Particle Straight Line Kinematics: Ex Prob 2

This is an $a = f(t)$ problem. I call it a "total distance" problem.
Variations: $v = f(t)$, $s = f(t)$….

A particle moves along a straight line with an acceleration of $a = (2t-6)\ m/s^2$. Initially (at $t = 0$), the position of the particle is $s_0 = 1$ m, and its velocity is $v_0 = 5$ m/s. For the time interval $0 \le t \le 6$ sec, please do the following:

(a) Draw a displacement plot.

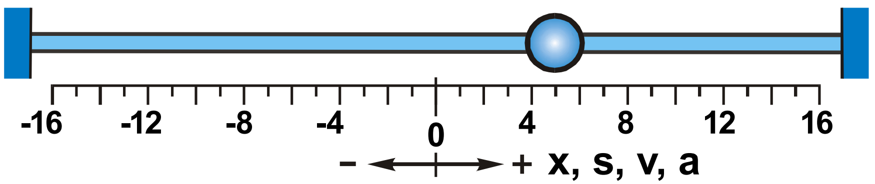
Calculate the particle's:

(b) Displacement, Δs.

(c) Average Velocity, v_{avg}.

(d) Total Distance Traveled, d.

(e) Average Speed, v_{sp}.



Typical Rectilinear Motion Coordinate System

A particle moves along a straight line with an acceleration of $a = (2t-6)\ m/s^2$. Initially (at $t = 0$), the position of the particle is $s_0 = 1$ m, and its velocity is $v_0 = 5$ m/s. Time interval $0 \le t \le 6$ s.

Step 1: Integrate the acceleration equation:

$a = (2t - 6)\ m/s^2$.

$v = t^2 - 6t + 5$ m/s.

$s = 1/3\ t^3 - 3t^2 + 5t + 1$ meters.

Note: If given the s(t) eqn, then differentiate. If given the v(t) eqn, then differentiate for a(t) and integrate for s(t)…..

Step 2: Determine the roots of the velocity eqn: (A key step!)

$0 = v = t^2 - 6t + 5 = (t-5)(t-1)$; Thus v = 0 at t = 1,5 seconds.

What are these v = 0 roots (times)? The particle has at least stopped and is most likely turning around.

Step 1: Integrate the acceleration equation:

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$v = t^2 - 6t + 5\ \ m/s.$

$s = 1/3\ t^3 - 3t^2 + 5t + 1\ \ meters.$

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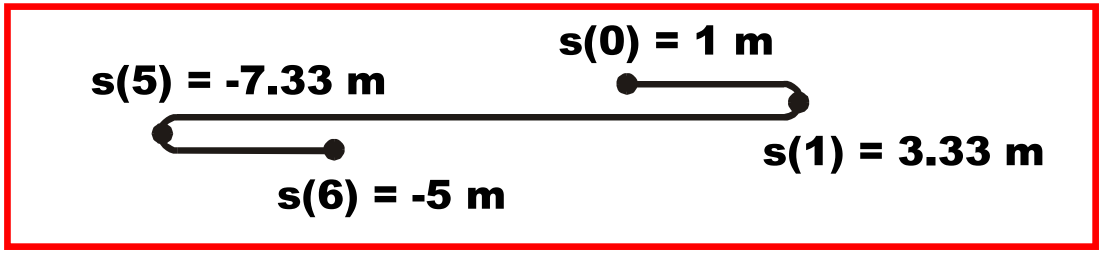
Step 3: Determine the particle's positions at key times. Key times are the start and finish times ($t = 0,6$ sec) and the turn-around times ($t = 1,5$ sec). Use the position equation: $s = 1/3\ t^3 - 3t^2 + 5t + 1\ \ m$

s(0) = 1 m	s(5) = -7.33 m
s(1) = 3.33 m	s(6) = -5 m

Step 4: Draw a Displacement Plot:



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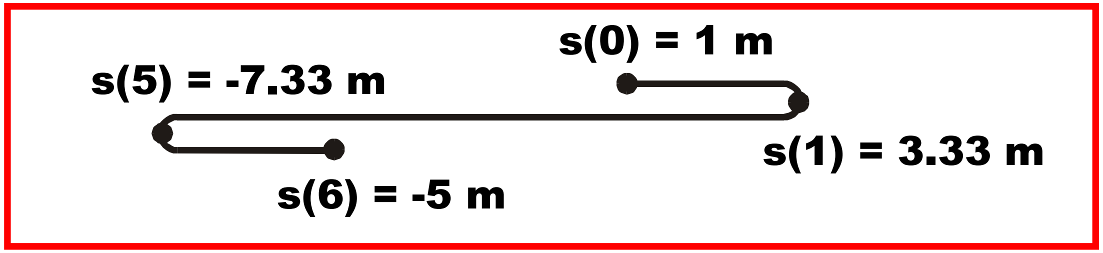

Step 5: Calculate Δs, d, v_{avg}, v_{sp}.

Displacement:	$\Delta s = s_{final} - s_{start} = -5 - 1 = -6$ meters
Total Distance:	d = 2.33 + 3.33 + 7.33 + 2.33 = 15.33 m

(Add lengths of the line segments of the displacement plot.)

Avg Velocity:	$v_{avg} = \Delta s/\Delta t = (-6)/(6 \text{ sec}) = -1$ m/s
Avg Speed:	$v_{sp} = d/\Delta t = (15.33)/(6 \text{ sec}) = 2.56$ m/s