Axisymmetric dynamics of composite spherical shells with active piezoelectric/composite stiffeners

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Summary. The paper presents a theoretical formulation for spherical shells reinforced by meridional and circumferential stiffeners. Active damping of the shell is introduced through control action of piezoelectric coupled pairs bonded to the meridional stiffeners. The induced loads can include radial pressure and a thermal field that are independent of the circumferential coordinate. Neglecting local deformations between adjacent meridional stiffeners, the response of the shell will be axisymmetric. The analysis employs the Donnell-Mushtari-Vlasov version of Love’s theory of shells together with a smeared stiffeners technique. The paper also considers a particular case of shell mounted piezoelectric coupled pairs without conventional stiffeners. A closed form solution is derived for spherical panels without conventional stiffeners within the range of the meridional coordinate between $75^\circ$ and $90^\circ$ using a version of the Geckeler approximation.

1 Introduction

The concept of piezoelectric stiffeners has been developed by Birman [1], [2], and Knowles et al. [3]. The recent paper of Murray et al. [4] presented an extension of the theory to axisymmetric dynamic problems of shallow spherical composite caps with meridional composite stiffeners. In this paper, pairs of piezoelectric actuators bonded to the stiffeners produced bending moments that reduced deformations and stresses in the cap. The problem was formulated based on the Donnell-Mushtari-Vlasov theory of shells and a smeared stiffeners technique. The solution was generated using the simplification suggested by Simites and Blackmon [5] who identified geometry of a shallow cap with the coordinate system based within the reference plane. The cap considered in the analysis was clamped along the boundary and its meridional curve was approximated according to Huang’s approach. The global axisymmetric deflections and the meridional displacements were approximated according to the approach of Timoshenko, while local asymmetric deformations between the stiffeners were neglected.

The present paper represents a generalization of the previous solution of Murray et al. [4]. The assumption of Simites and Blackmon [5] that replaced the spherical coordinate system with reference-plane based coordinates is omitted. This enables the authors to consider spherical caps with a larger meridional curvature. The solution is obtained for a general case where the actuators are bonded to continuous composite blade stiffeners. A particular design suggested in this paper employs pairs of piezoelectric actuators mounted on local supporting structures bonded to the skin. This method can be used to generate significant active control bending moments, while keeping the weight of the structure to a minimum.
2 Theoretical formulation

2.1 Equations of axisymmetric motion in terms of stress resultants and couples

Consider a spherical composite shell or panel reinforced by meridional and circumferential stiffeners and subject to an axisymmetric thermo-mechanical load. Additional reinforcement of the shell is provided by pairs of piezoelectric actuators bonded to the meridional stiffeners. The theory developed in this paper is based on the Love theory of shells using the Donnell-Mushtari-Vlasov approach. A treatment of the foundations of the Love theory of shells can be found elsewhere [6]. Bert and Kim [7] have shown that the results obtained using the Love theory are in agreement with the solutions available using the Sanders and other theories. Moreover, the governing equations of motion for a general asymmetric problem of a spherical shell [8] can be simplified if the problem is axisymmetric. According to the Donnell-Mushtari-Vlasov approach [9] further simplifications are possible if the shell model satisfies the following conditions:

(i) contributions of in-plane displacements to bending strains (changes of curvature and twist) are negligible;
(ii) in-plane inertia does not affect the response and
(iii) transverse shear stress resultants have a negligible effect on in-surface equilibrium.

It is also assumed in this paper that thermal effects on the properties of shell and actuator materials are negligible.

With these assumptions the axisymmetric equations of motion in the meridional and radial directions can be written as

\[ N_\phi' \sin \phi + (N_\phi - N_\theta) \cos \phi = 0 , \]
\[ (Q_\phi \sin \phi)' - (N_\phi + N_\theta) \sin \phi = (m\ddot{w} - q) a \sin \phi , \]
\[ (\ldots)' = \frac{\partial (\ldots)}{\partial \phi} , \quad (\ldots)'' = \frac{\partial^2 (\ldots)}{\partial \phi^2} . \tag{1} \]

In Eqs. (1) \( N_\phi \) and \( N_\theta \) are the meridional and circumferential stress resultants and \( Q_\phi \) is the transverse shear stress resultant. The spatial differentiation is performed with respect to the meridional coordinate \( \phi \). Furthermore, \( \dot{w} \) denotes a radial displacement, \( a \) is the radius of the middle surface of the skin, \( t \) denotes time, \( m \) represents the total mass of the shell per unit area, and \( q \) is the intensity of the radial pressure. Note that the equilibrium in the circumferential direction is identically satisfied.

The mass of the shell per unit area has to incorporate the contributions of the skin, stiffeners, piezoelectric actuators, and electrodes and wiring. In this problem, the stiffeners are oriented in the meridional and circumferential directions. The couples of actuators are bonded to meridional stiffeners only since the activation of forces or moments in the circumferential direction would violate axisymmetry of the problem and result in a less efficient design. Accordingly,

\[ m = \rho h + m_\phi + m_\theta , \quad \tag{2} \]

where \( \rho \) is the mass density of the skin, \( h \) is its thickness, \( m_\phi \) represent the mass per unit length for the meridional and circumferential stiffeners, \( (i = \phi, \theta) \) and for the actuators, electrodes and wiring \( (i = \phi) \). The spacing of the meridional and circumferential stiffeners are denoted
by \( s_\phi \) and \( s_\theta \), respectively. Note that in the case of a spherical shell, the spacing of meridional stiffeners is variable and the analytical solution becomes practically impossible. Accordingly, the equations of motion in terms of displacements are derived in the paper based on the simplification of Simitses and Blackmon [5], where it is assumed that the ratios of the cross-sectional area and the first and second area moments of meridional stiffeners to their spacing remains constant.

The transverse shear stress resultant is given by [9]

\[
Q_\phi = \left[ M_\phi' \sin \phi + (M_\phi - M_\theta) \cos \phi \right] / a \sin \phi ,
\]

where \( M_\phi \) and \( M_\theta \) are the stress couples. The substitution of this expression into the radial motion equation of (1) yields

\[
M_\phi'' + (2M_\phi' - M_\theta') \cot \phi - M_\phi' + M_\theta - a(N_\theta + N_\phi) = (m\ddot{w} - q) a^2 .
\]

The first equation (1) is presented in the form

\[
N_\theta' + (N_\theta - N_\phi) \cot \phi = 0 .
\]

The solution to equations of motion (4, 5) must satisfy the boundary conditions. In the case of a simply supported shell unrestrained in the meridional direction and experiencing axisymmetric deformations, these conditions are

\[
\phi = \phi_i \quad (i = 1, 2); \quad w = M_\phi = N_\phi = 0 .
\]

2.2 Constitutive relations

The stress couples and stress resultants in the equations of motion include the skin, stiffeners, and active moment contributions. The latter are generated by the induced action of the piezoelectric actuators that are discontinuously bonded to the stiffeners. Accordingly, their contribution to the stiffness of the shell can often be disregarded.

The skin is assumed to include multiple generally orthotropic layers that are symmetric about the middle surface. In the case of axisymmetric loading including axisymmetric temperature, the nonzero stress resultants and stress couples within the skin are

\[
\begin{pmatrix}
N_{\phi s} \\
N_{\theta s}
\end{pmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{12} & A_{22}
\end{bmatrix}
\begin{pmatrix}
\varepsilon_{\phi} \\
\varepsilon_{\theta}
\end{pmatrix}
- \begin{pmatrix}
N_{\theta s}^T \\
N_{\phi s}^T
\end{pmatrix},
\]

\[
\begin{pmatrix}
M_{\phi s} \\
M_{\theta s}
\end{pmatrix} =
\begin{bmatrix}
D_{11} & D_{12} \\
D_{12} & D_{22}
\end{bmatrix}
\begin{pmatrix}
\kappa_{\phi} \\
\kappa_{\theta}
\end{pmatrix}
- \begin{pmatrix}
M_{\theta s}^T \\
M_{\phi s}^T
\end{pmatrix},
\]

where the extensional and bending stiffness are defined by

\[
\{A_{ij}, D_{ij}\} = \int_{-h/2}^{h/2} Q_{ij} \{1, z^2\} dz \quad i, j = 1, 2 .
\]

In Eqs. (8), \( Q_{ij} \) and \( z \) are the reduced transformed stiffnesses and the (radial) thickness coordinate measured from the middle surface. The thermal contributions in Eqs. (7) can be calculated in terms of the coefficients of thermal expansion (\( \alpha \)) and temperature in excess of the
reference ambient value $(T)$, as

$$\{N_{kr}, M_{kr}\} = \sum_{j=1}^{2} \int_{-h/2}^{h/2} Q_{ij} \alpha_j \{1, z\} T \, dz,$$

(9)

where for $k = \phi, i = 1$ and for $k = \theta, i = 2$. Note that thermal variations $T = T(z)$ are measured from a stress-free reference value. It is assumed that for a nonuniform temperature distribution throughout the skin thickness the effect on the material properties is negligible so that the terms associated with the coupling stiffness matrix do not appear in Eq. (7).

The stiffeners are assumed thin, so that temperature is uniformly distributed throughout the material. Accordingly, the contribution of the stiffeners to the stress resultants and stress couples is given by

$$\{N_{kr}, M_{kr}\} = \frac{b_k}{s_k} \int_{z} \sigma_k \{1, z\} \, dz,$$

(10)

where $b_k$ is the thickness of the stiffener and $\sigma_k$ is the stress in the axial direction of the corresponding stiffener. This stress is found as

$$\sigma_k = Q_{kk} (\varepsilon_k^a - \alpha_k T + z \kappa_k).$$

(11)

In Eq. (11), $Q_{kk}$ is the transformed reduced stiffness of the material of the corresponding layer of the stiffener in the axial direction, $\alpha_k$ is the axial thermal expansion coefficient of the corresponding layer, the superscript “r” identifies the properties of the stiffener material, and $z$ indicates the distance from the centerline of the layer to the middle surface of the skin. In the case of an orthotropic stiffener, the stiffness is given by

$$Q_{kk}^r = \frac{E_k^r}{1 - \nu_{kz}^r \nu_{zk}^r},$$

(12)

where $E_k$, $\nu_{kz}$, and $\nu_{zk}$ are the modulus of elasticity and the Poisson ratios, respectively.

Substituting Eq. (11) into (10) yields the following expressions for the stress resultants and couples generated in orthotropic stiffeners:

$$N_{\phi r} = Q_{\phi \phi}^r \frac{A_k}{s_k} \varepsilon_\phi^0 + Q_{\phi \phi}^r \frac{F_\phi}{s_k} \kappa_\phi - N_{\phi r}^T,$$

$$N_{\theta r} = Q_{\theta \theta}^r \frac{A_k}{s_k} \varepsilon_\theta^0 + Q_{\theta \theta}^r \frac{F_\theta}{s_k} \kappa_\theta - N_{\theta r}^T,$$

$$M_{\phi r} = Q_{\phi \phi}^r \frac{F_\phi}{s_k} \varepsilon_\phi^0 + Q_{\phi \phi}^r \frac{I_\phi}{s_k} \kappa_\phi - M_{\phi r}^T,$$

$$M_{\theta r} = Q_{\theta \theta}^r \frac{F_\theta}{s_k} \varepsilon_\theta^0 + Q_{\theta \theta}^r \frac{I_\theta}{s_k} \kappa_\theta - M_{\theta r}^T.$$  

(13)

Here variables $A_k$, $F_k$ and $I_k$ denote the cross sectional area and the first and second moments of the stiffener about the middle surface of the skin, respectively. Note that Simitses and Blackmon [5] assumed that the width of the blade stiffeners varies along the meridional coordinate so that the ratios $A_k/s_k, F_k/s_k$ and $I_k/s_k$ remain constant. This simplified the equations of motion in terms of displacements and made the analytical solution feasible. While the practicality of design with a variable width of the blade stiffeners is questionable, the corresponding assumption may be adopted if the shell is supported by relatively close circumferential frames, i.e., a difference between the coordinates of the boundaries $\phi_1$ and $\phi_2$ is not large. The following solution utilizes the simplification of Simitses and Blackmon [5].
The thermal contributions in Eq. (13) can be obtained from
\[
\{N_{\theta r}^T, M_{\theta r}^T\} = \left[ \int_Q \rho \{1, z\} T \, dz \right] \frac{b_k}{s_k}, \tag{14}
\]

The corresponding stress resultants and stress couples for multilayered stiffeners can be expressed as
\[
\begin{align*}
N_{\theta r} &= (A'_{11} \epsilon^0_{\theta} + B'_{11} \varphi_{\theta}) \frac{b_k}{s_k} - N_{\theta r}^T, \\
N_{\theta z} &= (A'_{22} \epsilon^0_{\theta} + B'_{22} \varphi_{\theta}) \frac{b_k}{s_k} - N_{\theta z}^T, \\
M_{\theta r} &= (B'_{11} \epsilon^0_{\theta} + D'_{11} \varphi_{\theta}) \frac{b_k}{s_k} - M_{\theta r}^T, \\
M_{\theta z} &= (B'_{22} \epsilon^0_{\theta} + D'_{22} \varphi_{\theta}) \frac{b_k}{s_k} - M_{\theta z}^T,
\end{align*}
\tag{15}
\]

where
\[
\{A'_{i1}, B'_{i1}, D'_{i1}\} = \int_Q \{1, z, z^2\} \, dz, \quad i = 1, 2. \tag{16}
\]

The actuators are arranged in phase reversed pairs to produce bending moment and zero force. This can also be achieved by applying out-of-phase voltage to the “top” actuator relative to the “bottom” actuator. As was shown by Birman and Adali [10], the approach, utilizing the active bending moment is more efficient than a design based on utilization of in-surface active forces.

The actuators may contribute to the overall stiffness of structure and the previous paper [4] illustrated the method to incorporate this contribution into the analysis. However, due to a discontinuity of the actuators, it is often appropriate to disregard their contribution to the stiffness. An exception is the case, where the actuators, although discontinuous, are closely spaced in the meridional direction. Also note that the actuators must be symmetric about the stiffener plane to avoid inducing a torsional moment. The axial stress induced by a piezoelectric actuator bonded at a distance from the middle surface of the skin is given as
\[
\sigma_k = E_p (\epsilon_k^0 - \alpha_p T) - S, \tag{17}
\]
where \(E_p\) is the modulus of elasticity of the material in the axial (meridional) direction, \(\alpha_p\) is the corresponding coefficient of thermal expansion and \(S\) is the active stress generated by the actuator. The stress couple contributed by the pair of actuators can be determined using a modification of Eq. (10), where the width represents the dimension \(b\), as shown in Fig. 1.
Consider a single piezo-pair that is either phase reversed or activated by out-of-phase voltage. The substitution of the expression for the stress (17) into Eq. (10), yields

\[ N_{\theta \rho} = \left[ E_p \frac{A_p}{s_\phi} \varepsilon_{\theta}^0 + E_p \frac{F_p}{s_\phi} \chi_{\theta}^0 - N_{\rho \rho}^T \right] \frac{l_a}{l + l_a}, \]

\[ M_{\theta \rho} = \left[ E_p \frac{F_p}{s_\phi} \varepsilon_{\theta}^0 + E_p \frac{I_p}{s_\phi} \chi_{\theta}^0 - M_{\rho \rho}^T - M_{\rho}(E) \right] \frac{l_a}{l + l_a}, \]

(18)

where \( l_a \) is the length of the actuator, \( l \) is the distance between the actuator pair (as shown in Fig. 1), \( A_p \) is the total cross sectional area of the pair of actuators and \( F_p \) and \( I_p \) are the first and second moments of the actuators pair about the middle surface of the skin. The thermal and active contributions in Eq. (18) can be evaluated as follows:

\[ \{N_{\rho}^T, M_{\rho}^T\} = E_p \alpha_p T \{ A_p, F_p \} / s_\phi, \]

\[ M_{\rho}(E) = \frac{SA_p d}{2s_\phi}, \]

(19)

where \( T \) is a constant temperature and \( d \) is the arm, as shown in Fig. 1. Note that the net active stress resultant produced by the pair of actuators is zero.

The contribution of the actuators to the stiffness varies with the meridional coordinate, except for the case where the geometry is such that the ratios of \( A_p, F_p \), and \( l_a \) to the spacing \( s_\rho \) remain constant or nearly constant. In addition, as suggested above, if supporting circumferential frames are close to each other, variations of the contribution of the actuators to the stiffness can be disregarded. Note that if the distance between the actuators is large, i.e. \( l \gg l_a \), the contribution of the actuators to the stiffness given by the corresponding terms in Eqs. (18) can be neglected altogether, while the thermal and active terms given by Eqs. (19) should be applied at the corresponding meridional locations. Accordingly, in this case,

\[ \{N_{\rho}^T, M_{\rho}^T\} = \delta(\phi - \phi_j) E_p \alpha_p T \{ A_p, F_p \} / s_\phi, \]

\[ M_{\rho}(E) = \frac{1}{2} \delta(\phi - \phi_j) SA_p d, \]

(20)

where \( \delta \) is the Dirac delta function and \( \phi_j \) is the coordinate of the corresponding actuators pair.

Combining the contributions of the skin (Eqs. (7)), the stiffeners (Eqs. (13) or (15) and the actuators (Eqs. (18)), one can obtain the constitutive equations:

\[
\begin{bmatrix}
N_{\phi} \\
N_{\rho} \\
M_{\phi} \\
M_{\rho}
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} & B_{11} & 0 \\
A_{12} & A_{22} & 0 & B_{22} \\
B_{11} & 0 & D_{11} & D_{12} \\
0 & B_{22} & D_{12} & D_{22}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{\phi}^0 \\
\varepsilon_{\rho}^0 \\
\chi_{\phi} \\
\chi_{\rho}
\end{bmatrix} -
\begin{bmatrix}
N_{\phi}^T \\
N_{\rho}^T \\
M_{\phi}^T \\
M_{\rho}^T
\end{bmatrix} -
\begin{bmatrix}
0 \\
0 \\
0 \\
M(E)
\end{bmatrix}.
\]

(21)

The elements of the stiffness matrix that appear in Eqs. (21) can be calculated by the following formulae:

\[ A_{11} = A_{11}^e + \frac{b_{12}}{s_{\phi}^2} + \frac{A_p l_a}{s_\phi (l + l_a)}, \]

\[ A_{12} = A_{12}^e, \]

\[ A_{22} = A_{22}^e + \frac{b_{22}}{s_{\phi}^2}, \]

(22)
\[ B_{11} = B_{11}^p \frac{b_0}{s_0} + E_p F_p \frac{I_p}{s_0 (l + l_a)} , \]
\[ B_{22} = B_{22}^p \frac{b_0}{s_0} , \]
\[ D_{11} = D_{11}^s + D_{11}^p \frac{b_0}{s_0} + E_p \frac{I_p}{s_0 (l + l_a)} , \]
\[ D_{12} = D_{12}^s , \]
\[ D_{22} = D_{22}^s + D_{22}^p \frac{b_0}{s_0} . \]

The expressions in Eqs. (22) are presented for the general case of continuous stiffeners in both meridional and circumferential directions. It is assumed that the pairs of piezoelectric actuators are bonded to meridional stiffeners with a relatively small spacing \( l \). In this situation, the thermal terms and active stress couples are given by
\[ N_{\theta}^T = N_{\theta\theta}^T + N_{\theta r}^T + N_{r \theta}^T \frac{l_a}{(l + l_a)} , \]
\[ N_{\phi}^T = N_{\phi\phi}^T + N_{\phi r}^T , \]
\[ M_{\theta}^T = M_{\theta\theta}^T + M_{\theta r}^T + M_{r \theta}^T \frac{l_a}{(l + l_a)} , \]
\[ M_{\phi}^T = M_{\phi\phi}^T + M_{\phi r}^T , \]
\[ M(E) = \frac{l_a}{(l + l_a)} M_p(E) . \]

For the situation shown in Fig. 2, where the piezo-actuated coupled pair is installed without a conventional stiffener, the stiffness contribution of the supporting element is negligible. The active stress couples can be calculated using Eq. (20). In this case, Eqs. (22) and (23) can be simplified by omitting all terms associated with the contribution of the stiffeners, except for the active control moment \( M(E) \) and thermally-induced stress resultant and stress couple. The stiffness of the actuators can also be disregarded. Note that out-of-plane stability of the supporting element should be considered in actual design.

2.3 Strain-displacement relationships and equations of motion in terms of displacements

The Love’s strain-displacement relationships for a spherical shell undergoing an axisymmetric deformation are [8]:
\[ \varepsilon_{\theta} = \frac{1}{a} (u' + w) , \]
\[ \varepsilon_\phi = \frac{1}{a} (u \cot \phi + w) \]
\[ \varepsilon_{\phi \theta} = \kappa_{\phi \theta} = 0, \]
\[ \kappa_\phi = \frac{w''}{a^2}, \]
\[ \kappa_\theta = \frac{w'}{a^2} \cot \phi, \]

where \( u \) is a displacement in the meridional direction. The meridional displacement is neglected in the expressions for the changes of curvature in Eqs. (24), according to the Donnell-Mushtari-Vlasov theory.

The substitution of Eqs. (24) into Eqs. (21) with the result substituted into Eqs. (4), (5), and (6) yields the equations of motion and the boundary conditions in terms of displacements. Note that since \( s_\phi \) is variable, these equations become very complicated. Therefore, in the following, the equations are presented for the case where the design of the stiffeners and the actuators is such that the coefficients at all terms in the constitutive equations are constant, as explained in the previous discussion. In this case, the equations of motion and the boundary conditions become

\[ k_1 u'' + k_2 u''' + k_3 u'' + k_4 u' + k_5 u + \kappa_1 u'' + \kappa_2 u'' + n_1 u'' + n_2 u' + n_3 u + R = (m \ddot{w} - q) a^2, \]
\[ s_1 u'' + s_2 u'' + s_3 u + s_4 u + p_1 u'' + p_2 u' + p_3 u + L = 0, \]

where

\[ k_1 = -\frac{D_{11}}{a^2}, \]
\[ k_2 = -\frac{D_{11}}{a^2} \cot \phi, \]
\[ k_3 = \frac{2B_{11}}{a^2} [D_{11} - D_{12} + 2D_{12} \csc^2 \phi + (D_{22} - 2D_{12}) \cot^2 \phi], \]
\[ k_4 = \frac{\cot \phi}{a} \left( 2B_{11} + \frac{(D_{12} - D_{22})}{a} - \frac{D_{22}}{a} \csc^2 \phi \right), \]
\[ k_5 = \frac{B_{11}}{a} - (A_{11} + 2A_{12} + A_{22}), \]
\[ n_1 = \frac{B_{11}}{a}, \]
\[ n_2 = \frac{B_{11}}{a} \cot \phi, \]
\[ n_3 = -\left( A_{11} + A_{12} + \frac{B_{11}}{a} + \frac{B_{22}}{a} \cot^2 \phi \right), \]
\[ n_4 = \cot \phi \left[ -(A_{12} + A_{22}) + \frac{B_{22}}{a} (1 + \csc^2 \phi) \right], \]
\[ R = -(M_\phi)^n - [M(E)]'' - \cot \phi \left( 2(M_\phi^T)' - (M_\phi^T)' + 2 [M(E)]' \right) + M_\phi^T \]
\[ + M(E) - M_\phi^T + a(N_\phi^T + N_\theta^T), \]
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\[ p_1 = A_{11}, \]
\[ p_2 = A_{11} \cot \phi, \]
\[ p_3 = (A_{12} - A_{22}) \cot^2 \phi - A_{12} \csc^2 \phi, \]
\[ s_1 = -\frac{B_{11}}{a}, \]
\[ s_2 = -\frac{B_{11}}{a} \cot \phi, \]
\[ s_3 = A_{11} + A_{12} + \frac{B_{22}}{a} \cot^2 \phi, \]
\[ s_4 = (A_{11} - A_{22}) \cot \phi, \]
\[ L = -[(N^{\phi T}_\phi - N^{\phi T}_\phi) \cot \phi + (N^{\phi T}_\phi)'] a. \]

The boundary conditions, (6) at \( \phi = \phi_i \) can be rewritten as \( w = 0, \)
\[ D_{11} w'' + D_{12} w' \cot \phi - B_{11} a w - B_{11} a u' + a^2 [M^{\phi T}_\phi + M(E)] = 0, \]
\[ B_{11} w'' - (A_{11} + A_{12}) a w - A_{11} a u' - A_{12} a \cot \phi u + a^2 N^{\phi T}_\phi = 0. \]

Note that the coefficients of the equations of motion are variable. Therefore, the solution should be obtained numerically. For example, a finite difference method could be an attractive option for a numerical analysis. An example of such analysis of shallow spherical isotropic panels subjected to impulse loading was published by Skurlatov et al. [11]. However, it is possible to obtain an exact solution of the equations of motion in an important particular case where the boundaries of the shell correspond to relatively large values of the meridional coordinate, i.e., \( 75^\circ < \phi < 90^\circ \). In this case it is possible to assume that \( \cot \phi = 0 \), while \( \sin \phi = 1 \). This approach is similar to the so-called Geckeler approximation that has been proven a useful tool for generating approximate solution for spherical shells [12], [13]. A more general case corresponding to the Geckeler approximation is found if the range of variations of the meridional coordinate is small, i.e., the spherical panel is narrow. In this case it is possible to assume that trigonometric functions of the meridional coordinate remain constant.

3 Numerical analysis

Consider a particular case discussed in the previous section where the meridional coordinate varies in the range between \( 75^\circ \) and \( 90^\circ \). In addition, thermal loading is discounted. Then equations of motion can be simplified:
\[ k_1 \ddot{u} + k_3 \dddot{u} + k_5 u + n_3 \dot{u}' + R = (m \ddot{u} - q) a^2, \]
\[ s_3 \dot{u}' + p_1 u'' + p_3 \ddot{u} = 0, \]
where \( k_1 \) and \( p_1 \) are given by Eqs. (26), while other coefficients are
\[ k_3 = \frac{D_{11} + D_{12}}{a^2}, \]
\[ k_5 = -(A_{11} + 2A_{12} + A_{22}) , \]
\( n_3 = -(A_{11} + A_{12}), \)
\( \dot{R} = -[M(E)]'' + M(E), \)
\( s_3 = A_{11} + A_{12}, \)
\( p_3 = -A_{12}. \)

The solution of Eqs. (28) is available in the closed form. The first derivative of the meridional displacement can be obtained from the first equation (28) and substituted into the second equation (28) to yield
\[
\begin{align*}
p_1 [(m \dddot{u} + q') a^3 - R'' - k_1 \dddot{u} - k_3 \dddot{v} - k_5 u''] + p_3 [(m \dddot{u} - q) a^3 - R - k_1 \dddot{u} - k_3 \dddot{v} - k_5 u'] + s_3 \dddot{u}'' &= 0. \tag{30}
\end{align*}
\]
Consider the case where a periodic pressure is uniformly distributed over the surface, i.e.,
\( q = q_0 \sin \omega t, \)
\( \omega \) being the driving frequency.

The induced stress couples should be generated at the modal frequency of the disturbance load with zero phase angle differential. Then the corresponding term in the equation of motion becomes
\( M(E) = -M \sin \omega t, \)
where \( M = \max |M(E)| \) and the sign is chosen to reduce forced vibrations of the shell. The substitution of the pressure and the active stress couples given by Eqs. (31) and (32) and the radial deflection in the form
\( w = W \sin \omega t, \quad W = W(\phi), \)
into equation of motion (30) yields a single sixth-order differential equation with respect to the function \( W: \)
\[
\begin{align*}
b_6 W^{iv} + b_4 W^{iv} + b_2 W'' - b_0 W &= C, \tag{34}
\end{align*}
\]
where
\[
\begin{align*}
b_6 &= p_1 k_1, \\
b_4 &= p_1 k_3 + p_3 k_1, \\
b_2 &= p_1 (m \omega^2 a^2 + k_5) + p_3 k_3 - s_3 n_3, \\
b_0 &= p_3 (m \omega^2 a^2 + k_5), \\
C &= p_3 (-q_0 a^2 + M).
\end{align*}
\]

The solution of Eq. (34) has to satisfy the boundary conditions corresponding to a simple support along the edge \( \phi = \phi_i: \)
\[
\begin{align*}
W &= 0, \\
D_{11} W'' &= M a^2, \\
k_1 W^{iv} + k_3 W''' + \left[ k_5 + m \omega^2 a^2 - n_3 \left(1 + \frac{A_{12}}{A_{11}}\right)\right] W + R + q_0 a^2 &= 0. \tag{36}
\end{align*}
\]
The calculations were conducted for a panel manufactured from AS/3501 graphite/epoxy. The design incorporated 16 symmetrically laminated layers with the lamination angles [90/0/90/45/-45/45]. The matrices of transformed reduced stiffnesses for the layers are found in [14]. The mass density of the material is 1.55 g/cm³. The calculations have illustrated that the behavior of the panel can accurately be modeled by the transversely isotropic model. This means that the response to a uniform pressure can be assumed axisymmetric. The radius of the panel considered in the examples was $a = 0.5$ m, while the boundary coordinate corresponded to $\phi = 75^\circ$.

The piezoceramic actuators considered in this paper were manufactured from PZT-5A that has $C_{31} = -11.8$ N/(m²V). The voltage amplitude was chosen equal to 200 V. The effect of the driving frequency on the nondimensional amplitude of the radial displacement of the apex normalized with respect to the amplitude of the displacement in the absence of the active control, i.e., $W_a = W(\phi = 0, M)/W(\phi = 0, M = 0)$, was small. Of course, the absolute value of the radial displacement is affected by the value of the driving frequency, but in this analysis the emphasis was on the normalized value $W_a$ since it illustrates the effectiveness of the control.

The results generated for various values of nondimensional parameters
\[
A = \frac{bd}{as_0}, \quad L = \frac{l_a}{l + l_a},
\]
are presented in Figs. 3 and 4. As shown in Fig. 3, the amplitude of vibrations can be significantly reduced, particularly if the distance between the pairs of the stiffeners is small (small
values of $\phi$). Note that the ratios $A$ selected in the examples correspond to geometries that are feasible in practical design. As follows from Fig. 4, even if the amplitude of the pressure is relatively high, a significant reduction of the radial displacement is still possible. An additional conclusion available from the numerical analysis was that the effectiveness of the active control increases proportionally to voltage.

4 Conclusions

The problem of active control of reinforced composite spherical shells is considered in the paper. Active control is implemented through pairs of piezoelectric actuators bonded to the stiffeners. The actuators produce zero net force and a bending moment that reduces radial displacements. This is achieved by either using phase reversed actuators in each pair or applying out-of-phase voltage to the actuators that form pairs. In all situations, the active moment generated with the frequency of the driving pressure reduces radial displacements and the stresses in the shell. An alternative design discussed in the paper utilizes pairs of actuators mounted on local supporting structures bonded to the shell. This eliminates a need in conventional stiffeners and may result in a noticeable weight reduction.

The theory is developed based on the Donnell-Mushtari-Vlasov version of the Love shell theory. The contribution of the stiffeners as well as the effect of piezoelectric actuators are introduced through a smeared stiffeners technique. General equations are obtained for the case where the geometry of meridional stiffeners is such that the ratios of the their cross sectional area and the first and second moments about the shell middle surface to the spacing remain constant. Similar restrictions are applied to the actuators. Without these simplifications, the coefficients in the constitutive relations are variable and the analytical solution becomes infeasible. Even in the case considered in the paper, the equations of motion and the boundary conditions include variable coefficients that depend on the meridional coordinate. While a numerical integration is possible, an exact solution can be obtained using a Geckeler-type approximation for narrow spherical panels and for the panels where the meridional coordinate varies in the range between $75^\circ$ and $90^\circ$.

Numerical results were generated for a flexible spherical panel manufactured from graphite/epoxy and controlled by pairs of commercially available piezoceramic actuators. Using the control voltage of 200 V which is within the allowable range, it was possible to achieve a significant reduction of radial displacements. In summary, the theory and results presented in the paper illustrate a feasibility of active control of spherical shells and panels using piezoelectric actuators.

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References

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