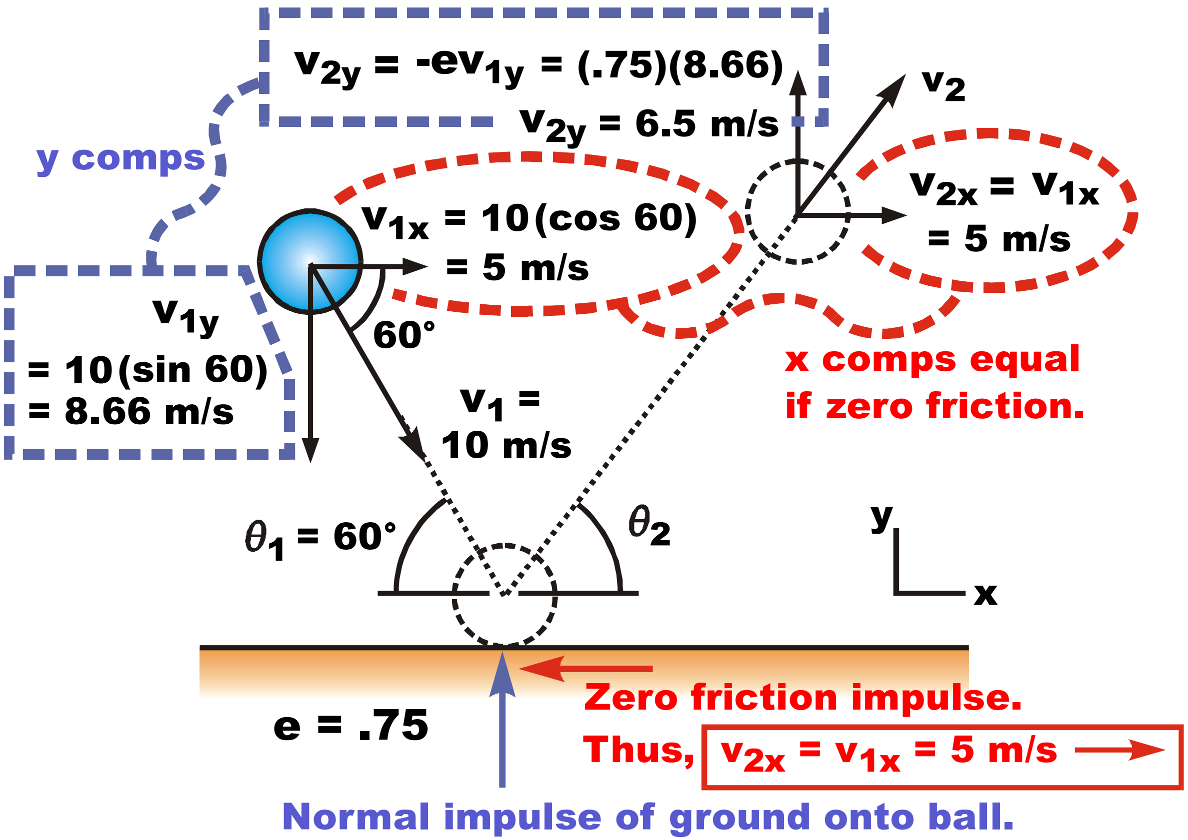
Key step: Resolve the v_1 components….

Resolve the v_1 vector into x and y components:

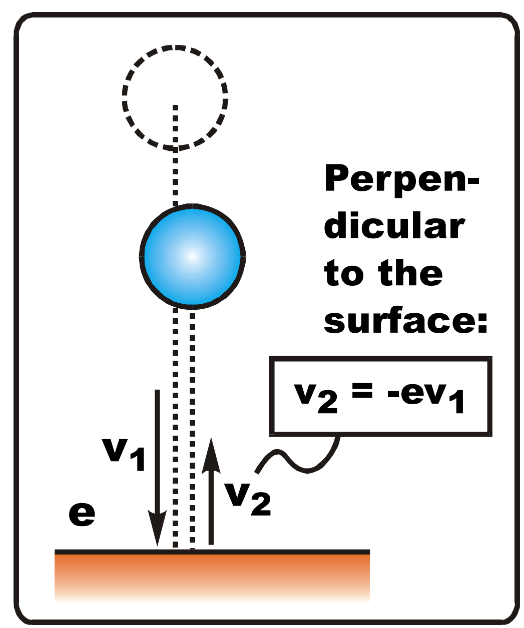
$v_{1x} = 5$ m/s; $v_{1y} = 8.66$ m/s.

(x direction): Along the surface: If there is zero friction, then there is no friction impulse in the x direction. Thus, $v_{2x} = v_{1x} = 5$ m/s.

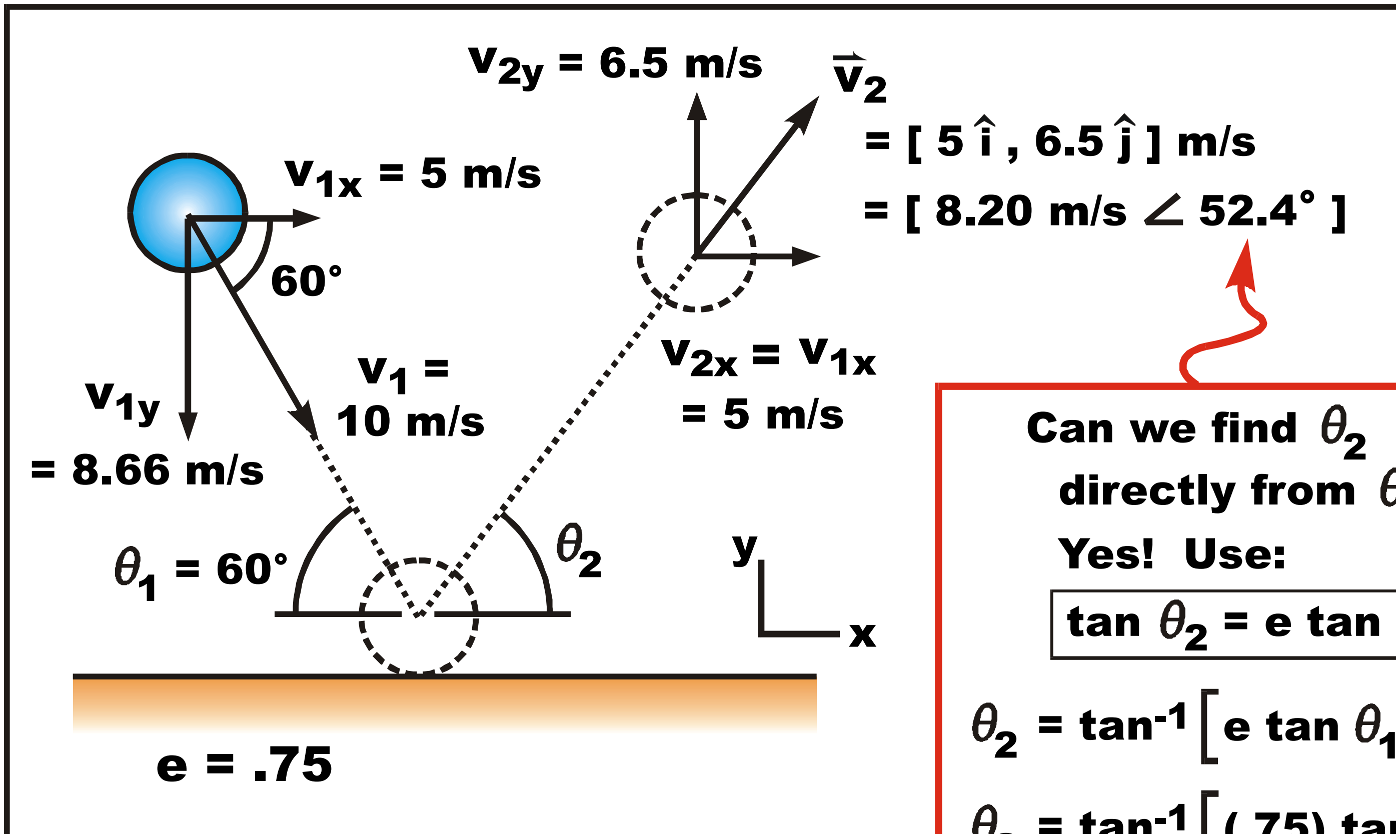
(y direction): Normal to the surface: Recall from the last example problem that $v_{2y} = -ev_{1y}$; thus, $v_{2xy} = ev_{1xy} = (.75)(8.66) = 6.5$ m/s.



Remember this:



Write the v_2 vector:



Can we find θ_2 directly from θ_1?

Yes! Use:

$$\boxed{\tan\theta_2 = e \tan\theta_1}$$

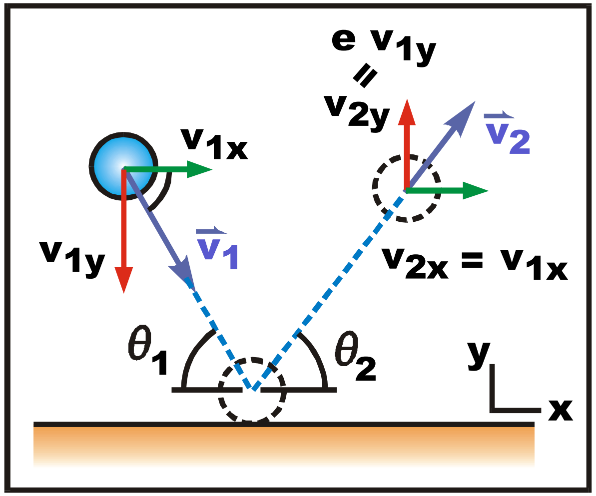
$$\theta_2 = \tan^{-1}\left[e \tan\theta_1\right]$$

$$\theta_2 = \tan^{-1}\left[(.75) \tan 60\right]$$

$$\boxed{\theta_2 = 52.4^\circ}$$

See the next page to learn where this equation comes from….

Let's generalize e vs. θ_1 and θ_2 :



$$\tan\theta_2 = \frac{v_{2y}}{v_{2x}} = \frac{ev_{1y}}{v_{1x}}$$

$$\frac{ev_{1y}}{v_{1x}} = \frac{e\cancel{v_1}\sin\theta_1}{\cancel{v_1}\cos\theta_1} = e\tan\theta_1$$

Thus: $\boxed{e\tan\theta_1 = \tan\theta_2}$

or: $\boxed{\tan\theta_2 = e\tan\theta_1}$

Conclusion: Rebound angles θ_2 for particles striking smooth surfaces are less than incident angles θ_1 for $e < 1$.