Rigid Body F=ma Intro (Gen Plane B): Example 1

(Last class we worked a drum and weight connected by a cable problem, except the cable attached to the center of the drum. In this problem the cable is not at the center and we must use a relative acceleration equation for kinematics.

A 20 kg disk A is connected to a 10 kg mass C by a massless cable which wraps around an inner hub on A and around massless, frictionless pulleys. If the system is released from rest in the position shown, please determine the tension in the cable and the component accelerations.
Draw the FBD and KD for the drum and the mass:

FBD for A

\[ m_{Ag} = (20)(9.81) = 196.2 \text{ N} \]

KD for A

\[ I_G \alpha = 0.45 \alpha \]

\[ I_G = m_k^2 \]

\[ I_G = (20)(0.15^2) = 0.45 \text{ kg-m}^2 \]

Important! Do not set \( F = \mu N \) unless the problem says it slips. Treat \( F \) as unknown.

FBD for C

\[ (10)(9.81) = 98.1 \text{ N} \]

KD for C

\[ 10a_c \]
Write the Equations of Motion:

Drum A:

\[
\begin{align*}
196.2 \text{ N} & \quad \text{FBD} \\
F & \quad T \\
N = 196.2 \text{ N} & \quad \text{KD}
\end{align*}
\]

Equations of Motion (Drum A):

\[
\begin{align*}
\sum F_y &= m_{gy} ; \quad N = 196.2 \text{ N} \\
\sum F_x &= m_{gx} ; \quad T - F = 20a_A \\
\sum M_G &= I_G \alpha ; \quad T(.1) + F(.2) = .45 \alpha
\end{align*}
\]

Equation of Motion (Mass C):

\[
\begin{align*}
\sum F_y &= m_{gy} ; \quad 98.1 - 2T = 10a_C
\end{align*}
\]

Count the unknowns in these THREE equations. How many do you get?

I count FIVE!

We need TWO more equations.
We need two additional equations...Use kinematics:

Assume no slip: \( a = \alpha r \)

Relative Accel Equation for Drum A:

Assume no slip: \( a = \alpha r \)

\( a_A = \alpha r \)

Note: This “no slip” assumption must be validated with your answers.

But \( a_B \) is not one of our current unknowns. How can we relate \( a_C \) to \( a_A \)? Use the relative accel equation.

\[
\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}
\]

\( a_B = .2\alpha + .1\alpha = .3\alpha \)

Finally: \( a_C = .15\alpha \)
Set up the matrix and solve the system of eqns:

1. \( T - F = 20a_A \)
2. \( T(.1) + F(.2) = .45 \alpha \)
3. \( 98.1 - 2T = 10a_C \)
4. \( a_A = .2 \alpha \)
5. \( a_C = .15 \alpha \)

Solve with calculator:
\[
\begin{align*}
T &= 41.6 \text{ N} \\
F &= 1.66 \text{ N} \\
a_A &= 2.0 \text{ m/s}^2 \\
a_C &= 1.50 \text{ m/s}^2 \\
\alpha &= 9.98 \text{ rad/s}^2
\end{align*}
\]

One last step: Check the no slip assumption...
\((F/N)_{\text{CALC}} < \mu = .2\) ? Clearly true because F is small.