Work-Energy (WE) for Rigid Bodies

From last class: The WE equation for a **system of particles** also applies to a **system of rigid bodies**.

$$\sum T_1 + \sum U_{1\text{-}2} = \sum T_2$$

Work terms ($\Sigma U_{1\text{-}2}$): The same ones for particles (force, weight, spring) also apply to rigid bodies. But there is one new term, the work of a couple. (Rotation is not defined for particles.)

Work of a force:

$$U_{Force} = (P \cos \theta)(d)$$

Work of weight:

$$U_{Weight} = \pm (mg)(\Delta h)$$

Work of friction:

$$U_{Friction} = -F\,(d)$$

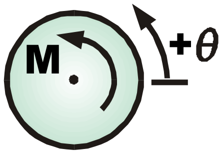
Work of a spring:

$$U_{Spring} = -\frac{1}{2} k \left[s_2^2 - s_1^2 \right]$$

Work of a couple: M = couple or torque, lb-ft or N-m

$$U_M = M \cdot \theta$$

θ = angular displacement, in radians



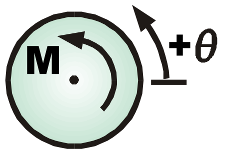
Work-Energy (WE) for Rigid Bodies

More on the work of a couple: **If a couple, M, is a function of θ, like the torsional spring on a mouse or rat trap, the energy stored in the spring is the area under the M vs. θ curve.**

Work of a couple: **M = couple or torque, lb-ft or N-m**

$$U_M = M \cdot \theta$$

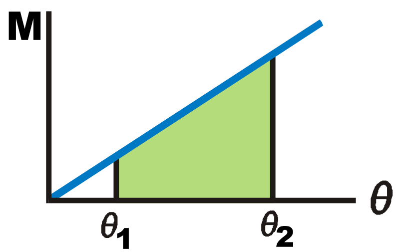
θ = angular displacement, in radians



If the couple, M, varies with θ like for a torsional spring:

$$U_M = \int M \, d\theta$$

Area under M(θ) curve is energy:



Examples:

Mouse trap

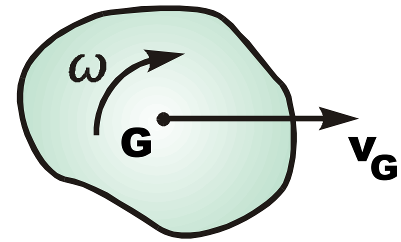
Some hinges

Work-Energy (WE) for Rigid Bodies

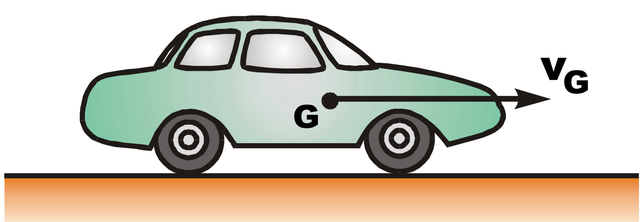
Rigid bodies in general plane motion store kinetic energy in both translation AND rotation, so they have two KE terms.

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

KE stored in translation of mass center | **KE stored in rotation**



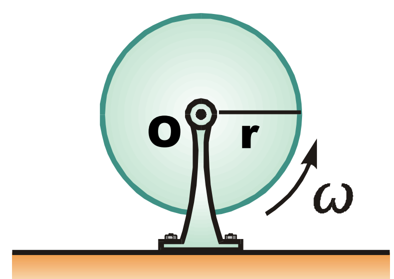
Examples: Translation, FA Rotation



Translation $T = \frac{1}{2} m v_G^2$

(neglecting tire rotation)

Fixed Axis Rotation



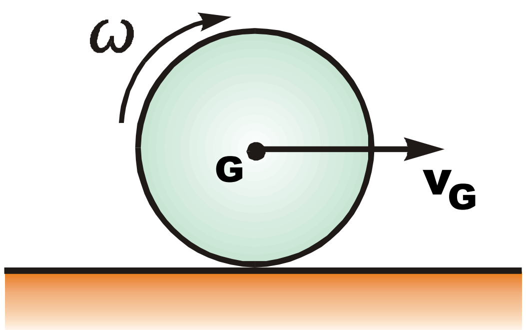
$$T = \frac{1}{2} I_G \omega^2$$

$v_G = 0$ at O.

Examples: Bench grinder
Hand-held grinder
Spin awhile after turned off due to stored KE.

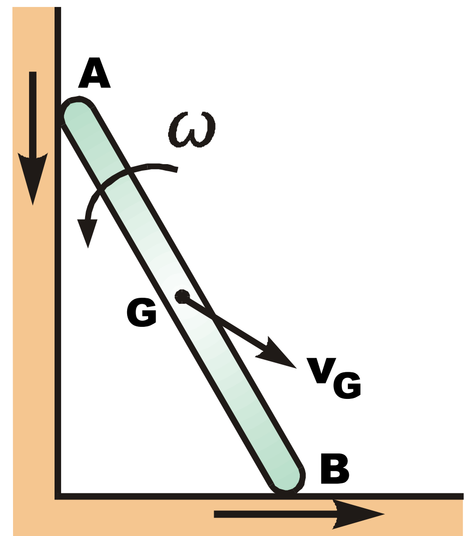
Examples of general plane motion

Rolling Wheel
(Slip or no slip)



$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$$

Bar sliding down wall

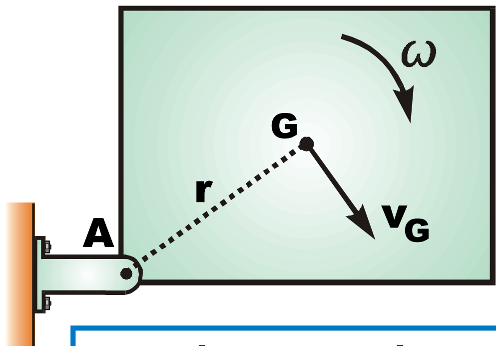


$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$$

Kinetic energy is stored in both the translation of the mass center and the rotation of the body. Kinematics can be a challenge because you need to relate v_G's and ω's.

Example of fixed axis rotation where C_G is not at pin:

Fixed Axis Rotation
c_G not at the pin.

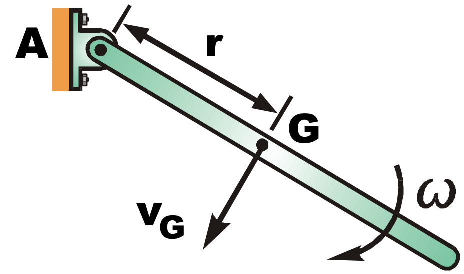


$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$$

Kinematics: $v_G = r\omega$

OR $T = \frac{1}{2}I_{Pin}\omega^2$

Fixed Axis Rotation
c_G not at the pin.



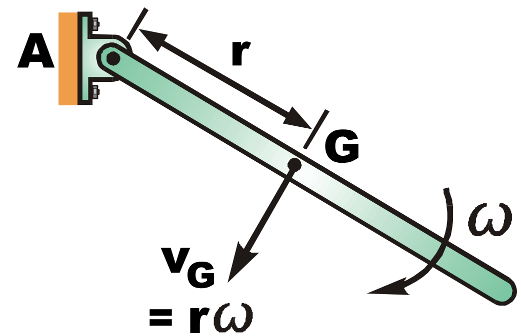
$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$$

Kinematics: $v_G = r\omega$

OR $T = \frac{1}{2}I_{Pin}\omega^2$

These show that a body in fixed axis rotation whose c_G is *not* at the pin will have both v_G and ω terms. However, there is a simpler equation, $T = \frac{1}{2}I_{Pin}\omega^2$, which can be used for this case.

From where do we get the $T = \frac{1}{2} I_{Pin} \omega^2$ equation?



Consider the slender bar:

Its stored KE is given by:

$$T = \frac{1}{2} m v_G^2 + \frac{1}{2} I_G \omega^2$$

Sub in kinematics: $v_G = r\omega$

Factor out $\frac{1}{2}$ and ω^2

$$T = \frac{1}{2}\left[I_G + mr^2\right]\omega^2$$

Parallel Axis Thm: $I_G + mr^2 = I_{Pin}$!

Result:

$$T = \frac{1}{2} I_{Pin} \omega^2$$

Only use this equation for fixed axis rotation where $v_G = r\omega$ applies.