Problem 1 (40pt)
Consider a weighted graph $G = (V,E)$, we want to find a MAXIMUM SPANNING TREE, i.e. a spanning tree of maximum weight. Design a greedy algorithm, based on Kruskal's algorithm, that at each iteration selects the edge with maximum weight. Provide the pseudo code (15pt) and prove the algorithm correctness (25pt).

Pseudo code

Proof of correctness
Termination:
The partial solution is always included in an optimal solution:

The final solution is optimal:
**Problem 2 (20pt)**
Show the execution of the Breadth First Search (BFS) algorithm on the following graph. To this purpose, write the value of the attributes on each node as set by the BFS visit. Clearly highlight the sequence of visited nodes and the resulting BFS tree.
Problem 3 (20pt)
Describe the Minimum Spanning Tree (MST) problem and the Prim’s algorithm for such problem. A complete answer should include:
(i) Definition of the MST problem (5pt)
(ii) Description of the Prim’s algorithm (5pt)
(iii) The Pseudo-code of the Prim’s algorithm (10pt)
Problem 4 (20pt)
Show the execution of the algorithm to find the Strongly Connected Components on the following graph. Clearly highlight the sequence of visited nodes and the resulting strongly connected components.

**Note:** it is not necessary to draw multiple times the graph during the visits.