Problem 1 (40pt)
Consider the Set Cover Problem, a set of elements $U = \{u_1, \ldots, u_n\}$, called universe, and a set $I$ of subsets of $U$, that is $I = \{Q_1, \ldots, Q_m\}$, where $Q_i \subseteq U$. We want to find the subset $S^*$ of $I$, with minimum cardinality, such that it covers all the elements in $U$. Formally:

$$S^* = \text{arg} \min_{S \subseteq I} |S|$$

s.t. $U \subseteq S Q_i = U$

Provide the pseudo code of a greedy algorithm for this problem (25pt), and a counter example (15pt).
Problem 2 (60pt)
Consider the following simplified version of the knapsack problem. A knapsack of size W, a set of n items with values v₁,...,vₙ such that the weight of each item is exactly 1. The thief wants to find the set of objects with maximum value.

Provide the pseudo code of the greedy algorithm that selects at each iteration the item with maximum value until the knapsack is full (15pt), and prove that it finds an optimal solution (45pt).