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**EXPERIMENTAL METHODS**  
**IN**  
**CHEMICAL ENGINEERING**

**STATISTICS, EXPERIMENTAL  
DESIGN, AND MODELING**

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## List of Symbols

<b>A</b>	heat transfer area used in Example 4.1 [ $\text{m}^2$ ]
<b>[A]</b>	concentration of chemical species A used in Example 4.2 [ $\text{kmol}/\text{m}^3$ ]
$b_i$	i-th model parameter
<b>b</b>	parameter vector ( $p \times 1$ )
<b>C</b>	C-matrix or $(\mathbf{X}^t\mathbf{X})^{-1}$ ( $p \times p$ )
$C_p$	heat capacity [ $\text{kJ}/\text{kg}\cdot\text{K}$ ]
<b>e</b>	dimensional error vector ( $(n-p) \times 1$ )
$E(y)$	expected value of the statistic y
$F_{\text{cal}}$	calculated F-ratio, $\text{ms}_i/\text{s}^2$
$F_{\text{tab}}$	tabulated F-ratio dependent on $\alpha$ , $v_1$ , and $v_2$
<b>F*</b>	matrix defined by Eq. (2.9)
$k_1, k_{-1}, k_2$	reaction rate constants used in Example 4.2
<b>K</b>	flow consistency used in Eq. (1.2) [ $\text{Pa}\cdot\text{s}^{n+1}$ ]
<b>m</b>	mass of liquid used in Example 4.1 [ $\text{kg}$ ]
$\text{ms}_i$	mean square, $S_i/v_i$
<b>n</b>	flow index used in Eq. (1.2)
<b>n</b>	total number of observations
$n_i$	number of observations at the i-th level
<b>p</b>	number of model parameters
$p^{\text{sat}}$	vapor pressure [ $\text{kPa}$ ]
$Q_s$	power supplied used in Example 4.1 [ $\text{kJ}/\text{s}$ ]
<b>r</b>	residual degrees of freedom, $n-p$
$r_B$	rate of formation of chemical species B
<b>R</b>	response used in Chapter 3
<b>R</b>	residual, $y - \hat{y}$

$s$	sample standard deviation
$s^2$	sample variance defined by Eq. (3.14)
$s_i$	sample standard deviation at the $i$ -th level defined by Eq. (3.15)
$s_p^2$	pooled sample variance defined by Eq. (3.16)
$s(y)$	standard error of statistic $y$ , such as parameters or prediction
$S$	total sum of squares
$S_A$	model sum of squares due to the average of all responses
$S_M$	model sum of squares
$S_{M'}$	model sum of squares without the average included, $S_M - S_A$
$S_R$	residual sum of squares, $S - S_M$
$S_E$	sum of squares, pure error
$S_L$	sum of squares, terms-left-out, $S_R - S_E$
$t$	time[s]
$t$	$t$ -statistic defined by Eq. (5.22)
$T$	temperature [K]
$U$	overall heat transfer coefficient used in Example 4.1 [ $\text{kJ}/\text{m}^2 \cdot \text{s} \cdot \text{K}$ ]
$V(R)$	variance of a response
$V(x_i)$	variance of an individual measurement
$x$	factor or independent variable
$x_i$	$i$ -th factor
$X_i$	coded $i$ -th factor defined in Appendix 2.A
$X$	$X$ -matrix ( $n \times p$ )
$y$	response or dependent variable
$y_{ij}$	$j$ -th response at the $i$ -th level
$\bar{y}$	sample average defined in Eq. (3.13)
$\mathbf{y}$	data vector ( $n \times 1$ )
$\hat{\mathbf{y}}$	model vector ( $p \times 1$ )
$z$	normal deviate defined by Eq. (5.23)

## Greek Symbols

$\alpha$	risk of making wrong decision
$a_i$	scalar value defined by Eq. (3.A.2)
$b_j$	j-th parameter in nonlinear model defined by Eq. (2.7)
$\beta$	parameter vector
$\beta^*$	initial guess to parameter vector
$dR$	instantaneous deviation of the response
$dx_i$	instantaneous deviation of the measurement $x_i$
$\dot{g}$	shear rate [ $s^{-1}$ ]
$e$	experimental error
$\eta$	apparent viscosity [Pa-s]
$\eta$	true mean of population
$\eta_0$	zero shear viscosity [Pa-s]
$q_i$	coded temperature defined by Eq. (2.2)
$\Theta$	temperature rise defined by $T - T_\infty$ [K] in Chapter 4
$v_i$	number of replicates at i-th level equal to $n_i - 1$
$v_E$	pure error degrees of freedom or total number of replicates
$v_L$	degrees of freedom, terms-left-out
$v_M$	number of parameters in model, which equals p
$v_R$	residual degrees of freedom, which equals $n - p$
$x_i$	i-th factor in nonlinear model used in Eq. (2.7)
$X$	nonlinear design matrix
$r$	polymer density [ $kg/m^3$ ]
$\sigma$	standard deviation of population
$\sigma^2$	population variance
$f_j$	partial derivative defined by Eq. (2.10)