EQUATIONS AND CONSTANTS

CONSTANTS

g_c = 32.2 lbm-ft/lbf-sec^2
K_{air} = 1.395

R = 53.34 lbf-ft/lbm- ˚R
C_p air = 0.240 BTU/lbm- ˚R

C_p water at 70EF = 0.998 BTU/lbm-˚F

C_p water at 160EF = 1.000 BTU/lbm-˚F

1 HP = 42.4 BTU/min = 0.746 Kwatts = 550 ft-lbf/sec

1 Watt = 1 J/sec = 0.05688 BTU/min

1 BTU = 778 ft-lbf

1 in. H_20 = 0.03611 lbf/in^2 = 5.199840 lbf/ft^2

1 in.hg. = 0.491 lbf/in^2

1 gal = 0.1337 ft^3

1 ton ref. = 200 BTU/min

1 BTU/lbm-EF = 1 cal/g-˚C

1 slug = 32.2 1bm

ρ g H_2O = 62.4 lbf/ft^3

air at STP = 0.07654 lbm/ft^3

T(˚C) = 5/9(˚F-32)

T(˚R) = T(˚F) + 460

T(˚K) = T(˚C) + 273

˚F = degrees Fahrenheit

˚C = degrees Celsius

˚K = degrees Kelvin

˚R = degrees Rankine

DYNAMIC BALANCING EQUATIONS

$$\Delta A_1 = \sqrt{\frac{(A_1^0)^2 + (A_1^{180})^2 - 2A^2}{2}} = \sqrt{\frac{(A_1^{90})^2 + (A_1^{270})^2 - 2A^2}{2}}$$

$$\Delta A_2 = \sqrt{\frac{(A_2^0) + (A_2^{180})^2 - 2A^2}{2}} = \sqrt{\frac{(A_2^{90})^2 + (A_2^{270})^2 - 2A^2}{2}}$$

$$\Delta B_1 = \sqrt{\frac{(B_1^0)^2 + (B_1^{180})^2 - 2B^2}{2}} = \sqrt{\frac{(B_1^{90})^2 + (B_1^{270})^2 - 2B^2}{2}}$$

$$\Delta B_2 = \sqrt{\frac{(B_2^0)^2 + (B_2^{180})^2 - 2B^2}{2}} = \sqrt{\frac{(B_2^{90})^2 + (B_2^{270})^2 - 2B^2}{2}}$$

ψ = The angle of original unbalance

$$\psi = Tan^{-1}\left[\frac{A_1^{90})^2 - (\Delta A_1)^2 - A^2}{(A_1^0)^2 - (\Delta A_1)^2 - A^2}\right]$$

$$\psi_B = Tan^{-1}\left[\frac{(B_1^{90})^2 - (\Delta B_1)^2 - B^2}{(B_1^0)^2 - (\Delta B_1)^2 - B^2}\right]$$

$$\overline{M}_A = \overline{R}_A M_{trial} \qquad \overline{M}_B = \overline{R}_B M_{trial}$$

$$\overline{R}_A \Delta A_1 + \overline{R}_B \Delta A_2 = -\overline{A} \qquad \overline{R}_A \Delta B_1 + \overline{R}_B \Delta B_2 = -\overline{B}$$

$$R_{Ax} = \frac{B(\Delta A_2)\cos\psi_B - A(\Delta B_2)\cos\psi_A}{(\Delta A_2)(\Delta B_2) - (\Delta A_2)(\Delta B_1)} \qquad R_{Bx} = \frac{-A\cos\psi_A - R_{Ax}\Delta A_1}{\Delta A_2}$$

$$R_{Ay} = \frac{B(\Delta A_2)\sin\psi_B - A(\Delta B_2)\sin\psi_A}{(\Delta A_1)(\Delta B_2) - (\Delta A_2)(\Delta B_1)} \qquad R_{By} = \frac{-A\sin\psi_A - R_{Ay}\Delta A_1}{\Delta A_2}$$

Beam Data:

$E = 69GPa$

$\rho_b = 2700 \frac{kg}{m^3}$

$L_b = 30cm$

$b = 0.0127m$ (Width of the beam)

$h_b = 0.00317m$ (Thickness of the beam in the bending direction)

$\beta_n L_b = 1.875$

Equations:

$$f_n = \frac{(\beta_n L_b)^2}{2\pi L_b{}^2} \sqrt{\frac{EI}{\rho_l}}, \; I = \frac{b h_b{}^3}{12}, \; \rho_l = \rho_b b h_b$$

$$U_n(x) = \cosh(\beta_n x) - \cos(\beta_n x) + \left(\frac{B_2}{B_1}\right)_n (\sinh(\beta_n x) - \sin(\beta_n x))$$

$$\left(\frac{B_2}{B_1}\right)_n = \frac{\sin(\beta_n L_b) - \sinh(\beta_n L_b)}{\cos(\beta_n L_b) + \cosh(\beta_n L_b)}$$

$$f_{mass-added} \approx \frac{f_n}{\sqrt{1 + \frac{M U_n(x^*)^2}{M_b}}}$$

$$\delta_k = \ln\left(\frac{x_1}{x_k}\right), \; \zeta = \frac{\delta_k}{\sqrt{\delta_k{}^2 + 4(k-1)^2 \pi^2}}$$

$$f_d = f_n \sqrt{1 - \zeta^2}$$

1-D TRIANGULAR FIN

1-D Fin Equation

$$\frac{d^2\theta}{dx^2} + \frac{1}{x} \cdot \frac{d\theta}{dx} - \frac{\theta}{x} \cdot p^2 = 0$$

where $\theta = T - T_\infty$, $\frac{x'}{L'} = x$, $L = \frac{L'}{l}$, $Bi = h \cdot \frac{l}{k}$ and $p = \sqrt{BiL^2 f}$

x = 0, θ = finite

x = 1, $\theta = \theta_0 = T_w - T_\infty$

$$\frac{\theta}{\theta_0} = \frac{I_0(2\sqrt{Bx})}{I_0(2\sqrt{B})} \text{ where } B = \frac{hl}{k} L^2 \sqrt{\frac{1}{L_2} + 1}$$

Least Squares Methodology

General

$F_i = A \cdot f_i$

$\rho_i = F_i - (A \cdot f_i)$

$S = \sum \rho^2$

$= \sum f_i^2 - 2A \sum F_i f_i + \sum A^2 f_i^2$

$\frac{\partial S}{\partial A} = 0$ results in

$$A = \frac{\sum F_i f_i}{\sum f_i^2}$$

$$\sigma^2 = \frac{\sum (\rho_i - \overline{\rho})^2}{n}$$

$$\overline{\rho} = \frac{\sum \rho_i}{n}$$

Specific

$\theta_i = D \cdot I_0(2\sqrt{Bx_i}) = D \cdot I_i$

$\rho_i = \theta_i - D \cdot I_i$

$S = \sum \rho^2$

$= \sum \theta_i^2 - 2D \sum \theta_i I_i + D^2 \sum I_i^2$

$\frac{\partial S}{\partial D} = 0$ results in

$$D = \frac{\sum \theta_i I_i}{\sum I_i^2} = \frac{\sum \theta_i I_0(2\sqrt{Bx_i})}{\sum I_0^2(2\sqrt{Bx_i})}$$

$$\sigma^2 = \frac{\sum (\rho_i - \overline{\rho})^2}{n}$$

$$\overline{\rho} = \frac{\sum \rho_i}{n}$$

ACOUSTICS

$$\rho = 1 - \alpha ; c = f \lambda ; SWR = 10^{\frac{\Delta SPL}{20}} , c^2{}_{THEORY} = KTRg_c , U = \frac{p}{\rho c}$$

$$p_t = p_i + p_r = A \sin \omega t + B \sin (\omega t - 2\frac{\omega}{c} x - \theta)$$

$$| p_t | = \sqrt{A^2 + B^2 + 2AB \cos (2\frac{\omega}{c} x + \theta)}$$

$$\alpha_n = \frac{A^2 - B^2}{A^2} = 1 - \frac{B^2}{A^2}$$

$$| p_t |_{MAX} = \sqrt{A^2 + B^2 + 2AB} = A + B$$

$$| p_t |_{MIN} = \sqrt{A^2 + B^2 - 2AB} = A - B$$

$$\alpha_n = \frac{4 | p_t |_{MAX} | p_t |_{MIN}}{(| p_t |_{MAX} + | p_t |_{MIN})^2}$$

$$L_{p,max} - L_{p,min} = 20 \log_{10} \left[\frac{| P_t |_{max}}{| P_t |_{min}} \right] dB$$

$$SWR = \frac{p_{MAX}}{p_{MIN}} = \frac{A + B}{A - B} = \frac{1 + B/A}{1 - B/A} = 10^{\frac{\Delta SPL}{20}}$$

$$\alpha_n = \frac{4 SWR}{(1 + SWR)^2}$$

PUMP EXPERIMENT

$$\dot{m} = \rho V A$$

$$H = \frac{1}{\rho g} (P_{out} - P_{in})$$

$$P_w = \rho g Q H$$

$$P_s = 2 \pi N T$$

$$\eta = \frac{P_w}{P_s}$$

$$\eta_v = \frac{Q}{Q + Q_L}$$

$$u_d = \sqrt{u_o^2 + u_c^2}$$

$$u_y = \sqrt{\left(\frac{\partial f}{\partial x_1} u_{x_1}\right)^2 + \left(\frac{\partial f}{\partial x_2} u_{x_2}\right)^2}$$