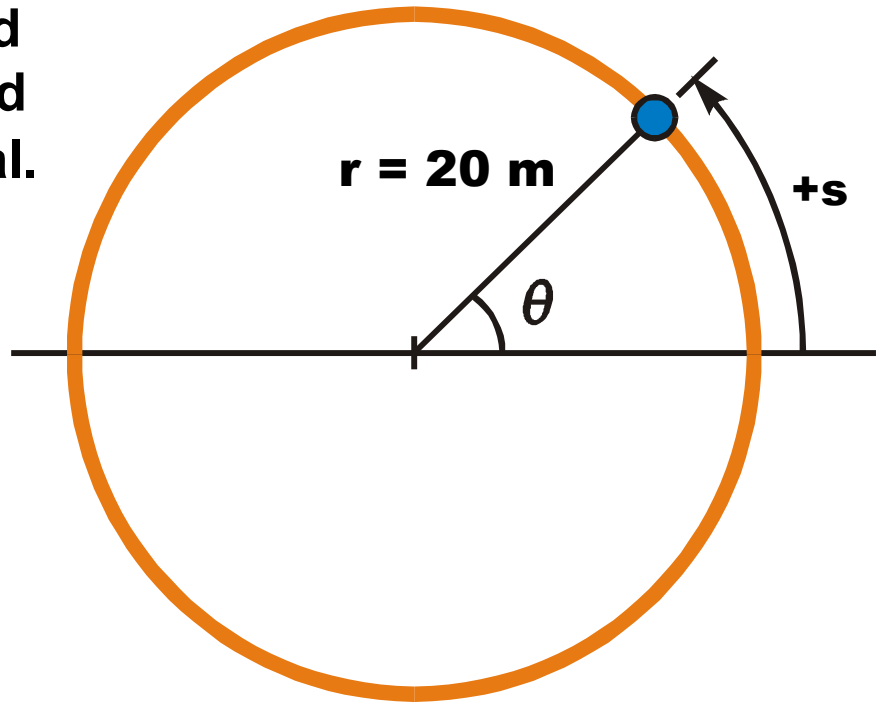
Circular Motion Example Problem 3: $a_t = f(t)$

A bead moves along a circular wire. Its speed increases at $a = 2t - 4$ m/s^2. Its initial (at t = 0) position and speed are s(0) = 0 m and v(0) = 3 m/s. At t = 5 sec, please determine:

(a) The magnitude of the bead's acceleration.

(b) The position of the bead along the wire (give both arc length, s, and angle, θ.

(c) The total distance traveled along the wire by the bead in the 0-5 sec time interval.



Circular Motion Ex Prob 3: at = f(t) (a total dist problem)

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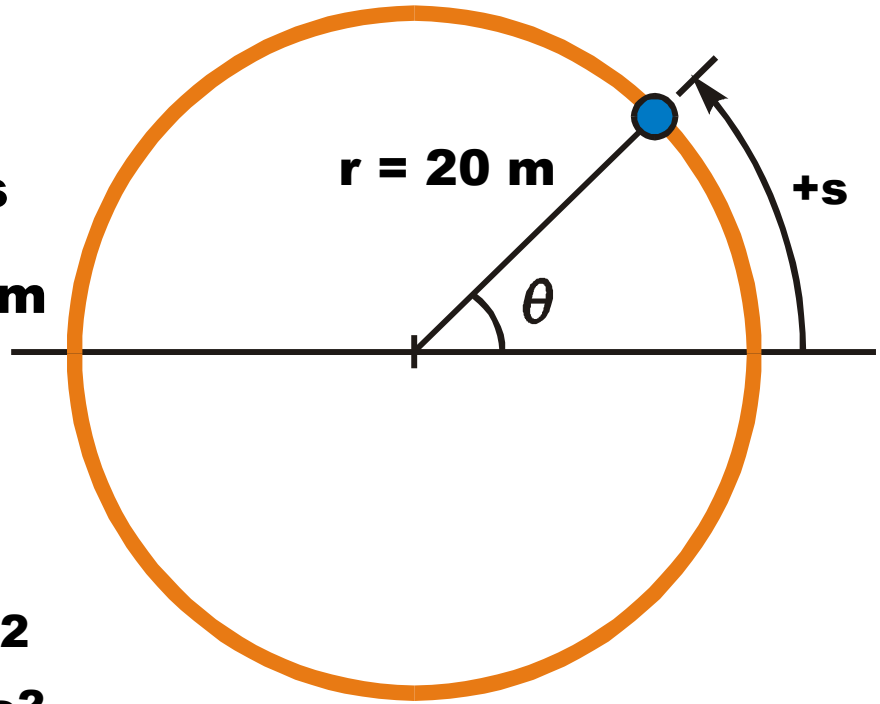


Solution:

Step 1: Integrate the a_t function:

$a_t = 2t - 4\ m/s^2$ $\qquad v_0 = 3$ m/s

$v = t^2 - 4t + 3\ m/s$ $\qquad s_0 = 0$ m

$s = \frac{1}{3}t^3 - 2t^2 + 3t + 0\ m$

Step 2: Evaluate at t = 5 sec

$s(5) = 6.67$ m $\qquad a_t(5) = 6\ m/s^2$

$v(5) = 8$ m/s $\qquad a_n(5) = \frac{v^2}{r} = \frac{8^2}{20}$

$= \frac{64}{20} = 3.2\ m/s^2$

Circular Motion Ex Prob 3: $a_t = f(t)$ (a total dist problem)

A bead moves along a circular wire. Its speed increases at $a = 2t - 4$ m/s^2….

Step 3: Further investigate the bead's motion…
Find roots of the velocity equation….

$v(t) = t^2 - 4t + 3$ m/s

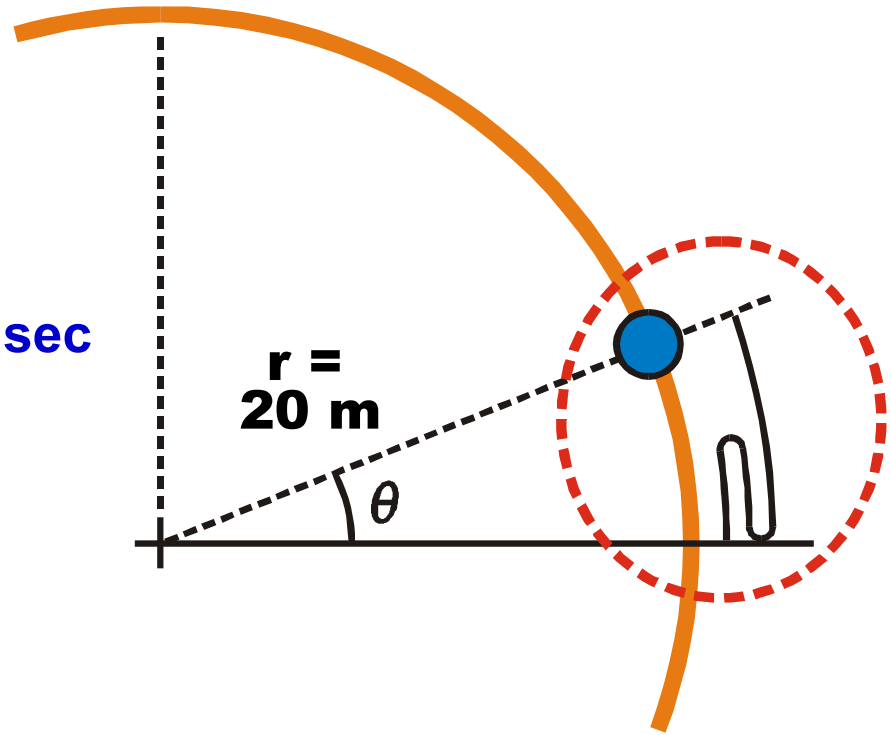
$v = 0 = (t - 1)(t - 3)$

$v = 0$ at $t = 1, 3$ seconds

Step 4: Evaluate s(t) at 0, 1, 3, 5 sec

$s(t) = \frac{1}{3}t^3 - 2t^2 + 3t$ m

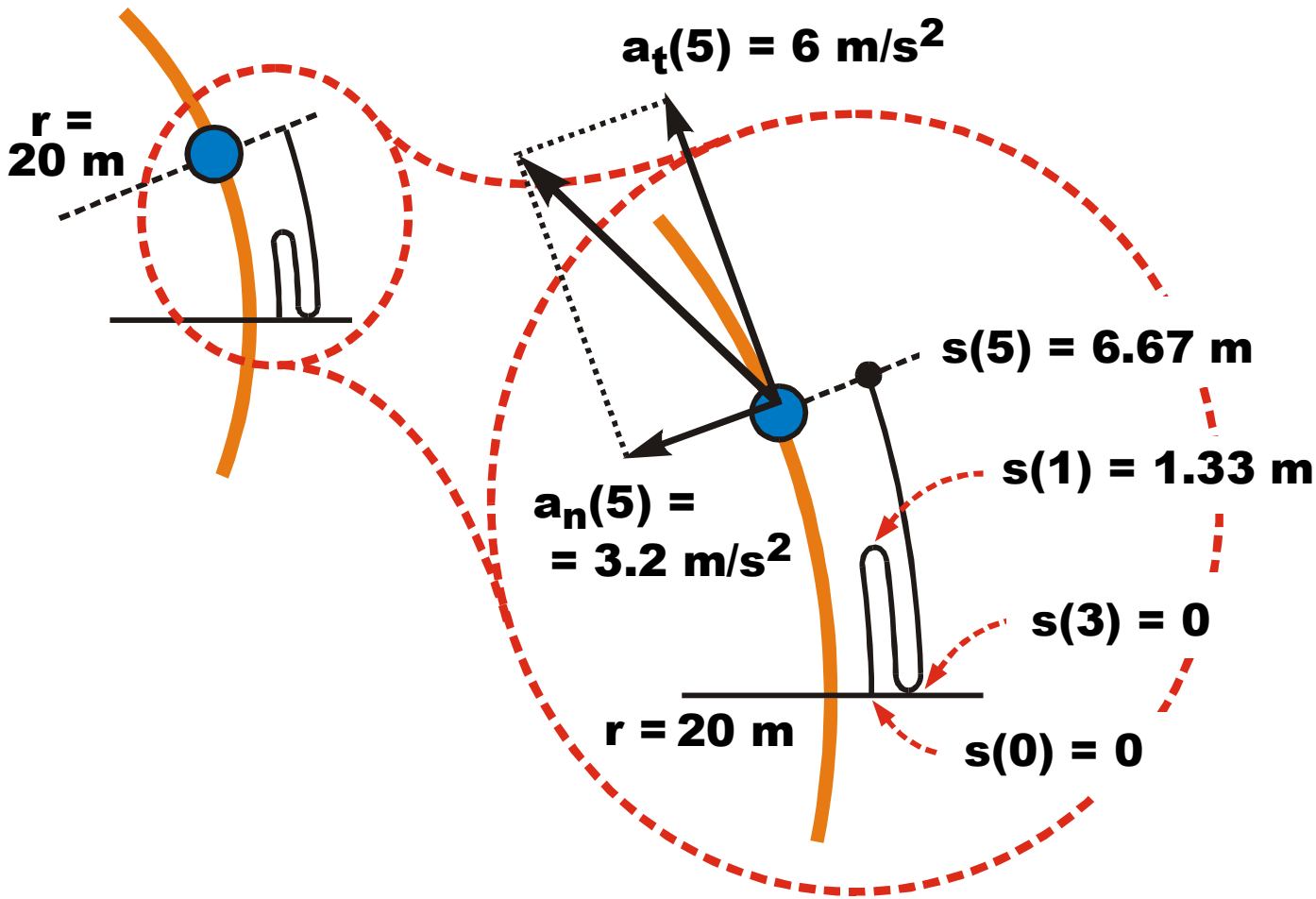
t	s(t)
0	0 m
1	1.33
3	0
5	6.67



Circ Motion Ex Prob 3: $a_t = f(t)$ (a total dist problem)

Step 5: Plot the bead's displacement along the wire…

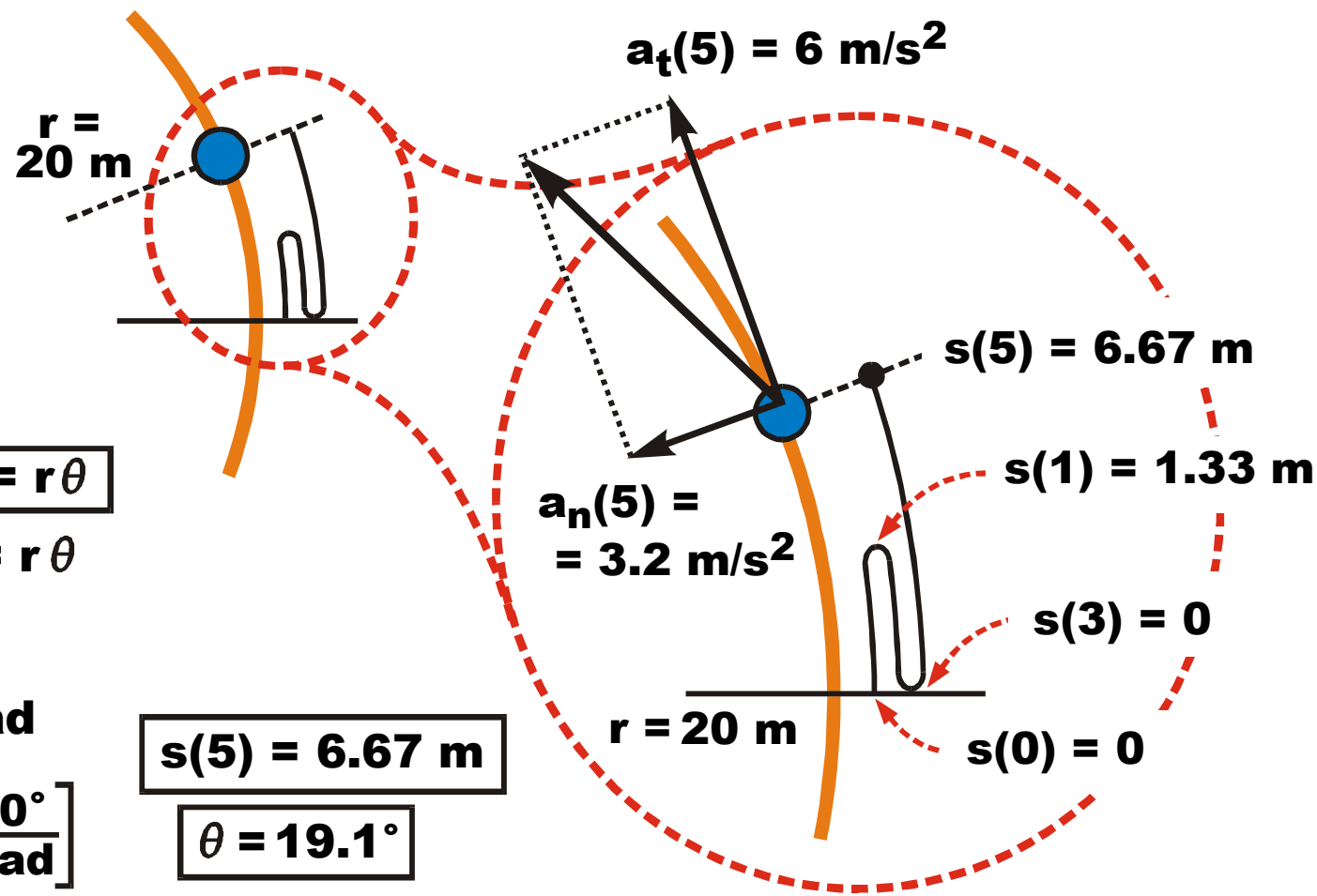
t	s(t)
0	0 m
1	1.33
3	0
5	6.67



Circ Motion Ex Prob 3: $a_t = f(t)$ (a total dist problem)

Step 6: Bead's position s (in meters) and θ (in degrees) at t = 5 sec

t	s(t)
0	0 m
1	1.33
3	0
5	6.67



Arc Length: $s = r\theta$

$s(5) = 6.67 \text{ m} = r\theta$

$6.67 = 20\theta$

$$\theta = \frac{6.67}{20} = \frac{1}{3} \text{ rad}$$

$$\theta = \left[\frac{1}{3} \text{ rad}\right]\left[\frac{180°}{\pi \text{ rad}}\right]$$

$s(5) = 6.67$ m

$\theta = 19.1°$

Circ Motion Ex Prob 3: $a_t = f(t)$ (a total dist problem)

Step 7: Acceleration magnitude at t = 5 sec

Magnitude of the $\vec{a}$ vector:
(the "total acceleration")

$$|\vec{a}| = \sqrt{a_t^2 + a_n^2}$$

$$|\vec{a}| = \sqrt{6^2 + 3.2^2}$$

$$|\vec{a}| = 6.80 \text{ m/s}^2$$