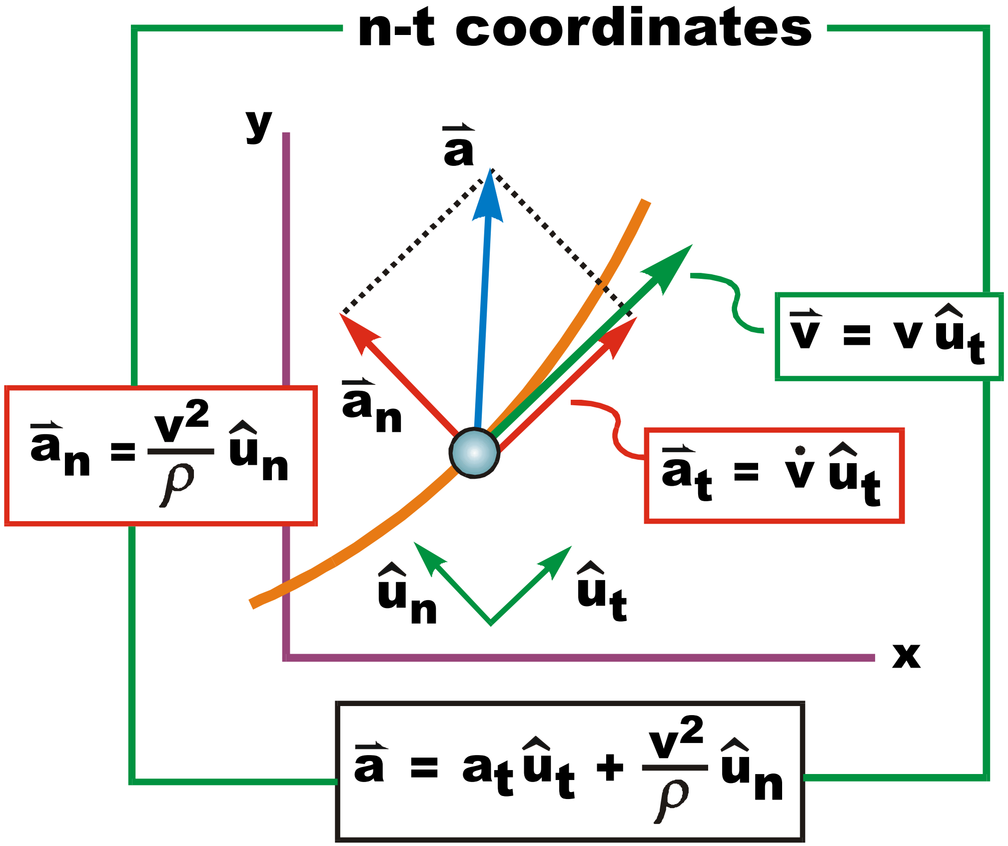
Particle Kinematics n-t Coordinates Intro

We've already discussed and used accelerations tangential (a_t) and normal (a_n) to the path of motion. We've calculated a_n from $a_n = v^2/r$ (circular motion) or $a_n = v^2/\rho$ (non-circular).

n-t coordinates



$$\vec{v} = v\,\hat{u}_t$$

$$\vec{a}_n = \frac{v^2}{\rho}\,\hat{u}_n$$

$$\vec{a}_t = \dot{v}\,\hat{u}_t$$

$$\vec{a} = a_t\hat{u}_t + \frac{v^2}{\rho}\,\hat{u}_n$$

In this section I'll show you from where the a_n term arises (it arises from the time derivative of the u_t unit vector).

And we'll work some example n-t problems.

n-t coordinates:

Position vector: Not defined

[n-t coordinates are attached to and move with a particle.]

Velocity vector: $\boxed{\vec{v} = v\,\hat{u}_t}$

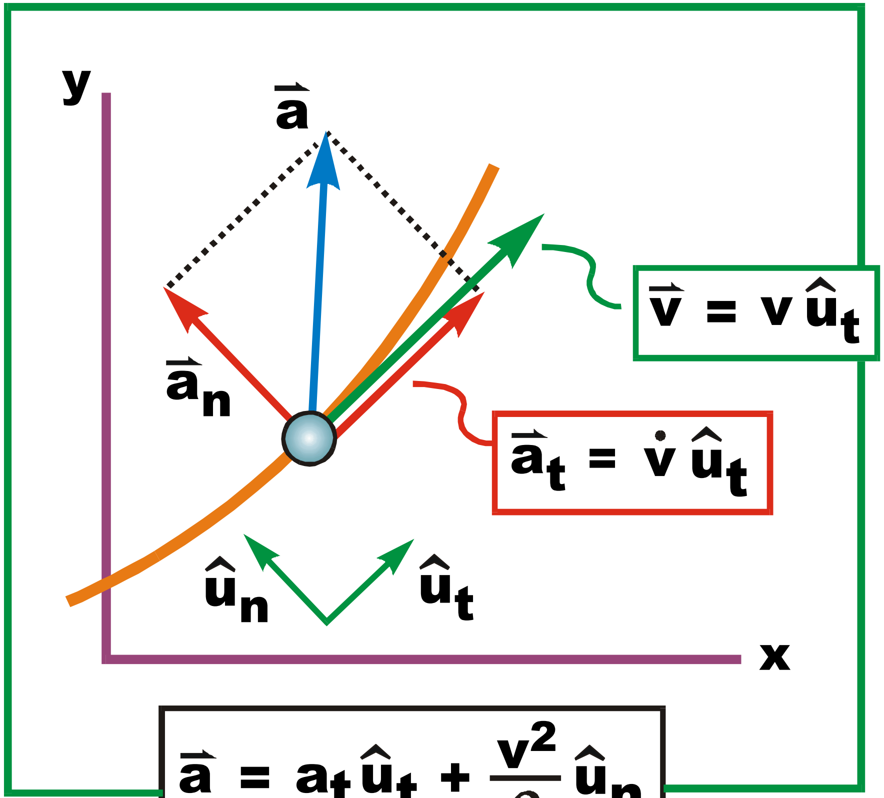
Speed

Direction

Acceleration vector:

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{\vec{v}} = \underbrace{\dot{v}\,\hat{u}_t + v\,\dot{\hat{u}}_t}_{\text{Product Rule}}$$

$$\vec{a} = \underbrace{a_t\,\hat{u}_t}_{\text{tangential acceleration}} + \underbrace{v\,\dot{\hat{u}}_t}_{\text{what's this?}}$$



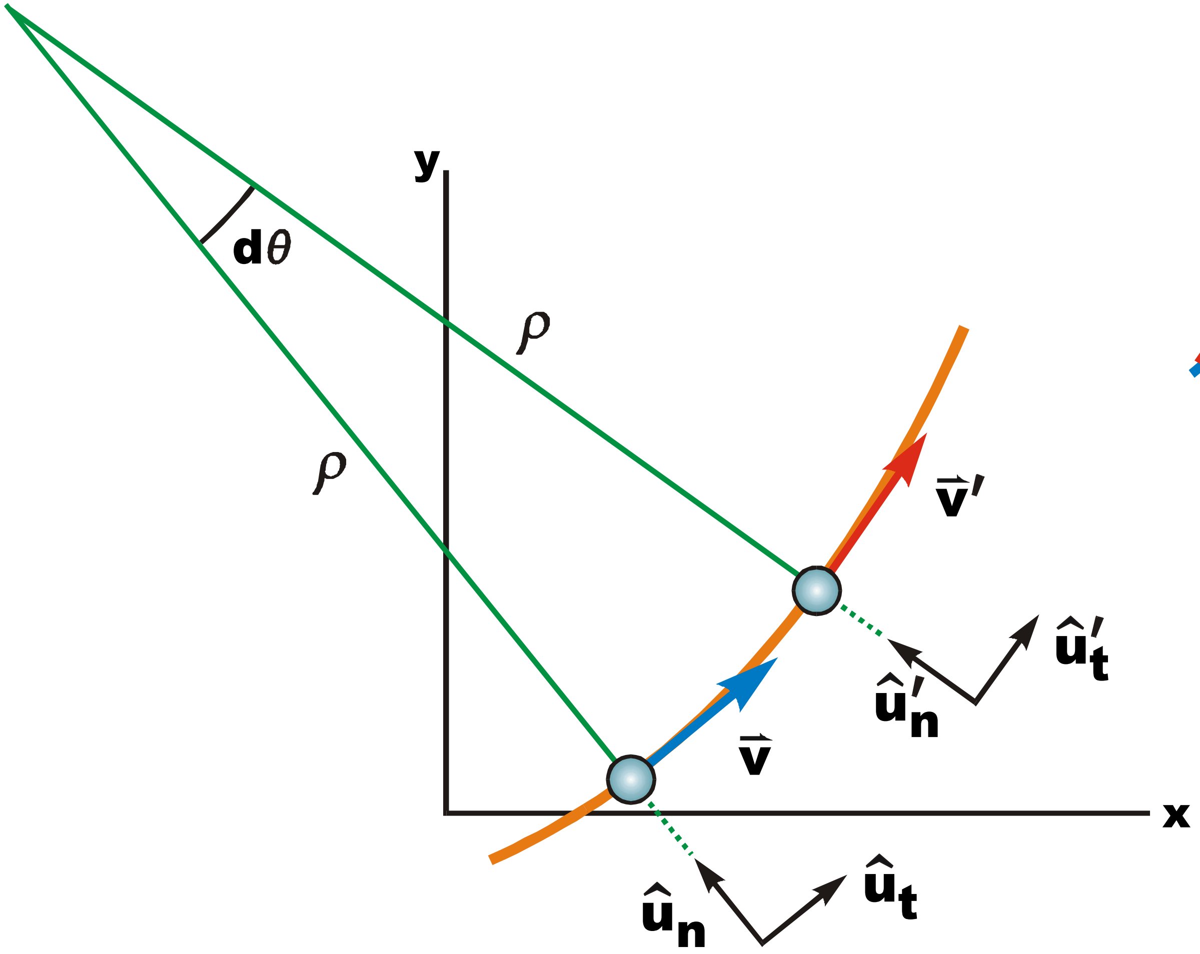
$\boxed{\vec{v} = v\,\hat{u}_t}$

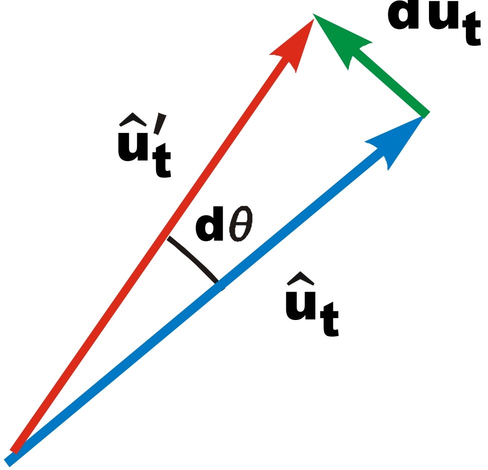
$\boxed{\vec{a}_t = \dot{v}\,\hat{u}_t}$

$$\boxed{\vec{a} = a_t\,\hat{u}_t + \frac{v^2}{\rho}\,\hat{u}_n}$$

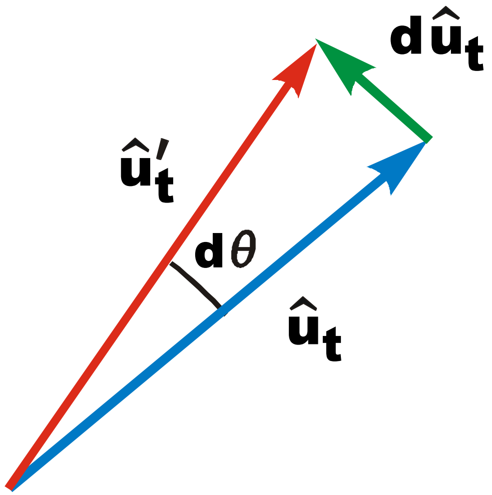
$$\boxed{\vec{a}_n = \frac{v^2}{\rho}\,\hat{u}_n}$$

The *direction* of the u_t and u_n unit vectors change, because they are attached to and move with the particle. Their lengths are constant (as unit vectors, their length is 1, of course!).





Think about this $d\hat{u}_t$ vector:



Length = arc length of sweeping a unit vector through $d\theta$.

Direction = normal to the $\hat{u}_t$ vector.

This direction is given by $\hat{u}_n$.

Thus, $d\hat{u}_t = 1 \cdot d\theta \cdot \hat{u}_n$

Dividing by dt gives $\dot{\hat{u}}_t$:

$$\dot{\hat{u}}_t = \frac{d\hat{u}_t}{dt} = \dot{\theta}\,\hat{u}_n = \frac{v}{\rho}\,\hat{u}_n$$

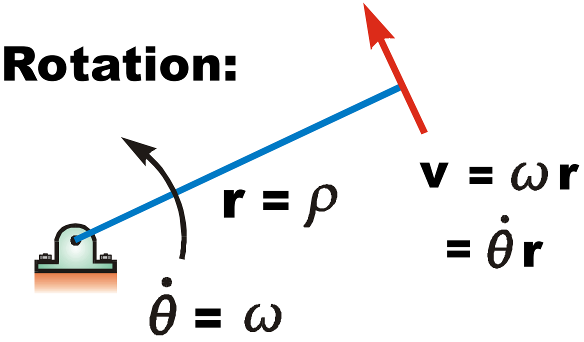
Finally:

$$\boxed{\dot{\hat{u}}_t = \frac{v}{\rho}\,\hat{u}_n}$$

Note that:

$$\frac{d\theta}{dt} = \dot{\theta} = \frac{v}{\rho}$$

Rotation:



n-t coordinates (continued)

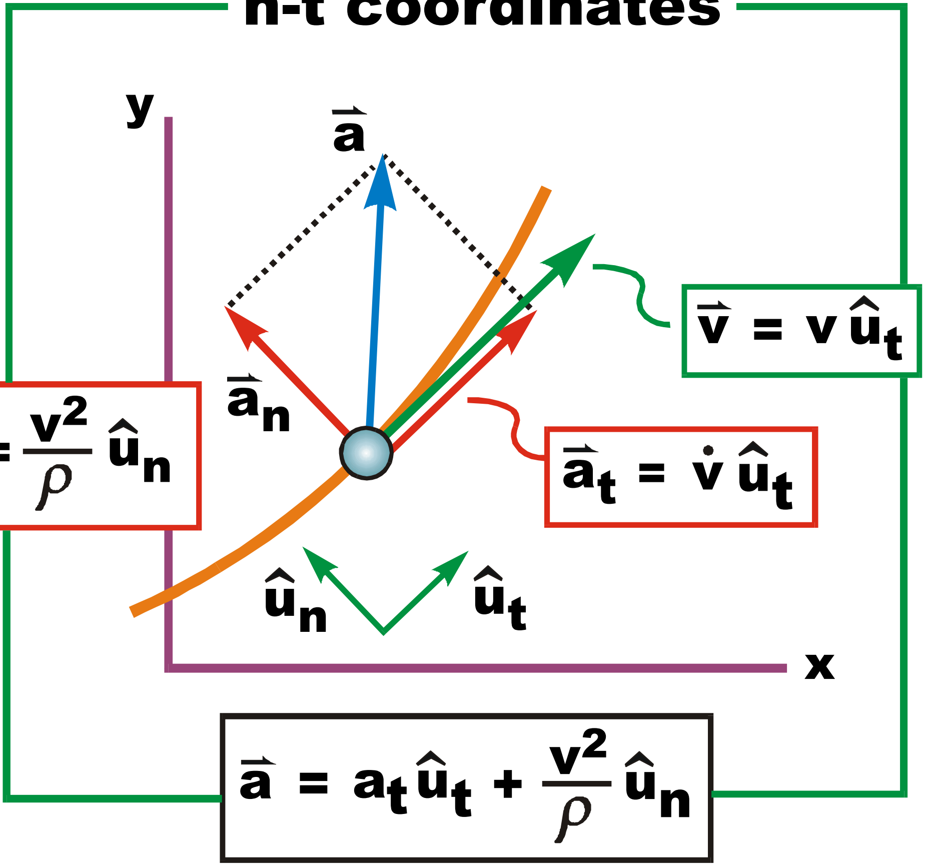
$$\vec{a} = \underbrace{a_t \hat{u}_t}_{\text{tangential acceleration}} + \underbrace{v \dot{\hat{u}}_t}_{\text{what's this?}}$$

$$\boxed{\dot{\hat{u}}_t = \frac{v}{\rho}\hat{u}_n}$$

$$\vec{a} = a_t \hat{u}_t + v\left[\frac{v}{\rho}\hat{u}_n\right]$$

$$\underbrace{\boxed{\vec{a} = a_t \hat{u}_t + \frac{v^2}{\rho}\hat{u}_n}}$$

tangential acceleration — **normal acceleration**



n-t coordinates

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