Work-Energy (WE) Equation for Particles

Important: The WE equation is not a radically new concept. It is an integrated form of F = ma.

Work of a Force: Work = (Force)(Distance)

Units: ft-lb or N-m = Joule

Work is a dot product:

Work is a dot product

$$dU = \vec{F} \cdot d\vec{r}$$

of force times displacement

Properties of a dot product:

Magnitude: $dU = (F \cos\theta)(dr)$

Where, $F \cos\theta$ is the component of F acting in the direction of motion.

The component of $\vec{F}$ $\perp$ to $d\vec{r}$ does no work.

Derivation of the Work-Energy Equation

Work definition: $dU = \vec{F} \cdot d\vec{r}$

In scalar form, with F acting along ds:

Three familiar concepts combined and integrated.

Work: $dU = F \cdot ds = m\,a\,ds$

F = ma: $F = ma$

Kinematics: $a\,ds = v\,dv$

$$dU = m\,v\,dv$$

Integrate: $$\int_1^2 dU = \int_{v_1}^{v_2} m\,v\,dv$$

$$U_{1\text{-}2} = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

For a single particle, with multiple work terms:

$$\frac{1}{2}mv_1^2 + \sum U_{1\text{-}2} = \frac{1}{2}mv_2^2$$

Initial Kinetic Energy | Sum of Work Done on Particle | Final Kinetic Energy

Two other ways to write Work-Energy Equation:

Let T_1 and T_2 represent initial and final KE

$$T_1 + \sum U_{1\text{-}2} = T_2$$

W-E equation for multiple particles with KE:

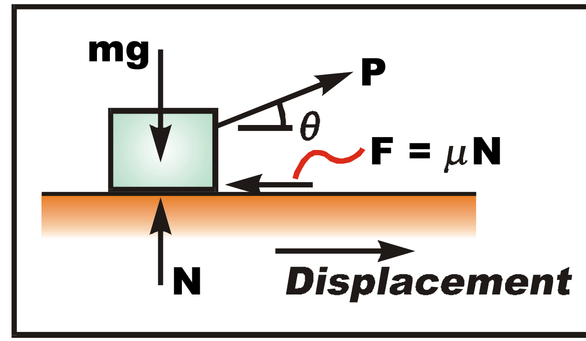
$$\sum T_1 + \sum U_{1\text{-}2} = \sum T_2$$

Now that we have the Work-Energy Equation…

What are the work terms?

Work of a force:

$$U_{Force} = (P\cos\theta)(d)$$



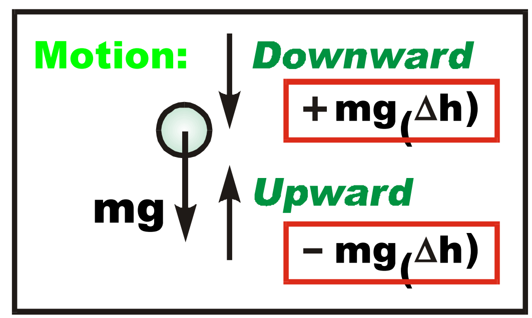
Work of friction:

$$U_{Friction} = -F\,(d)$$

(work of friction is always negative because it always acts opposite of motion)

Work of weight:

$$U_{Weight} = \pm(mg)(\Delta h)$$



Work of a spring:

$$U_{Spring} = -\frac{1}{2}k\left[s_2^2 - s_1^2\right]$$

s_1 and s_2 are *stretch*!

$s_2 = L_2 - L_0$

$s_1 = L_1 - L_0$

k = spring constant

L_0 = unstretched length

L_1 = original spring length

L_2 = final spring length

Where does the spring work term come from?

The Work Energy Equation....

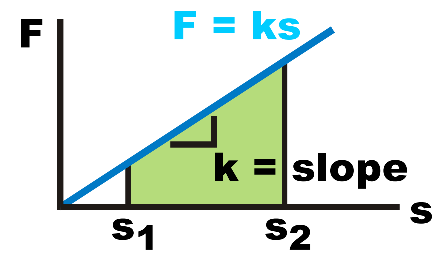
$$\frac{1}{2}mv_1^2 + \sum U_{1\text{-}2} = \frac{1}{2}mv_2^2$$

can also be written, when force varies with s, as:

$$\frac{1}{2}mv_1^2 + \int F\,ds = \frac{1}{2}mv_2^2$$

Area under the F-ds curve. (force vs. displacement)

For a linear spring:



Recall our work definition: $dU = F\,ds$, with $F = ks$

Integrate:

$$\int_1^2 dU = \int_{s_1}^{s_2} ks\,ds = \frac{1}{2}ks^2\Big|_{s_1}^{s_2}$$

$$U_{Spring} = -\frac{1}{2}k\left[s_2^2 - s_1^2\right]$$

This work is the green area, the area under the F vs. s curve.

Always write with a neg sign. The s_2 and s_1 values will interact with this sign to produce the correct overall sign for U_{spring}.

Other applications of:

$$\frac{1}{2}mv_1^2 + \int F\,ds = \frac{1}{2}mv_2^2$$

Area under the F-ds curve.

Traditional long bow:

Compound bow:

Other applications of:

$$\frac{1}{2}mv_1^2 + \int F\,ds = \frac{1}{2}mv_2^2$$

Area under the F-ds curve.

Potato gun:

Slingshot: