Particle $F=ma$ (n-t): Example Problem 5

An automobile drives around a banked curve on a rural road. (a) What direction does friction act? (b) Obtain an equation that relates car speed $v$, bank angle $\theta$, curve radius $r$, and friction $\mu$.
Again, what direction does friction act?

Answer: “It depends!”

At high speed, if auto slipped, it would slide up slope.

At low speed, if auto slipped, it would slide down slope.

An auto as a particle, with one N force, one F force.
Optimal Design: Hwy banked so that zero friction req’d to keep auto on curve.
Obtain an equation that relates car speed $v$, bank angle $\theta$, curve radius $r$, and friction $\mu$.

FBD

\[ \begin{align*}
mg & \quad \text{(normal force)} \\
N & \quad \text{(normal force)} \\
F & \quad \text{(friction force)}
\end{align*} \]

$\mathbf{FBD}$

KD

\[ ma_n = \frac{mv^2}{r} \]

Eqns of Motion from FBD

\[ \begin{align*}
N(sin \theta) + F(cos \theta) &= \frac{mv^2}{r} \\
N(cos \theta) - F(sin \theta) - mg &= 0
\end{align*} \]
\[
\begin{align*}
\text{Assume on} & \quad \text{verge of slip.} \\
\text{Sub in} & \quad F = \mu N \\
\begin{aligned}
\text{①} & \quad N(\sin \theta) + \mu N(\cos \theta) = \frac{mv^2}{r} \\
\text{②} & \quad N(\cos \theta) - \mu N(\sin \theta) = mg
\end{aligned}
\end{align*}
\]

Divide: \[
\frac{\frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta}}{\frac{v^2}{gr}} = \frac{v^2}{gr}
\]

(\(N\) and \(m\) cancel and drop out...) 

Car on banked curve, high speed case, slip impending UP-slope.

If you know \(\theta\), \(\mu\) and \(r\), and need to find the car speed \(v\), this is an easy problem.

If you know \(v\), \(r\) and \(\mu\) and need to find \(\theta\), then you have to iterate or use an equation solver to get \(\theta\).
If you know $\theta$, $\mu$ and $r$, and need to find the car speed $v$, this is an easy problem.

For example, let $\theta = 20$, $\mu = .3$, $r = 200$ ft, $g = 32.2$ fps$^2$; find the speed $v$ at which the car is on the verge of slipping upslope.
If you know v, r and \( \mu \) and need to find \( \theta \), then you have to iterate or use an equation solver to get \( \theta \). Let \( \mu = .3 \), \( r = 100 \) m, \( g = 9.81 \) m/s\(^2\), and \( v = 20 \) m/s; find the angle \( \theta \) at which the car is on the verge of slipping upslope.
High Speed Case (Car on Verge of Slipping Upslope)

\[ \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta} = \frac{v^2}{gr} \]

Car on banked curve, high speed case, slip impending UP-slope.

Low Speed Case (Car on Verge of Slipping Downslope)

(Signs reverse on \( \mu \) terms...)

\[ \frac{\sin \theta - \mu \cos \theta}{\cos \theta + \mu \sin \theta} = \frac{v^2}{gr} \]

Car on banked curve, low speed case, slip impending DOWN-slope.
Zero Friction Case (set $\mu = 0$):

This is the highway design case. For a given speed, $v$, find the bank angle, $\theta$, at which zero friction is needed to keep the car on the curve.

$$\frac{\sin \theta}{\cos \theta} = \tan \theta = \frac{v^2}{gr}$$

Car on banked curve, zero friction case. (Highway design case.)

Curve Not Banked ($\theta = 0$) Case:

Car on level curve ($\theta = 0$) case.

$$\frac{0 - \mu \cdot 1}{1 + \mu \cdot 0} = \mu = \frac{v^2}{gr}$$

Simplified...

$$\mu = \frac{v^2}{gr}$$

Solving for $v_{\text{max}}$

$$v_{\text{max}} = \sqrt{\mu gr}$$