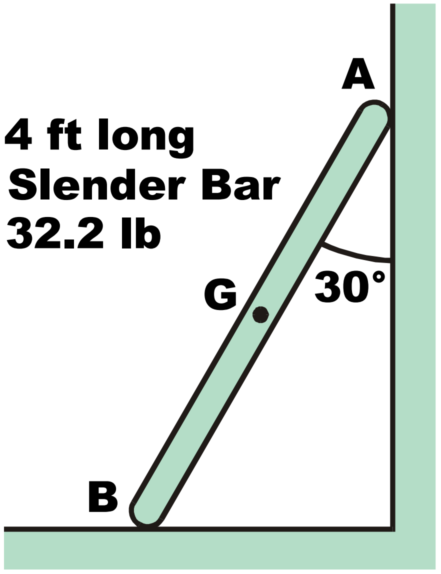
# Rigid Body F=ma Intro (Gen Plane B): Example 1

(This bar down wall problem was a poster child for simplicity for rigid body kinematics. Now, for rigid body F = ma, if we release the bar to move, finding the accelerations is a challenge requiring us to write FIVE equations for five unknowns.)

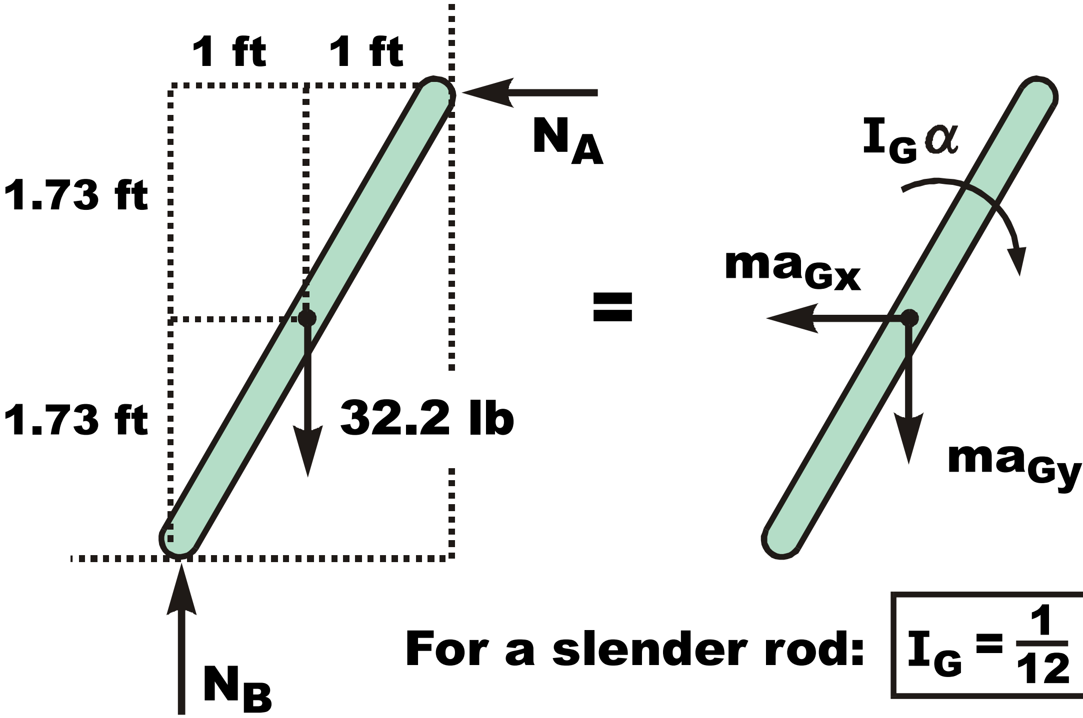



Surfaces are smooth.
Released from rest.

A 4 ft long 32.2 lb slender bar AB is released to move from the position shown. The surfaces are smooth. Please determine the normal reactions and the acceleration components for bar AB.

Before going to the next page, think about the unknowns that will appear on the FBD and on the KD. How many will there be? What are they?

# Draw the FBD and KD for bar AB:

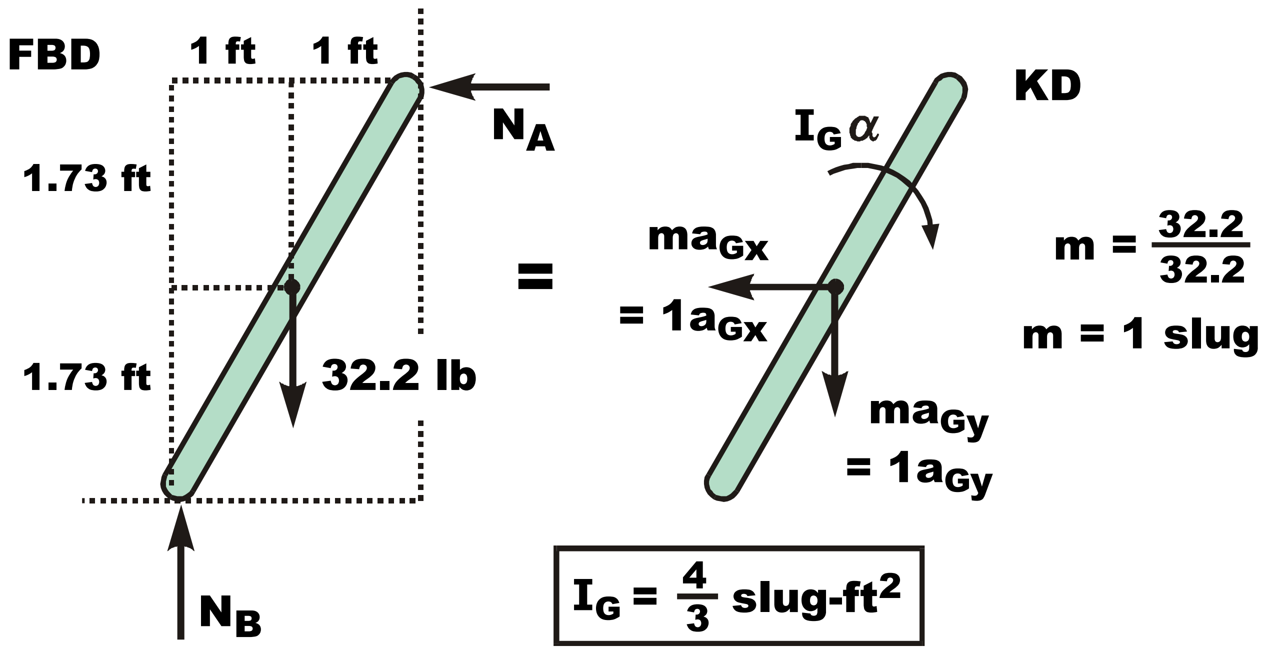



For a slender rod: $\boxed{I_G = \frac{1}{12} m L^2}$

$$I_G = \frac{1}{12}\left(\frac{32.2}{32.2}\right)4^2 = \frac{4}{3}\ \text{slug-ft}^2$$

Note that there are five unknowns! $N_B$, $N_A$, $ma_{Gx}$, $ma_{Gy}$, and $\alpha$. But we can only write *three* equations of motion. Where will we get two additional equations?

# Write the Equations of Motion:



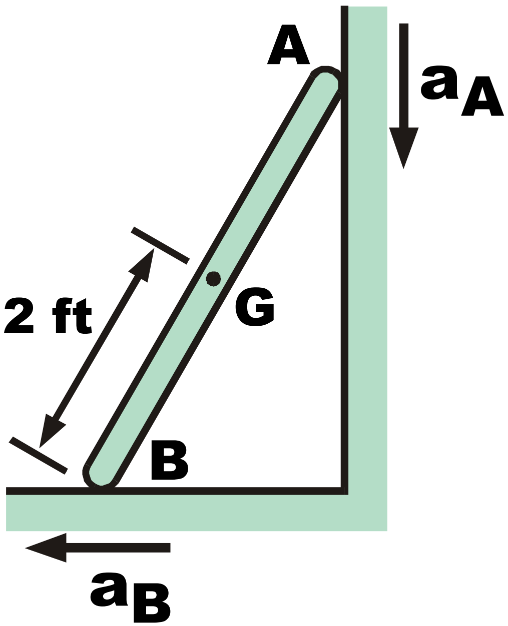

**Equations of Motion:**

$+\uparrow \sum F_y = ma_{Gy}$ ;  $N_B - 32.2 = -a_{Gy}$  ①

$\xleftarrow{+} \sum F_x = ma_{Gx}$ ;  $N_A = a_{Gx}$  ②

$\overset{+}{\curvearrowright} \sum M_G = I_G\alpha$ ;  $N_B(1) - N_A(1.73) = 1.33\alpha$  ③

# We need two additional equations…From where?:

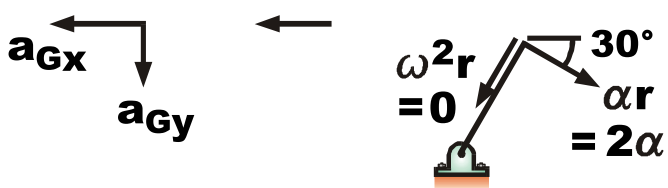



Look at the picture at left. What constraints are present?

Correct! End B slides horizontally, so $a_B$ is horizontal.

Also: End A slides along a vertical wall, so $a_A$ is vertical.

$$\vec{a}_G = \vec{a}_B + \vec{a}_{G/B}$$




$+\downarrow \quad a_{Gy} = 2\alpha \sin 30$

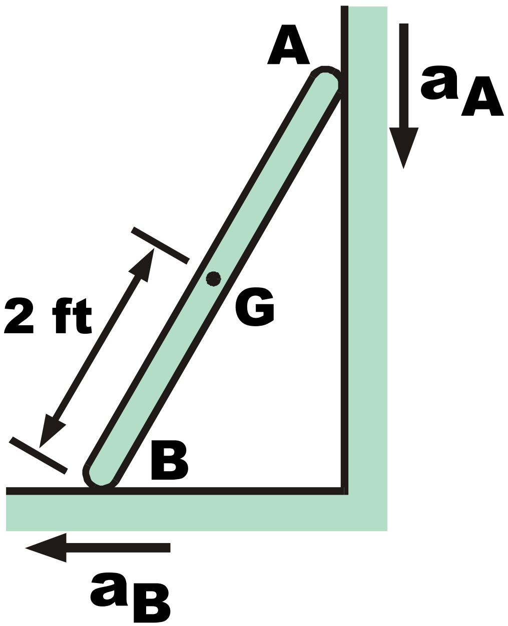
$\boxed{a_{Gy} = \alpha}$ (4)

This may not seem like much, but it's more than you realize. These two constraints are two "pieces" of information that you can use to get two kinematics equations.

Write a rel accel equation from B to G and use the y scalar equation *only* (*not* the x equation) to get $a_{Gy} = \alpha$ .

That's equation 4. We need one more.

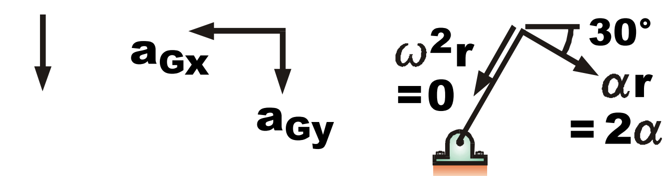
# One last equation from kinematics….




Look at the picture at left. What constraints are present? Which one have we not used?

End A slides along a vertical wall, so $a_A$ is vertical.

$$\vec{a}_A = \vec{a}_G + \vec{a}_{A/G}$$




$$\overset{+}{\leftarrow} \quad 0 = -a_{Gx} + 2\alpha \cos 30$$

$$\boxed{a_{Gx} = 1.732\,\alpha} \quad 5$$

Like we did between B and G, now write a rel accel equation from G to A and use the x scalar equation *only* (*not* the y equation) to get $a_{Gx} = 1.73\alpha$ .

That's our 5th equation.

See the next page to see how we use a matrix solution for the 5 equations and five unknowns.

## Set up the matrix and solve the system of eqns:

1. $N_B - 32.2 = -a_{Gy}$
2. $N_A = a_{Gx}$
3. $N_B(1) - N_A(1.73) = 1.33\,\alpha$
4. $a_{Gy} = \alpha$
5. $a_{Gx} = 1.732\,\alpha$

$$
\begin{array}{c}
 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} N_B & N_A & a_{Gx} & a_{Gy} & \alpha \end{array} \\
\begin{bmatrix}
1 & 0 & 0 & 1 & 0 \\
0 & 1 & -1 & 0 & 0 \\
1 & -1.73 & 0 & 0 & -1.33 \\
0 & 0 & 0 & 1 & -1 \\
0 & 0 & 1 & 0 & -1.73
\end{bmatrix}
\end{array}
\begin{bmatrix} N_B \\ N_A \\ a_{Gx} \\ a_{Gy} \\ \alpha \end{bmatrix}
=
\begin{bmatrix} 32.2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

**Solve with calculator:**

$N_B = 26.2$ lb $\uparrow$

$N_A = 10.46$ lb $\leftarrow$

$a_{Gx} = 10.46$ fps$^2$ $\leftarrow$

$a_{Gy} = 6.04$ fps$^2$ $\downarrow$

$\alpha = 6.04$ rad/s$^2$ $\circlearrowright$