Mass Moment of Inertia, I_G

I_G is the "mass moment of inertia" for a body about an axis passing through the body's mass center, G.

I_G is defined as: $\boxed{I_G = \int r^2\, dm}$ **Units: kg-m^2 *or* slug-ft^2**

I_G is used for several kinds of rigid body rotation problems, including:

> **(a) F=ma analysis moment equation ($\Sigma M_G = I_G \alpha$).**
>
> **(b) Rotational kinetic energy ($T = \frac{1}{2} I_G \omega^2$)**
>
> **(c) Angular momentum ($H_G = I_G \omega$)**

I_G is the resistance of the body to angular acceleration. That is, for a given net moment or torque on a body, the larger a body's I_G, the lower will be its angular acceleration, α.

I_G also affects a body's angular momentum, and how a body stores kinetic energy in rotation.

Mass Moment of Inertia, I_G (cont'd)

I_G for a body depends on the body's mass and the location of the mass.

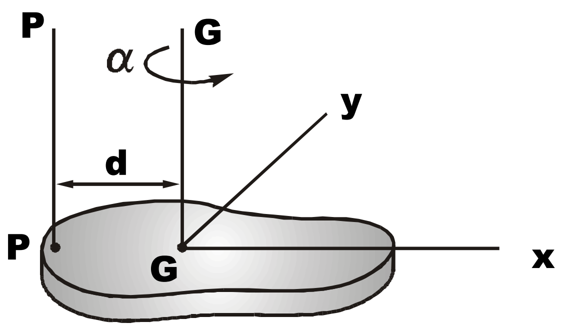
The greater the distance the mass is from the axis of rotation, the larger I_G will be.

For example, flywheels have a heavy outer flange that locates as much mass as possible at a greater distance from the hub.

If I is needed about an axis other than G, it may be calculated from the "parallel axis theorem."

Parallel Axis Theorem (PAT) for $\mathbf{I}$ about axes other than G.

Parallel Axis Theorem



$$I_P = I_G + md^2$$

If you know I_G about the G axis, and need I_P about another axis (parallel to the G axis) use the "parallel axis theorem."

I_G = I about center of mass, G

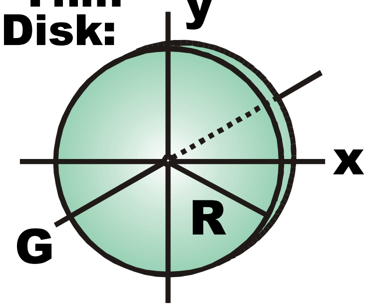
I_P = I about an axis passing through P (parallel to the G axis)

md^2 = "transfer term"; m = mass of body, d = distance between axes

Important: This equation cannot be used between *any* two parallel axes. One axis must be G, about the center of mass.

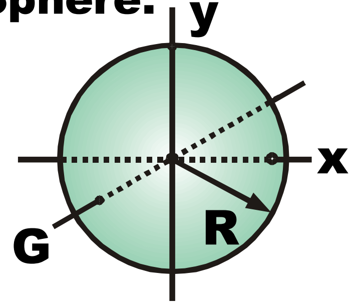
I_G's for Common Shapes

Thin Disk:



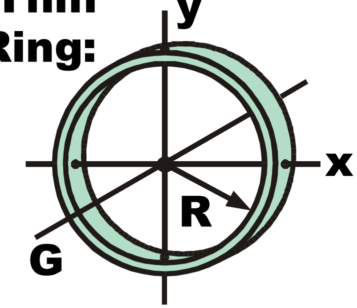
$$I_G = \frac{1}{2} mR^2$$

Sphere:



$$I_G = \frac{2}{5} mR^2$$

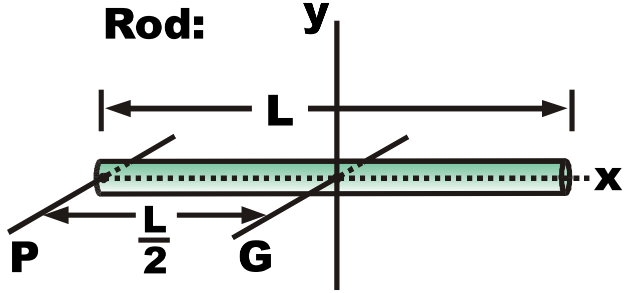
Thin Ring:



$$I_G = mR^2$$

(All mass is at the same radius, R)

Slender Rod:



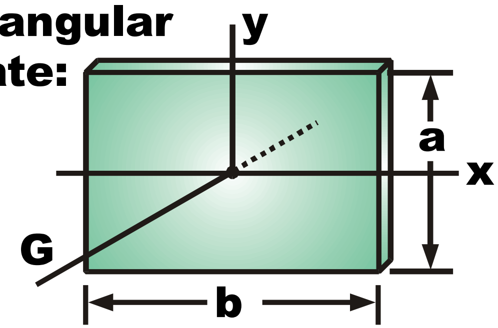
$$I_P = \frac{1}{3} mL^2$$

About P (end of rod)

$$I_G = \frac{1}{12} mL^2$$

About G (center of rod)

Rectangular Plate:



$$I_G = \frac{1}{12} m(a^2 + b^2)$$

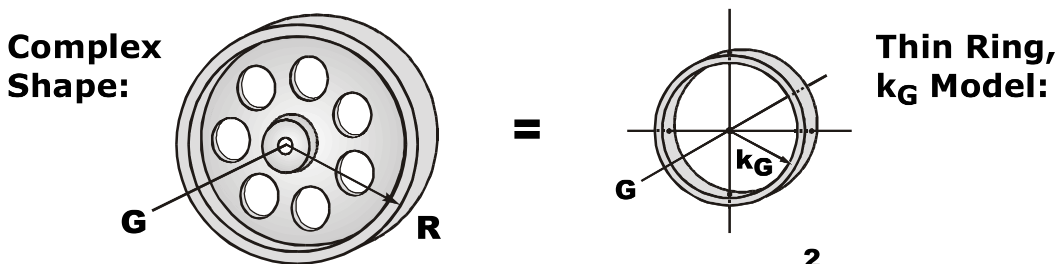
Radius of Gyration, k_G, for Complex Shapes

Some problems with a fairly complex shape, such as a drum or multi-flanged pulley, will give the body's mass m and a radius of gyration, k_G, that you use to calculate I_G.

If given these, calculate I_G from: $$I_G = mk_G^2$$

As illustrated below, using k_G in this way is effectively modeling the complex shape as a thin ring.

Radius of Gyration, k_G



$$I_G = mk_G^2$$

Some problems involving a complex shape with mass, m, and an outer radius, R, will give a "radius of gyration", k_G , that can be used to determine I_G for that shape. The equation, $I_G = mk_G^2$, indicates that the complex shape is being modeled dynamically by a thin ring with mass, m, and a radius, k_G.