1. Definition of A Fluid

A fluid is a substance in which molecules within are free to move relative to each other and it *deforms continuously when subjected to shear stress*. This fluid can be made to flow if it is acted upon by a source of energy (for example, pressure or heat) which is used by the molecules to overcome attractive forces that bind them together. This can be made clear by assuming the fluid consists of layers parallel to each other and letting a force act upon one of the layers in a direction parallel to its plane (Figure 3-1). This force, divided by the area of the layer, is called shear stress. As long as this shear stress is applied, the layer will continue to move relative to its neighboring layers (Chasteau, 1982).

Imagine a circular cross-section of pipe containing a fluid such as water. For flow to occur without slippage, the various layers must move at different velocities. The fluid layer adjacent to the pipe wall is virtually stationary, while the layers further out move at increasingly higher velocities until a maximum velocity is attained at the center (Figure 3-1). In addition, a fluid is always a continuous medium without voids. However, the properties of a fluid, e.g., density, may vary from place to place in the fluid.

In the aforementioned pipe, if the neighboring layers offer no resistance to the movement of fluid, this fluid is said to be a *frictionless fluid* or *ideal fluid*, although, practically speaking, ideal fluids do not exist in nature. In many practical problems, the resistance caused by air molecule layers is either small or is not important, and therefore can be ignored. In addition to shear force, fluid also may be subjected to *compressive forces*. These compressive forces tend to change the volume of the fluid, and in turn its density. If the fluid yields to the effect of the compressive forces and changes its volume, it is *compressible*, otherwise it is *incompressible*.

In a stationary, incompressible fluid where no shear forces are applied, pressure exerted by a column with height $H$ (ft) with density $w$ (lb/ft$^3$) can be calculated by,
\[ P = w g H \]  

where \( g \) is the local acceleration due to gravity in \( \text{ft/sec}^2 \).

In addition to pressure, air quantity is another important measurement used to access the performance of a ventilation system. Most measurements of airflow are based on \textit{volume flow}, which is the amount of air (\( \text{ft}^3 \)) that passes through a given cross-section of an airway in a unit time (min.), or cfm. This is valid as long as there is little variation in air density throughout the system, which is true in most conditions. But under certain circumstances, the composition and energy content of air do vary as it circulates through underground airways, resulting in density changes, and it is preferable to work in mass flow. \textit{Mass flow} \((M)\) is measured in the air mass (lb) passing through a specific cross-sectional area \((A)\) in one minute, or lb/min.

\[
\begin{align*}
 w & = \frac{\text{mass}}{\text{volume}} \frac{\text{lb}}{\text{ft}^3} \\
  & = \frac{\text{mass flow}}{\text{volume flow}} \frac{\text{lb}}{\text{min}} \frac{\text{m}^3}{\text{m}^3} (= \frac{\text{lb}}{\text{min}} \frac{\text{ft}}{\text{sec}}) \\
 M & = Qw \frac{\text{lb}}{\text{min}} \frac{\text{m}^3}{\text{m}^3} (= \frac{\text{ft}^3}{\text{min}} \frac{\text{lb}}{\text{ft}^3})
\end{align*}
\]

For constant air density, \( Q = VA \), where \( V \) is average air velocity in \( \text{ft/min} \).

\[
M = Qw = VAw \frac{\text{lb}}{\text{min}} \frac{\text{ft}^3}{\text{m}^3} (= \frac{\text{ft}}{\text{min}} \frac{\text{ft}^3}{\text{ft}^3}) \quad (3-2)
\]

According to the conservation of mass, the mass flow \((M)\) passing through all cross-sections along its length should remain constant in any continuous airway, provided there are no inflows or outflows or air or other gases,

\[
M = Q_1w_1 = Q_2w_2 = Q_3w_3 \ldots = \text{constant}, \frac{\text{lb}}{\text{min}} \quad (3-3)
\]

where \( Q_1, Q_2, Q_3, \ldots \) etc. are air volume at locations 1, 2, 3, etc. respectively and \( w_1, w_2, w_3, \ldots \) etc. are air densities at locations 1, 2, 3, \ldots respectively. For all practical purposes, assuming air to be an incompressible flow will not cause any appreciable error, unless air velocity exceeds 19,685 \( \text{ft/min} \) at about atmospheric pressure or there is an elevation change of more than 1,400 \( \text{ft} \) (See §4.10: The Compressibility of Air).

2. \textbf{Turbulent and Laminar Flows}

In a flowing fluid, each particle changes its position with a certain velocity. The magnitudes and directions of the velocities of all particles may vary with position as well as with time. Streamline is used to illustrate this concept.
A streamline is an imaginary line in a fluid, the tangent to which gives the direction of the flow velocity at that position, as shown in Figure 3-2, and the distance between two streamlines is an inverse measure of the magnitude of the velocity. If the streamlines are smoothly curved and almost parallel to each other, as illustrated in Figure 3-3, the flow is known as streamlined flow, or laminar flow. On the other hand, if the streamlines are arranged haphazardly, the flow is known as turbulent flow, as illustrated in Figure 3-4.
If the streamline pattern of a flow remains constant with time, the flow is steady. If it does not, the flow is unsteady. In this case, a streamline picture is an instantaneous one, valid only for a particular instant of time. A greatly enlarged view of any small region of a turbulent flow shows that the flow is a randomly unsteady laminar flow. If the mean flow values are unchanged over a period of time, it is called a steady turbulent flow.

It was recognized earlier that the type of flow depended upon the velocity and viscosity of the fluid. It was not until the 1880s that this relationship was mathematically expressed in terms of the ratio of inertial and viscous forces by Professor Osborne Reynolds, who performed many experiments and dimension analyses,

\[ N_{Re} = \frac{\rho D V}{\mu} = \frac{DV}{\nu} \] (3–4)

\( \rho \) is the fluid mass density (=w/g), in lb/sec/ft\(^4\), \( \gamma = \rho/\mu \) is kinematic viscosity, in ft\(^2\)/sec; \( \mu \) is absolute viscosity (= \( \rho \nu \)), in lb.sec/ft\(^2\); \( D \) is diameter of conduit, in ft; and \( V \) is velocity, in ft/sec.

The Reynolds number \( (N_{Re}) \) is a dimensionless number in either British or SI units. If this number is less than 2000, viscous forces prevail and the flow will be laminar. The condition of the flow is less well defined for this number beyond 2000, at which value turbulence starts to occur. It has been common practice to regard the flow as turbulent when Reynolds number is larger than 4000,

- Laminar flow – \( N_{Re} < 2,000 \)
- Turbulent flow – \( N_{Re} > 4,000 \)

**Example 3-1:** A ventilation shaft of diameter 5 m passes an airflow of 200 m\(^3\)/sec at a mean density of 1.2 kg/m\(^3\) and a mean temperature of 18°C (64.4°F). Determine the Reynolds number for the shaft.

**Solution:** For air at 18°C, \( \mu_{air} = (17.0 + 0.045 \times t) \times 10^{-6} \) where \( t \) is temperature in °C

\[ \mu_{air} = (17.0 + 0.045 \times 18) \times 10^{-6} = 17.81 \times 10^{-6} \text{ N.sec/m}^2 \]

Air velocity = \( V = \frac{Q}{A} = \frac{200}{\pi (2.5)^2} = 10.186 \text{ m/sec} \)

\[ N_{Re} = \frac{\rho D V}{\mu} = \frac{1.2 \times 5 \times 10.186}{17.81 \times 10^{-6}} = 3.432 \times 10^6 \text{ (dimensionless)} \]

The Reynolds number indicates that the flow will be turbulent in the shaft. The same result can be obtained using British units.

3. Fluid Pressures

Pressure in a fluid (gas or liquid) is developed when fluid molecules bounce around the container. Because of the vast number of molecules in a unit volume, pressure developed is always considered to be uniformly distributed over the entire volume. Such a pressure is called a static pressure,
usually expressed in force in a unit area, or lb/in² (psi.). In the SI system, the unit of measure is Newton per square meter (N/m²), or Pascal (Pa).

If a vertical container, with height \( h \), contains liquid with density \( w \) (Figure 3-5), volume of the liquid is \( hA \) (ft\(^3\)). Since mass = volume x density, then

\[
\text{Fluid mass} = (hA)w \text{ lb}
\]

![Figure 3-5. Fluid pressure at the bottom of a container.](image)

The weight of the liquid will exert a force \( F \) on the base of the tube equal to mass x gravitational acceleration (g),

\[
F = hAwg
\]

Re-arranging the equation,

\[
\text{Pressure} = \frac{F}{A} = hwg \text{ psi}
\]

If the density of the fluid is given, then pressure at a point can be expressed as a column height \( h \).

4. Bernoulli’s Equation

Mine ventilation normally is an example of a steady flow process in which none of the variables of flow changes with time. Transitions and losses in energy are involved in such a process. Energy changes and their mathematical expressions are basic to the calculation of mine air quantity and pressure.

According to Newton’s Second Law, the force \( F \) used to accelerate a body must be equal to the product of the mass \( m \) of the body and its acceleration \( a \). If this law is applied to an ideal (frictionless), incompressible (constant–density) fluid – Euler’s Law of conservation of momentum is obtained.
Picture a fluid moving steadily through an imaginary fixed volume element \( xyz \), in the \( x \)-direction (across the \( yz \)-plane), as shown in Figure 3-6. If pressure \( p_1 \) and \( p_2 \) act upon the two opposite faces, the resultant force on the fluid in the volume element considered is:

\[
F = P_1yz - P_2yz + Bωxyz
\]

where:  
- \( B \) = body or mass forces per unit mass  
- \( w \) = constant fluid density

if \( x \) is very small distance, the acceleration along it is constant,  
\[
a = \frac{(V_2 - V_1)}{t}
\]

where \( t \) is the time for the fluid through distance \( x \).

Since the mass of the fluid element is \( M = wxyz \), with Newton's Law \( F = ma \) :

\[
P_1yz - P_2yz + Bwx = wxyz \left( \frac{V_2 - V_1}{t} \right)
\]

or,  
\[
P_1 - P_2 + Bwx = wx \left( \frac{V_2 - V_1}{t} \right)
\]

(3 - 6)

The mean velocity over the distance \( x \) is \( (V_2 + V_1)/2 = x/t \)

Substituting:

\[
P_1 - P_2 + Bw = w \left( \frac{V_2^2 - V_1^2}{2} \right)
\]

This is Euler's Equation for the \( x \)-direction, also known as the momentum equation.
If the x–direction is taken as vertically upwards as shown in Figure 3-7, such that \( P_1 \) \& \( V_1 \) are at height \( Z_1 \) and \( P_2 \) \& \( V_2 \) at height \( Z_2 \), the distance is \( x = Z_2 - Z_1 \).

![Figure 3-7. Illustrations of Euler's Law for vertical control body.](image)

The force \( B \) per unit mass would now be due to gravitation, and \( B = -g \)

\[
p_1 - p_2 - gw(Z_1 - Z_2) = w \frac{V_1^2 - V_2^2}{2} \]

or

\[
p_1 + w \frac{V_1^2}{2} + gwZ_1 = p_2 + w \frac{V_2^2}{2} + gwZ_2 \quad (3-7)
\]

This is **Bernoulli’s Equation** for the ideal incompressible fluid in terms of pressure (Conservation of Momentum). The **Second Law of Thermodynamics** limits the directions in which energy can be transformed or transferred. It is well known that friction causes a degradation of useful "pressure" energy. So the heat will flow from a hot body to a cold body. In terms of the Second Law it is impossible for the reverse to occur,

\[
\text{or } p_1 \frac{V_1^2}{wg} + Z_1 = p_2 \frac{V_2^2}{2g} + Z_2 \quad (3-8)
\]

This is **Bernoulli’s Equation** (in SI unit) for the ideal incompressible fluid in terms of pressure head (m). The pressures \( P_1 \) and \( P_2 \) are referred to as *static* pressures \((H_s)\), in N/m\(^2\) (Pa) which act in all directions regardless of the direction of the flow. The terms \( V_1^2/2g \) and \( V_2^2/2g \) are referred to as *velocity* or *dynamic* pressures \((H_v)\), in m/sec; \( Z_1 \) and \( Z_2 \) are referred to as potential pressures \((H_z)\), in m; \( w \) is density in kg/m\(^3\); and \( g \) is gravitational force, 9.8 m/sec\(^2\). The same equation can also be expressed in British units,
where the pressures $P_1$ and $P_2$ are in lb/ft$^2$; $V_1$ and $V_2$ are in ft/min.; $Z_1$ and $Z_2$ are in ft; $w$ is in lb/ft$^3$; and $g$ is 32.2 ft/min$^2$. Each term in the equation is actually specific energy, in units of ft-lb/lb, or ft. (Hartman, et al., 1997). The summation of these three pressures is referred to as the total pressure of the oncoming fluid stream, also called the facing pressure,

$$H_{T1} = H_{T2}$$

$$H_{s1} + H_{v1} + H_{z1} = H_{s2} + H_{v2} + H_{z2}$$

(3–9)

Each term in the equation is a specific energy, in units of ft-lb/lb, or ft. Since ft is a measure of fluid head, these terms also can be called pressure heads, or head.