1 Introduction and Rationale

Dynamic absorbers, also called tuned mass dampers, are widely applied to attenuate vibration at a specific frequency [1]. The terms “tuned mass damper” and “dynamic absorber” may be considered synonymous for the most part, but the former denotes the explicit addition of a loss mechanism – usually a viscous or friction damper. Most often, dynamic absorbers are employed when the frequency where vibration reduction is desired lies at or near a system resonance or natural frequency. Dynamic absorbers have a long history; Den Hartog published an exact solution of system combining both Coulomb (friction) and viscous damping in 1931 [2]. Most of the literature on dynamic absorbers describes their application in buildings and other civil structures such as bridges [3].

Because of the importance of dynamic absorbers, and the fact that few undergraduate students in engineering encounter them in their studies, the development of a small-scale dynamic absorber experiment is proposed here.

1.1 Basic theory

\[
\begin{align*}
M_1 \ddot{x}_1 &= f(t) + C_2 (\dot{x}_2 - \dot{x}_1) + K_2 (x_2 - x_1) - K_1 x_1 - C_1 \dot{x}_1 \\
M_2 \ddot{x}_2 &= -C_2 (\dot{x}_2 - \dot{x}_1) - K_2 (x_2 - x_1)
\end{align*}
\]

or

\[
M_1 \ddot{x}_1 + (C_1 + C_2) \dot{x}_1 + (K_1 + K_2) x_1 - C_2 \dot{x}_2 - K_2 x_2 + f(t) = 0
\]

Assuming \( f(t) = F_0 \sin \Omega t = F_0 \Im \{ e^{j\Omega t} \} \) and thus assuming \( \mathbf{x}(t) = X \Re \{ e^{j\Omega t} \} \), where,

\[
\Re \{ e^{j\Omega t} \} = \sin \Omega t
\]

denotes the Imaginary part of \( e^{j\Omega t} \), we have

\[
\begin{bmatrix}
K_1 + K_2 - M_1 \Omega^2 + j(C_1 + C_2) \Omega & -jC_2 \Omega - K_2 \\
-jC_2 \Omega - K_2 & K_2 - M_2 \Omega^2 + jC_2 \Omega
\end{bmatrix}
\begin{bmatrix}
X_1 \\
X_2
\end{bmatrix} = \begin{bmatrix}
F_0 \\
0
\end{bmatrix}
\]
By Cramer's rule

\[ X_1 = \frac{\det \begin{bmatrix} F_0 & -jC_2\Omega - K_2 \\ 0 & K_2 - M_2\Omega^2 + jC_2\Omega \end{bmatrix}}{\det \begin{bmatrix} K_1 + K_2 + j(C_1 + C_2)\Omega - M_1\Omega^2 - (jC_2\Omega + K_2) \\ -j(C_2\Omega + K_2) & jC_2\Omega - M_2\Omega^2 \end{bmatrix}} \]

(5)

\[ = \frac{(K_2 - M_2\Omega^2 + jC_2\Omega)F_0}{\Delta} \]

(6)

where

\[ \Delta = M_1M_2\Omega^4 - [C_1C_2 + (K_1 + K_2)M_2 + K_2M_1]\Omega^2 + K_1K_2 \]

+ \( j[(K_1C_2 + K_2C_1)\Omega - (M_1C_2 + (C_1 + C_2)M_2)\Omega^3] \)

(7)

\[ \Delta = M_1M_2\Omega^4 - [C_1C_2 + (K_1 + K_2)M_2 + K_2M_1]\Omega^2 + K_1K_2 \]

+ \( j[(K_1C_2 + K_2C_1)\Omega - (M_1C_2 + (C_1 + C_2)M_2)\Omega^3] \)

= \Re \{\Delta\} + j\Im \{\Delta\} \]

(8)

where

\[ \Re \{e^{j\Omega t}\} = \cos \Omega t \]

(9)

is the Real part of \( e^{j\Omega t}\). Hence, the magnitude of \( X_1 \) is given by

\[ |X_1| = F_0 \sqrt{(K_2 - M_2\Omega^2)^2 + C_2^2\Omega^2 \over \Re^2 \{\Delta\} + \Im^2 \{\Delta\}} \]

(10)

where

\[ \Re \{\Delta\} = M_1M_2\Omega^4 - [C_1C_2 + (K_1 + K_2)M_2 + K_2M_1]\Omega^2 + K_1K_2 \]

(11)

and

\[ \Im \{\Delta\} = (K_1C_2 + K_2C_1)\Omega - (M_1C_2 + (C_1 + C_2)M_2)\Omega^3 \]

(12)

### 1.2 Vibration Absorber Numerical Example

The following is an example of the amount of vibration attenuation possible through the use of a dynamic absorber. A numerical simulation was performed via Maple 16 using the following parameters:

\( M_1 = 10 \) kg, \( K_1 = 3,947,842 \) N/m, \( C_1 = 3,948 \) N-s/m

\( M_2 = 2 \) kg, \( K_2 = 7,895,681 \) N/m, and

\( F_0 = 3,948 \) N.

The natural frequency before the addition of the dynamic absorber was \( f_n = 100 \) Hz.

The operating frequency was 100 Hz.

Resonance frequencies after the addition of the absorber were: \( f_1 = 80 \) Hz, and \( f_2 = 124 \) Hz.

A remarkable 99% reduction at 100 Hz was predicted by the simulation.

Based on Equation (10), it would seem that the amplitude of \( x_1(t) \) is mostly a function of the ratio of \( K_2 \) to \( M_2 \) and \( C_2 \). If that were the case, one could pick a tiny absorber to put on a large machine or even a building as long as the correct absorber stiffness to mass ratio were correct, and fully expect it to absorb all of the vibrational energy of the original structure! Alas, life is not so simple... The absorber mass and stiffness must be reasonably large in order to separate the resulting two modes sufficiently, but more importantly, they must be chosen large enough to absorb an appreciable amount of the kinetic and potential energy of the original structure.
Figure 2. Amplitude of $x_1(f)$ before and after addition of vibration absorber.

Figure 3. Schematic of proposed small-scale dynamic absorber experimental apparatus.
2 Experimental Plan

A small-scale dynamic absorber experimental apparatus will be designed and built to attach to the small shaker in the ME242 lab. A schematic of the proposed system is shown in Figure 3. The system will be constructed using readily obtainable parts and materials. Hacksaw blades will be used for the absorber spring, and small movable weights will be constructed to clamp on to the hacksaw blades to allow precise positioning. The spring stiffness will thus be controlled by the distance between the absorber mass and the spring (hacksaw blade) attachment according to the approximate stiffness formula:

\[ K_b \approx \frac{3EI}{L^3} \]  

(1)

where \( E \), \( I \), and \( L \), are the Young’s modulus, second moment of area of the beam cross section, and distance between the spring attachment and the mass, respectively.

To determine the effect of absorber mass, the system will be tested using three different masses: 10 grams, 20 grams, and 30 grams. The basic theory, as detailed in section 1.1 does not predict any difference in attenuation as long as the stiffness to mass ratio is held constant. However, energy considerations predict greater attenuation with increasing mass, and this is the hypothesis that will be tested in this study. The resulting resonances and before-and-after vibration amplitude will be recorded, and the resulting percent attenuation calculated.

3 Budget

Table 1. Bill of required supplies and materials.

<table>
<thead>
<tr>
<th>Item</th>
<th>Source</th>
<th>Cost</th>
<th>Number Required</th>
<th>Subtotal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hacksaw Blades: Greenlee-333-1232-Bi-Metal-Hacksaw-12-Inch</td>
<td>Amazon¹</td>
<td>$12.55/5 pack</td>
<td>1</td>
<td>$12.55</td>
</tr>
<tr>
<td>1/2 in. socket head 5-40 cap screws, part #60662061</td>
<td>MSC Industrial Supply²</td>
<td>$11.96/100</td>
<td>1</td>
<td>$11.96</td>
</tr>
<tr>
<td>1.75&quot; 3/8&quot; Dia hex head flange bolt</td>
<td>MSC³</td>
<td>$37.71/100</td>
<td>1</td>
<td>$37.71</td>
</tr>
<tr>
<td>0.25&quot;x1.25&quot; stainless steel bar stock</td>
<td>Online Metals⁴</td>
<td>$10.47</td>
<td>1</td>
<td>$10.47</td>
</tr>
<tr>
<td>Miscellaneous aluminum stock and steel bolts</td>
<td>ME Shop</td>
<td>$0.00</td>
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<td>$0.00</td>
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<td>Poster production</td>
<td>S&amp;T Library</td>
<td>$10.00</td>
<td>1</td>
<td>$10.00</td>
</tr>
<tr>
<td><strong>Total:</strong></td>
<td></td>
<td><strong>$82.69</strong></td>
<td></td>
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</tr>
</tbody>
</table>

References


[Online]. Available: http://dx.doi.org/10.1016/j.jsv.2008.01.014

¹http://www.amazon.com/Greenlee-333-1232-Bi-Metal-Hacksaw-12-Inch/dp/B001HWALZE/ref=sr_1_3?s=hi&ie=UTF8&qid=1378917173&sr=1-3&keywords=hacksaw+blades+32+tpi
²http://www.mscdirect.com/product/60662061
³http://www.mscdirect.com/product/60078359
⁴http://www.onlinemetals.com/merchant.cfm?pid=20556&step=4&showunits=inches&id=943&top_cat=1
⁵Exclusive of shipping charges for parts.