

PUMP EXPERIMENT

Technical Advisor: Dr. J. A. Drallmeier

January 1996

1.1 OBJECTIVES

- 1) To determine the typical operating characteristics of an external gear pump and a rotary screw pump under various load and speed conditions.
- 2) To understand instrument accuracy and design error analysis.

1.2 BACKGROUND

In general, pumps may be divided into two major categories:

- 1) Dynamic, in which energy is continuously added to the fluid by means of fast-moving blades or vanes.
- 2) Positive displacement, in which energy is periodically added by application of force to one or more fluid volumes, resulting in an increase in pressure.

Dynamic pumps usually require some type of priming system to begin the pump's operation. An example of a dynamic pump is a centrifugal type, shown below, which depends on centrifugal force for the pumping action.

Fluid enters the pump through this eye of the impeller and travels outward between the vanes of the impeller to its edge. At this point fluid enters the casing of the pump and is discharged through the exit port. An increase in fluid pressure is obtained by the rotary action of the impeller. In the same way that fluid pressure increases toward the edge of a rotating tank due to the radial acceleration of the fluid (as studied in your undergraduate fluid mechanics course), the fluid pressure rises in the casing.

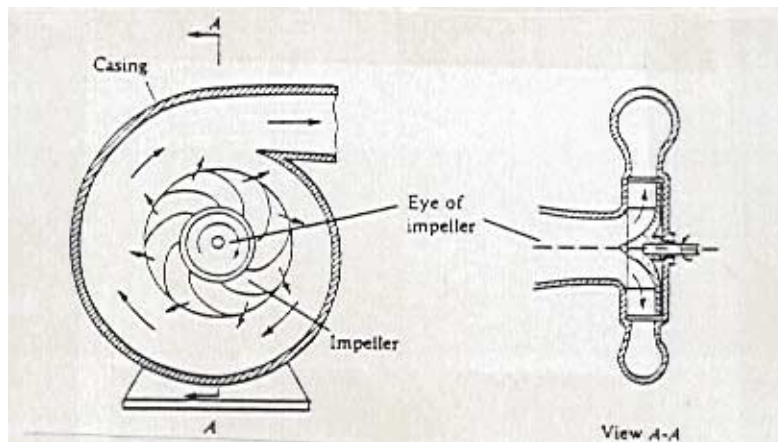


Figure 1 Centrifugal Pump.

Centrifugal pumps are typically used where high volumetric flow rates at low pressure are needed.

Positive displacement pumps are in general self-priming and may require some type of mechanical bypass valve to protect the system from excessive pressures. This class of pumps is further divided into reciprocating or piston type and rotary type. Both the gear pump and the rotary screw pump used in this experiment falls into the class of positive-displacement rotary pumps.

Rotary pumps are typically used for high pressure, low flow rate, applications. Excessive leakage is in many cases a problem since no satisfactory method of sealing the moving surfaces to compensate for wear has been developed. Consequently, rotary pumps are best suited for pumping oils and other liquids having lubricating value and sufficient viscosity to prevent excessive leakage.

The gear pump you will test in this experiment is an external gear type. The pump consists of two spur gears inside an enclosure and in mesh with each other, as shown in the figure below.



Figure 2 Gear pump.

The liquid is carried in the pockets formed between the gear teeth and the housing as the two gears rotate. The volume flow per revolution is approximately equal to the total volume of the liquid pockets. The place where the two gears mesh serves as a seal in addition to the tooth-casing interface, to prevent back flow from the high pressure discharge to the low pressure intake.

The rotary screw or progressive cavity pump is a positive displacement pump designed to handle solid particles in suspension such as abrasive slurries, water base paints, dirty oils, etc. The operating principle can be understood in reference to Figure 3. The rotor has an eccentric axial pitch while the elastic stator has an axial pitch one half that of the rotor. The rotor and stator form a constant sealing line creating a series of sealed cavities. When the shaft rotates, these sealed cavities progress axially down the stator to the discharge end. Meanwhile the opened cavities at the suction end pull in new material. Both the gear pump and the rotary screw pump suffer from pump leakage at high-back pressures due to imperfect sealing at the sealing interfaces.

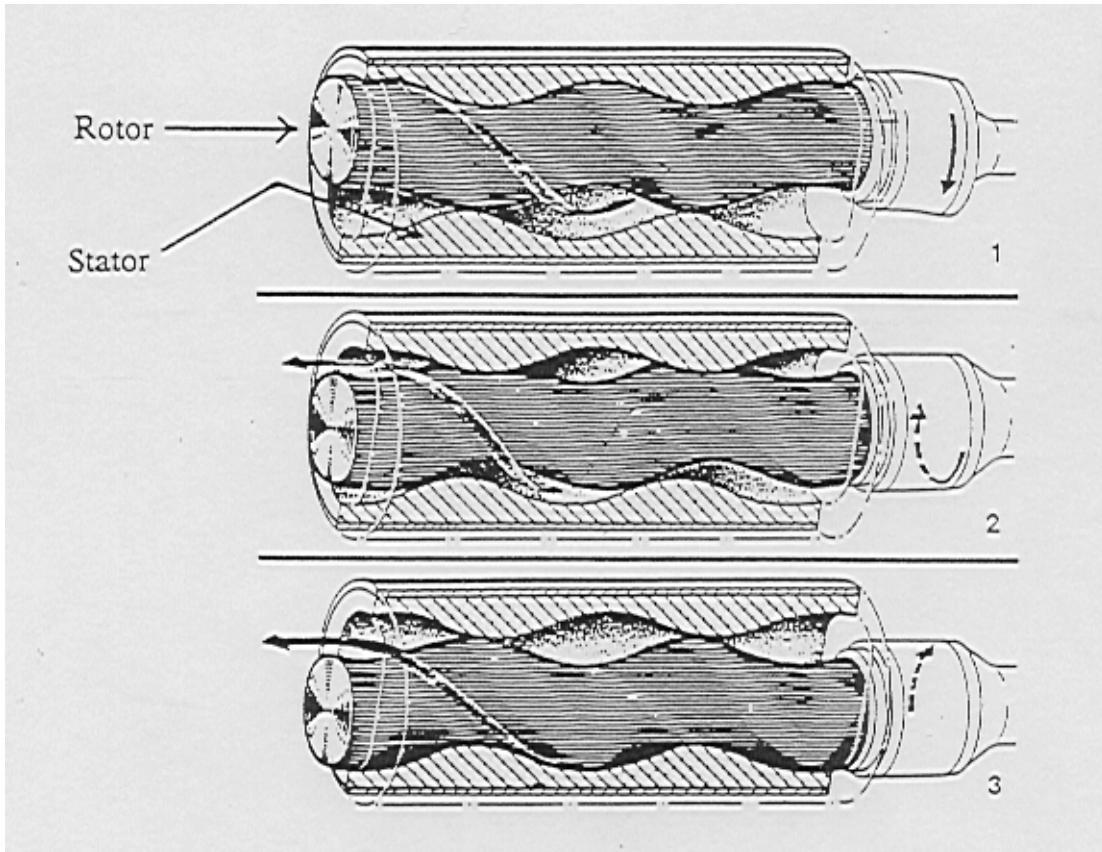


Figure 3 Rotary screw pump.

1.2 THEORY

Consider the energy transfer of a pump by applying the steady state form of the energy equation:

$$\dot{Q} + \dot{W}_x = \dot{m}_{out} \left[h + \frac{V^2}{2} + gz \right]_{out} - \dot{m}_{in} \left[h + \frac{V^2}{2} + gz \right]_{in} \quad (1)$$

By definition the enthalpy, h , is

$$h = u + \frac{P}{\rho} \quad (2)$$

and by conservation of mass

$$\dot{m}_{in} = \dot{m}_{out} \quad (3)$$

This implies that on a per unit mass basis,

$$q + w_s = \left[u + \frac{P}{\rho} + \frac{V^2}{2} + gz \right]_{out} - \left[u + \frac{P}{\rho} + \frac{V^2}{2} + gz \right]_{in} \quad (4)$$

Since the flow is considered incompressible, the inlet and outlet velocities are equal by Equation(3) assuming that the inlet and outlet areas are approximately equal. In addition, negligible change in potential energy occurs across the pump, resulting in

$$\left(\frac{P}{\rho g} \right)_{out} - \left(\frac{P}{\rho g} \right)_{in} = \frac{W_s}{g} - \frac{u_{out} - u_{in} - q}{g} \quad (5)$$

The last group of terms represent frictional losses and are grouped into a frictional head loss term, h_f . Similarly, expressing the shaft work put into the pump as a shaft work head term, h_s , gives

$$H = \frac{1}{\rho g} (P_{out} - P_{in}) = h_s - h_f \quad (6)$$

H is the "net" or "pump" head and is the primary output parameter for a pump. Note that this is the difference between the shaft work and the frictional head.

The energy ultimately delivered to the fluid results from the difference between the input energy and the energy lost to friction, both mechanical and viscous, and losses due to pump leakage. The power delivered to the fluid is by tradition termed the "water horsepower",

$$P_w = \rho g Q H \quad (7)$$

where Q is the volumetric flow rate of the fluid. In contrast, the power required to drive the pump is termed the "brake horsepower"

$$P_s = 2\pi N T \quad (8)$$

where T is the measured shaft torque and N is the shaft speed in revolutions per minute.

The overall efficiency of the pump is defined as the ratio between the power delivered to the fluid and the input power,

$$\eta = \frac{P_w}{P_s} = \frac{\rho g Q H}{2\pi N T} \quad (9)$$

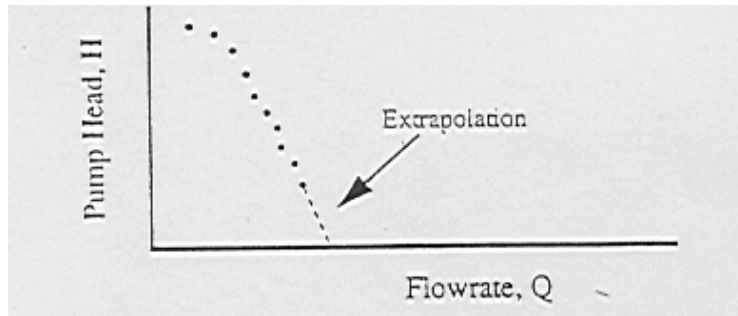
This overall efficiency is a function of both mechanical and fluid mechanical losses.

Contributing factors to the overall inefficiency are viscous frictional effects and mechanical frictional effects in the bearings, packing and other contact points in the pump.

For both the gear pump and the rotary screw pump we will also consider the volumetric efficiency, which compares the actual pumped fluid flow rate to the loss of flow, Q_L , due to leakage of the fluid around the gear and casing. The volumetric efficiency is defined as,

$$\eta_v = \frac{Q}{Q + Q_L} \quad (10)$$

The leakage flow rate is defined as the difference between the flow rate at a given head and speed and the flow rate at that speed if no head were developed, Q_0 (i.e. 0 back pressure). You will be collecting data on the head developed by the pump for various back pressures for a fixed speed. Hence, to find Q_0 , a plot of flow rate versus head need simply be extrapolated to zero head as shown below:



For lower head valves, the data will be fairly linear. Thus a method of linear least squares can be used to accurately extrapolate the plot to zero head. The general nature of the approach will be discussed here and applied in the discussion section.

If we assume the flow rate to be linearly proportional to the head (this should be true for most of your data at lower head values), then we can write:

$$Q = AH + Q_0 \quad (11)$$

where A is some constant. The method of linear least squares is based on finding the equation of a line (i.e. A and Q_0) which best fits the data by minimizing the sum of the square of the residual. The residual, ρ , is defined by

$$\rho = AH + (Q_0 - Q). \quad (12)$$

Hence we wish to minimize the sum of the square of residual, S,

$$\begin{aligned}
 S &= \sum_{i=1}^N \rho_i^2 = \sum_{i=1}^N (AH_i + (Q_0 - Q_i))(AH_i + (Q_0 - Q_i)) \\
 &= A^2 \sum_{i=1}^N H_i^2 + 2A \sum_{i=1}^N H_i(Q_0 - Q_i) \\
 &\quad + \sum_{i=1}^N (Q_0 - Q_i)^2.
 \end{aligned} \tag{13}$$

Here N is the number of data points.

To give us the best fit of our line (Equation 11) in the least squares sense, we must set the derivatives of Equation 13 to zero, or,

$$\frac{\partial S}{\partial Q_0} = 0, \quad \frac{\partial S}{\partial A} = 0. \tag{14}$$

Doing so gives,

$$A \sum_{i=1}^N H_i^2 + Q_0 \sum_{i=1}^N H_i = \sum_{i=1}^N H_i Q_i \tag{15}$$

$$A \sum_{i=1}^N H_i + N Q_0 = \sum_{i=1}^N Q_i. \tag{16}$$

This gives us two equations and two unknowns by which we can solve for A and the flow rate at zero head, Q_0 . Remember, apply this approach only for as much of the data as you can at lower head values over which the data appear linear. Obviously, the more data that can be used in the linear least squares, the better the result.

1.3 EXPERIMENTAL SETUP

A schematic of the apparatus used is given in Figure 4. The gear pump you will test is driven by a variable speed motor, through a coupling that gives a read out of torque and rotational speed. The torque is sensed by measuring torsional strain, using electrical resistance strain gages, and transmitting the signal through slip rings.

The liquid flowing through the pump is water. There are pressure gages to measure static pressure at the inlet and outlet of the pump. There is a valve on the outlet side of the pump to control the discharge pressure. Turning the valve handle clockwise closes the valve and increases the discharge pressure.

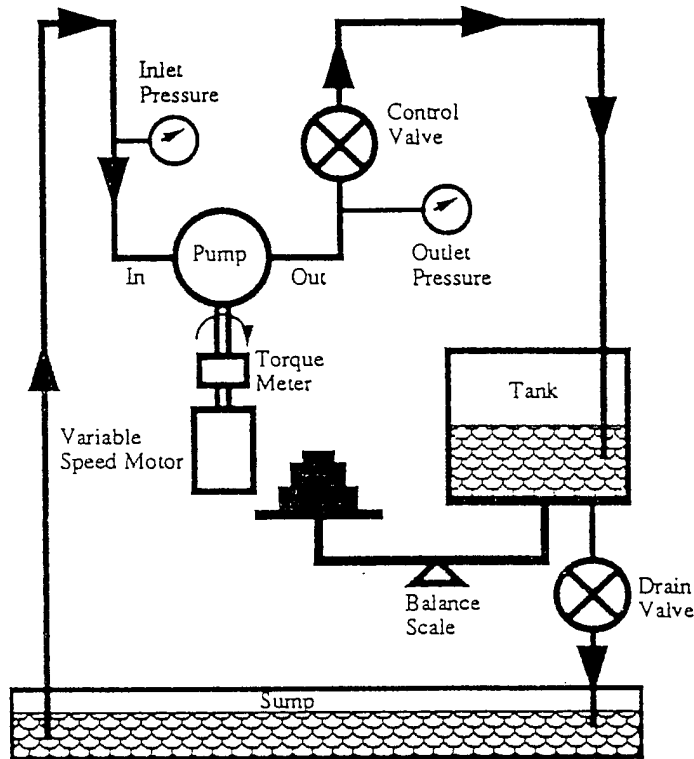


Figure 4. Pump experiment apparatus.

The flow rate is measured by passing the water into a tank mounted on a scale. A stopwatch will be used to determine the time for the scale reading to change by a noted amount (50 lb_m is suggested.)

1.4 DATA ACQUISITION PROCEDURE

A) Gear Pump

- 1) Familiarize yourself with the pressure gage scales, balance and stopwatch operation.
- 2) Check and confirm that the Outlet Pressure control valve is fully open (counterclockwise). Switch the variable speed motor ON.
- 3) Adjust the speed of the motor to 900 rpm.
- 4) Adjust the outlet pressure to 10 psig.
- 5) Set the initial balance on the scale such that the scale is balanced with the drain valve open.
- 6) Record the motor speed, inlet and outlet pressures, input torque and the time taken

to collect a given amount of water (suggest 50 lb_m).

- 7) Increase the outlet pressure by closing the control valve and repeat step 6). Continue until values have been recorded for outlet pressures shown on the data sheets.
- 8) Increase the pump speed and repeat steps 3) through 7) for 1300 rpm.

Caution: In step 7, when the outlet pressure is increased, check to be sure that the motor speed is still at the set point.

B) Rotary Screw Pump

- 1) Remove the gear-pump and replace it with the rotary screw pump. (Be sure that the quick disconnects are fully locked!).
- 2) Run the pump at 1500 RPM for one minute to insure stable operation.
- 3) We will conduct this test similar to the gear pump tests by fixing the speed and varying the outlet pressure. However, you will find that this pump cannot withstand as high back pressures as the gear pump before cavitation begins in the pump. Set the initial speed to 900 RPM.
- 4) Set the balance on the scale such that the scale is balanced with the drain valve open.
- 5) Record the motor speed, inlet and outlet pressures, input torque and the time taken to collect a given amount of water (use 50 lbs).
- 6) Increase the outlet pressure by closing the control valve and repeat step 5). Continue until values have been recorded for outlet pressures as shown on the data sheets.
- 7) Increase the pump speed and repeat steps 4) through 6) for 1300 rpm.

1.5 UNCERTAINTY ANALYSIS

For the pump experiment we would like to pay special attention to experimental errors and how we account for them in the final analysis. In this light, the next few sections are devoted to understanding sources of error and how to properly account for these.

1.5.1 Random Versus Systematic Errors

Experimental uncertainty or error can generally be divided into two categories; random and systematic. Random errors are associated with the random fluctuation of repeated measurements. True random errors are subject to statistical analysis and will typically follow a

normal distribution. Common examples of random error sources are insufficient equipment sensitivity and environmental effects such as temperature fluctuations. The second category is systematic errors. Systematic errors are due to effects that influence the result but with repeated measurements are not subject to statistical analysis. These are also called bias errors in that often their value remains roughly constant during a series of measurements under fixed operating conditions. Examples here include calibration errors, improper meter reading techniques or instrument resolution. Often systematic errors are established based on the instrument's manufacturer specifications and by the experience and intuition of the experimenter.

In an error analysis, random and systematic errors are treated differently. However, it is sometimes difficult to clearly establish whether an error is random or systematic. The best rule of thumb is to treat an uncertainty as a random error, if it can be treated statistically, otherwise treat it as a systematic error.

1.5.2 Propagation of Error

The measurement of each parameter in the lab (e.g. pump speed, pressure, etc.) has one or more errors associated with it. For example, in using a pressure transducer for making a pressure measurement, we may have a systematic error due to the resolution of the device in addition to random errors associated with fluctuation environmental conditions. To then estimate the total uncertainty associated with the pressure measurement we must have some scheme for combining these errors. The technique we will use is termed the root-sum-squares (RSS) method. The uncertainty, u_x , for the measurement of variable x is given by root-sum-square of all of the errors associated with that measurement,

$$u_x = \pm \sqrt{e_1^2 + e_2^2}$$

where for the previous example e_1 would be the systematic error and e_2 would be the random error due to fluctuating environmental conditions.

Often in experimental work, measurements of several variables (e.g. torque, speed, pressure, etc.) are combined using some functional relationship to give a final answer (e.g. water horsepower). We will again use the RSS method to combine all of the errors associated with the

different measurements. Let us assume we wish to calculate y and its associated uncertainty u_y based on measurements of x_1 , x_2 and x_3 :

$$y = f(x_1, x_2, x_3)$$

The best value for y would be based on the mean values for x_1 , x_2 , and x_3 . The uncertainty in y , u_y , is calculated by

$$u_y = \pm \sqrt{\left(\frac{\partial f}{\partial x_1} u_{x_1}\right)^2 + \left(\frac{\partial f}{\partial x_2} u_{x_2}\right)^2 + \left(\frac{\partial f}{\partial x_3} u_{x_3}\right)^2}.$$

The partial derivatives are often called sensitivities since they represent the sensitivity of the final result, y , to the measured parameter. As a final note, one must be very careful to use consistent units in this calculation.

1.5.3 Design Error Analysis

For this experiment we will only consider errors due to the measurement system's resolution. These instrumentation errors will be treated as systematic errors since we are unable to treat them statistically. Random errors will not be considered simply because we do not have the lab time to obtain statistically significant sample sizes for the parameters of interest. This brief discussion on design error analysis is focused on understanding how the limitations of an existing system, impacts the accuracy of the results. However, the strategy employed here could also be used to evaluate the accuracy of a system before it is built (design stage).

We will consider two types of errors that will contribute to the uncertainty of a parameter measured with a particular instrument: interpolation errors, u_o , and instrument errors, u_c . An interpolation error is due to the finite ability to resolve the information provided by the instrument. The resolution of the information is typically dependant on the scale provided on the instrument. The rule of thumb for interpolation error is to use $\frac{1}{2}$ of the resolution. For example, if a pressure gage has increments of 1 psi, the interpolation error, u_o , is $\pm\frac{1}{2}$ psi.

Instrument errors are based on the manufacturer's statement as to the accuracy of the instrument. Here we are depending on the experience of the manufacturer to establish the uncertainty. Often the uncertainty given in the specifications for the instrument will be in several parts (e.g. linearity, hysteresis, repeatability, etc.). What the manufacturer is saying is that all of these uncertainties will contribute to the total uncertainty in the reading. All of these instrument errors must then be combined using RSS (using the same units) to give the total instrument error, u_c .

Once we have established the interpolation and instrument errors, the design stage uncertainty for that particular measurement, u_d , is given by

$$u_d = \sqrt{u_o^2 + u_c^2}.$$

All of the design stage uncertainties from all of the measured parameters should then be combined as outlined in Section 1.5.2 to give the final uncertainty in the final answer.

Just as a final note, if a large enough number of samples of, say, pressure data were taken to account for random errors, these would simply be combined with the design stage uncertainty using the RSS method. Typically the uncertainty associated with the random errors would be based on the 95% confidence interval.

1.6 DATA REDUCTION AND DISCUSSION

A. To illustrate the accuracy of your results, develop:

- 1) A table showing inlet pressure, outlet pressure, flow rates, torques, heads, water horsepower, shaft horsepower for each motor speed and pump (see Table 1)
Note: Be sure to include with each measured value the design stage uncertainty based on the instrument specs given in Table 2. Also, for each calculated value, include the uncertainty based on the methods outlined in Section 1.5
Note: The error for the flow rate value should be estimated at 5% of the flow rate value since we do not have enough data for a statistical analysis.
- 2) A handwritten appendix of one sample calculation of each variable (including uncertainty calculations).
- 3) Let us assume that a design stage uncertainty of less than 15% for each measured value is acceptable. Based on this, which, if any, instruments are NOT acceptable for this experiment?

B. To describe the operating characteristics of each pump, plot:

- 1) The head developed by the pump, H (ft), versus volumetric flow rate, Q (ft^3/min) for both pumps. Plot this for all speeds on the same plot. On these plots include errors bars for each head data point as estimated from your error analysis. If your plotting routine does not allow you to do this, put them on by hand. Based on these results, could both pumps meet the needs of a 100 ft head at $0.6 \text{ ft}^3/\text{min}$? If so, what should the approximate operating speeds be?
- 2) Leakage flow rate versus total head. Recall the leakage flow rate, Q_L (ft^3/min), is defined as the difference between the flow rate at a given total head and speed and the flow rate at that speed if there were no head developed (i.e. 0 back pressure). The flow rate at zero head can be determined by using your plot of head versus flow rate and extrapolating to 0 head using the method of least squares outline in Section 1.2. Discuss why leakage flow increases with increasing head.
- 3) Pump volumetric efficiency versus volumetric flow rate for both pumps and speeds on the same plot.
- 4) Pump overall efficiency versus volumetric flow rate for both pumps at both speeds on the same plot. Under what conditions (speed, head and flowrate) is each pump best suited?

| Motor Speed | Inlet Pressure | Outlet Pressure | Torque | Water mass flow rate | Pump Head | Water Power | Shaft Power |
|-------------|----------------|-----------------|------------|----------------------|-----------|---------------|---------------|
| 900±1/2 rpm | -2±0.5 in Hg | 10±1.6 psig | 95±3 in-oz | 0.6±0.03 lbs/s | 25±3.7 ft | 0.03±0.004 Hp | 0.09±0.003 Hp |
| | | | | | | | |
| | | | | | | | |

Table 1. Sample Data Table

| | Motor Speed | Inlet Pressure | Outlet Pressure | Torque |
|-------------------------------|-------------|--------------------|--------------------|-------------------------|
| Resolution: | 1 rpm | 0.5" Hg | 1 psi | 1 in-oz |
| Accuracy: | | 1.5% of full scale | 1.5% of full scale | |
| Nonlinearity: | | | | 0.05% of rated capacity |
| Hysteresis: | | | | 0.1% of rated capacity |
| Repeatability: | | | | 0.05% of rated capacity |
| Zero Balance: | | | | 0.3% of rated capacity |
| Full Scale or Rated Capacity: | | 30 in Hg vacuum | 100 psig | 1,000 in-oz |

Table 2. Instrument Specifications

PUMP EXPERIMENTAL DATA SHEET

Date: _____

| Run No. | Motor Speed (RPM) | Inlet Pressure (in. Hg) | Outlet Pressure (PSIG) | Torque (in.-oz.) | Mass Water (lbm) | Time (sec) |
|-------------------|-------------------|-------------------------|------------------------|------------------|------------------|------------|
| Gear Pump | | | | | | |
| 1 | 900 | | 5 | | 50 | |
| 2 | | | 10 | | | |
| 3 | | | 20 | | | |
| 4 | | | 30 | | | |
| 5 | | | 40 | | | |
| 6 | | | 50 | | | |
| 7 | | | 60 | | | |
| 8 | | | 70 | | | |
| 9 | 1300 | | 5 | | | |
| 10 | | | 10 | | | |
| 11 | | | 20 | | | |
| 12 | | | 30 | | | |
| 13 | | | 40 | | | |
| 14 | | | 50 | | | |
| 15 | | | 60 | | | |
| 16 | | | 70 | | | |
| Rotary Screw Pump | | | | | | |
| 17 | 900 | | 5 | | | |
| 18 | | | 10 | | | |
| 19 | | | 20 | | | |
| 20 | | | 30 | | | |
| 21 | | | 40 | | | |
| 22 | 1300 | | 5 | | | |
| 23 | | | 10 | | | |
| 24 | | | 20 | | | |
| 25 | | | 30 | | | |
| 26 | | | 40 | | | |
| 27 | | | 50 | | | |