**Effect of Aerodynamic Heating on Deformation of Composite Cylindrical Panels in a Gas Flow**

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**ABSTRACT**

In this paper, motions of composite cylindrical panels in a gas flow are considered. It is shown that the main factor contributing to large static deformations is a nonuniform aerodynamic heating, while aerodynamic pressure is of secondary importance, at high Mach number.

It was found that the main factor resulting in the increase of deformations is the nonuniform distribution of temperature along the curved edges. Deformations decrease rapidly in shallower panels. Nonuniformly heated panels become unstable at the values of axial compressive load, which are much smaller than the static buckling value calculated in the absence of heating. The condition of panel flutter of nonuniformly heated composite panels in a gas flow is also formulated.

**1 INTRODUCTION**

Problems of panel flutter of composite cylindrical shells have been studied since the 1970s. At high flight velocities and with modern

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materials, aerodynamic heating has to be included in the analysis. The effect of temperature on supersonic panel flutter of aerospace structures has also been studied.\textsuperscript{4-9}

In this paper, the effect of aerodynamic heating on the behavior of a simply supported composite cylindrical panel in a supersonic gas flow is considered. Both aerodynamic and material damping effects are neglected. The aerodynamic pressure is described by the Ackeret theory. The temperature distribution is assumed to be a general quadratic function of both the $x$- and the $y$-position coordinates:

$$T=(a_0 + a_1 x + a_2 x^2)(b_0 + b_1 y + b_2 y^2) T_0$$

This assumption permits reduction to uniform, linear and parabolic distributions of the temperature in particular cases.

It is shown that cylindrical panels experience static deformations in a nonuniform thermal field. These deformations can exist even if the gas flow is absent. The presence of a gas flow can result in panel flutter, superimposed on static deformations. The effects of different geometries, temperature distributions and axial compression on static deformations of nonuniformly heated panels are considered in numerical examples.

2 ANALYSIS

Consider a cylindrical panel in a supersonic gas flow (Fig. 1). The axial length of the panel is $a$, the arc length of the curved edges is $b$, and the radius of the middle surface is $R$. The panel is formed of layers symmetrically oriented with respect to the middle surface. The number of layers is large so that the analysis can utilize the assumption that the panel is orthotropic. The analysis is based on Donnell's shell theory, which was shown to be sufficiently accurate for thin, shallow composite shells that are not too long.\textsuperscript{10}

The equations of motion of the panel can then be written in terms of displacements and thermal and external forces as follows:

$$L_{11} u + L_{12} v + L_{13} w - N_{1,x}^T - N_{2,y}^T = 0$$
$$L_{12} u + L_{22} v + L_{23} w - N_{6,x}^T - N_{2,y}^T = 0$$
$$L_{13} u + L_{23} v + L_{33} w - N_1 w_{,xx} - N_2 w_{,yy} = q - \rho h \ddot{w}$$

where $u$, $v$, and $w$ denote the axial, circumferential and radial displacements, respectively; $\ddot{w}$ is the second derivative of $w$ with respect to time; $N_1^T$, $N_2^T$, $N_6^T$ are thermal stress resultants; $N_1$ and $N_2$ are the stress
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resultants of external loads acting in the $x$ and $y$ directions, respectively; $\rho$ is the mean density of the material; $h$ is the total panel thickness; and $q$ is the intensity of aerodynamic loading. The differential operators $L_i$ corresponding to the generalization of Donnell's theory to orthotropic shells are\(^{10}\)

\[
L_{11} = A_{11} d_x^2 + A_{66} d_y^2; L_{12} = (A_{12} + A_{66}) d_x d_y \\
L_{13} = (A_{12}/R) d_x; L_{22} = A_{66} d_x^2 + A_{22} d_y^2; L_{23} = (A_{22}/R) d_y \\
L_{33} = D_{11} d_y^2 + 2(D_{12} + 2D_{66}) d_x^2 d_y + D_{22} d_y^4 + A_{22}/R^2
\] (3)

In eqn (3), $A_j$ and $D_j$ are extensional and bending stiffnesses, respectively; $d_x$ and $d_y$ denote differentiations with respect to the corresponding argument.

Following the well-known Ackeret supersonic theory,\(^{11}\) one can represent the aerodynamic pressure as

\[
q = -\frac{\rho_a c^2 M^2}{\sqrt{M^2 - 1}} \dot{w}_x
\] (4)

where $\rho_a$ is the gas density, $c$ is the sound velocity in the gas, and $M$ is the Mach number. The vector of thermal stress resultants is given by

\[
N^T = \sum_{k=1}^{N} Q^{(k)} \alpha^{(k)} h_k T
\] (5)

where

\[
N^T = \begin{bmatrix} N_1^T \\ N_2^T \\ N_6^T \end{bmatrix}, \quad \alpha^{(k)} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_6 \end{bmatrix}_k
\]
and

$$Q^{(k)} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{12} & Q_{22} & Q_{26} \\ Q_{16} & Q_{26} & Q_{66} \end{bmatrix}$$ (6)

Here, $Q^{(k)}$ is the matrix of the transformed plane-stress reduced stiffnesses of the $k$-th layer, $\alpha^{(k)}$ is the vector of the transformed thermal expansion coefficients of the same layer and $T$ is the temperature, which is assumed to be independent of time. The total number of layers is $N$, the thickness of the $k$-th layer is $h_k$.

The relationships between the elements of the vector $\alpha^{(k)}$ and $\alpha_L$ and $\alpha_T$, the coefficients of thermal expansion in the principal orthotropic directions of the layer, are

$$\alpha_1 = \alpha_L \cos^2\theta + \alpha_T \sin^2\theta$$
$$\alpha_2 = \alpha_L \sin^2\theta + \alpha_T \cos^2\theta$$
$$\alpha_6 = (\alpha_L - \alpha_T) \sin \theta \cos \theta$$ (7)

where $\theta$ is the lamination angle.

The temperature distribution is assumed to be a general quadratic function of the middle-surface coordinates given by eqn (1). This assumption permits consideration of uniform, linear and parabolic distributions, in particular cases. Under these circumstances

$$N^T = \beta T$$ (8)

where

$$\beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_6 \end{bmatrix}$$ (9)

is the vector of reduced thermal stiffness coefficients

$$\beta = \sum_{k=1}^{N} Q^{(k)}_{ij} \alpha^{(k)}_j h_k \quad (i, j = 1, 2, 6)$$ (10)

The boundary conditions at the edges $x=0$, $x=a$, $y=0$ and $y=b$ correspond to S2 conditions in the terminology of Almroth\textsuperscript{12} or SS3 conditions in the terminology of Hoff:\textsuperscript{13}

$$x=0, a \quad N_1 = v = w = M_1 = 0$$
$$y=0, b \quad N_2 = u = w = M_2 = 0$$ (11)
These conditions are satisfied if the displacements field is represented by the following series:

\[ u = \sum_n \left( U_{1n} \cos \frac{\pi x}{a} + U_{2n} \cos \frac{2\pi x}{a} \right) \sin \frac{n\pi y}{b} \]

\[ v = \sum_n \left( V_{1n} \sin \frac{\pi x}{a} + V_{2n} \sin \frac{2\pi x}{a} \right) \cos \frac{n\pi y}{b} \]

\[ w = \sum_n \left( W_{1n} \sin \frac{\pi x}{a} + W_{2n} \sin \frac{2\pi x}{a} \right) \sin \frac{n\pi y}{b} \]

The substitution of eqn (12) into the first two parts of eqn (2) and applying the Galerkin procedure yields the following two independent sets of algebraic equations:

\[ k_1 \ddot{U}_{1n} + k_2 \ddot{V}_{1n} = k_3 \ddot{W}_{1n} + T_1 \]

\[ k_2 \ddot{U}_{1n} + k_4 \ddot{V}_{1n} = k_5 \ddot{W}_{1n} + T_2 \]

and

\[ k_6 \ddot{U}_{2n} + k_7 \ddot{V}_{2n} = k_8 \ddot{W}_{2n} + T_3 \]

\[ k_7 \ddot{U}_{2n} + k_9 \ddot{V}_{2n} = k_{10} \ddot{W}_{2n} + T_4 \]

where

\[ (\ddot{U}_{1n}, \ddot{V}_{1n}, \ddot{U}_{2n}, \ddot{V}_{2n}) = (U_{1n}, V_{1n}, U_{2n}, V_{2n})/h \]

are the nondimensional amplitudes of in-surface displacements. The nondimensional coefficients \( k_i \) in eqns (13) and (14) are

\[ k_1 = (\pi \ddot{h})^2 \ddot{A}_{11} + (n\pi\lambda \ddot{h})^2 \ddot{A}_{66}; \quad k_2 = n\lambda (\pi \ddot{h})^2 (\ddot{A}_{12} + \ddot{A}_{66}); \quad k_3 = \pi \ddot{h}^2 \ddot{A}_{12} \]

\[ k_4 = (\pi \ddot{h})^2 \ddot{A}_{66} + (n\pi\lambda \ddot{h})^2 \ddot{A}_{22}; \quad k_5 = n\pi\lambda \ddot{h}^2 \ddot{A}_{22}; \]

\[ k_6 = (2\pi \ddot{h})^2 \ddot{A}_{11} + (n\pi\lambda \ddot{h})^2 \ddot{A}_{66} \]

\[ k_7 = 2k_2; \quad k_8 = 2k_3; \quad k_9 = (2\pi \ddot{h})^2 \ddot{A}_{66} + (n\pi\lambda \ddot{h})^2 \ddot{A}_{22}; \quad k_{10} = k_5 \]

where

\[ \ddot{a} = a/R, \ddot{h} = h/a, \lambda = a/b \]

and

\[ \ddot{A}_{ij} = A_{ij}/E_i \ddot{h} \]

(17)
$E_T$ being the transverse-direction modulus of elasticity of a layer. The nondimensional thermal terms in eqns (13) and (14) are given by the following relations:

\[
T_1 = 8\bar{h}\left\{2(a_2a^2)[b_0f_1(n) + b_1bf_2(n) + b_2b^2f_3(n)]/T_0 + \lambda(a_1a + a_2a^2)[b_1bf_2(n) + 2b_2b^2f_3(n)]/\pi^2 \right\} \\
T_2 = -8\bar{h}\left\{[a_0 + a_1a + a_2a^2(1 - 4/\pi^2)][b_2b^2f_4(n)] + \bar{\beta}_3[-a_1a + a_2a^2][b_1bf_4(n) + b_2b^2f_3(n)]/\pi \right\} \\
T_3 = -2(\lambda/\pi^2)\bar{h}(a_2a^2)[b_1bf_4(n) + 2b_2b^2f_3(n)]/\pi \\
T_4 = 4\bar{h}\left\{[a_0 + a_1a + a_2a^2][b_2b^2f_4(n)] + \bar{\beta}_3(a_2a^2)[b_1bf_4(n) + b_2b^2f_3(n)]/\pi \right\}
\]

where

\[
(\bar{\beta}_1, \bar{\beta}_2, \bar{\beta}_3) = (\beta_1, \beta_2, \beta_3)(T_0/E_T h)
\]

are nondimensional parameters, and

\[
f_1(n) = (1 - \cos n\pi)/n\pi \\
f_2(n) = -(\cos n\pi)/n \\
f_3(n) = -2/(n\pi)^3 - [1/(n\pi) - 2/(n\pi)^3] \cos n\pi \]

\[
f_4(n) = (\cos n\pi - 1)/(n\pi)^2; f_5(n) = 2 \cos n\pi/(n\pi)^2
\]

Note that the products $a_1a$, $a_2a^2$, $b_1b$ and $b_2b^2$ are nondimensional parameters.

From eqns (13) and (14) the nondimensional in-surface amplitudes can be expressed in terms of $\bar{W}_{1n}$ and $\bar{W}_{2n}$:

\[
\bar{U}_{1n} = S_1 \bar{W}_{1n} + S_2 \\
\bar{V}_{1n} = S_3 \bar{W}_{1n} + S_4 \\
\bar{U}_{2n} = S_5 \bar{W}_{2n} + S_6 \\
\bar{V}_{2n} = S_7 \bar{W}_{2n} + S_8
\]

where

\[
S_1 = (k_3k_4 - k_2k_5)/(k_1k_4 - k_2^2) \\
S_2 = (k_4T_1 - k_2T_2)/(k_1k_4 - k_2^2) \\
S_3 = (k_1k_5 - k_2k_3)/(k_1k_4 - k_2^2) \\
S_4 = (k_1T_2 - k_2T_1)/(k_1k_4 - k_2^2) \\
S_5 = (k_3k_4 - k_2k_5)/(k_1k_4 - k_2^2) \\
S_6 = (k_4T_1 - k_2T_2)/(k_1k_4 - k_2^2) \\
S_7 = (k_1k_5 - k_2k_3)/(k_1k_4 - k_2^2) \\
S_8 = (k_1T_2 - k_2T_1)/(k_1k_4 - k_2^2)
\]
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\[ S_5 = (k_8 k_9 - k_7 k_{10})/(k_6 k_9 - k_7^2) \]  
\[ S_6 = (k_9 T_3 - k_7 T_4)/(k_8 k_9 - k_7^2) \]  
\[ S_7 = (k_8 k_{10} - k_7 k_8)/(k_6 k_9 - k_7^2) \]  
\[ S_8 = (k_6 T_4 - k_7 T_3)/(k_6 k_9 - k_7^2) \]  

The substitution of eqns (4) and (12) into the third part of eqn (2) and applying again the Galerkin procedure yields the set of differential equations:

\[ (\rho h^2/E_T) \ddot{W}_{1n} + k_{11} \dot{U}_{1n} + k_{12} \dot{V}_{1n} + k_{13} W_{1n} = k_{14} \ddot{W}_{2n} \tilde{M} \]  
\[ (\rho h^2/E_T) \ddot{W}_{2n} + k_{15} \dot{U}_{2n} + k_{16} \dot{V}_{2n} + k_{17} W_{2n} = k_{18} \dot{W}_{1n} \tilde{M} \]  

In eqn (24)

\[ \tilde{M} = M^2/\sqrt{M^2 - 1} \]  

The nondimensional coefficients are given by

\[ k_{11} = -\pi \tilde{h}^2 \tilde{A}_{12} = -k_3 \]  
\[ k_{12} = -n \pi \lambda \tilde{h}^2 \tilde{A}_{22} = -k_5 \]  
\[ k_{13} = (\pi \tilde{h})^4 [\tilde{D}_{11} + 2(\tilde{D}_{12} + 2\tilde{D}_{66})(n\lambda)^2 + \tilde{D}_{22}(n\lambda)^4] + (\tilde{h}^2) \tilde{A}_{22} + \tilde{N}_1 + (n\lambda)^2 \tilde{N}_2 \]  
\[ k_{14} = (8/3) \tilde{h}(\rho a c^2/E_T) \]  
\[ k_{15} = 2k_{11}; k_{16} = k_{12} \]  
\[ k_{17} = (\pi \tilde{h})^4 [16\tilde{D}_{11} + 8(\tilde{D}_{12} + 2\tilde{D}_{66})(n\lambda)^2 + D_{22}(n\lambda)^4] + (\tilde{h}^2) \tilde{A}_{22} + 4\tilde{N}_1 + (n\lambda)^2 \tilde{N}_2 \]  
\[ k_{18} = -k_{14} \]  

where

\[ \tilde{D}_q = D_q/E_T h^3 \]  

and

\[ \tilde{N}_i = \left( \frac{\pi}{a} \right)^2 N_i h/E_T \]  

\[ (i = 1, 2) \]  

is the nondimensional axial load.
Equation (24) can be written in terms of radial deflections only:

\[
\left( \frac{\rho h^2}{E_T} \right) \ddot{W}_{1n} + R_1 \dot{W}_{1n} + R_2 = k_{14} \tilde{M} \dot{W}_{2n}
\]
\[
\left( \frac{\rho h^2}{E_T} \right) \ddot{W}_{2n} + R_3 \dot{W}_{2n} + R_4 = k_{18} \tilde{M} \dot{W}_{1n}
\]

(29)

where

\[
R_1 = k_{11}S_1 + k_{12}S_3 + k_{13}
\]
\[
R_2 = k_{11}S_2 + k_{12}S_4
\]
\[
R_3 = k_{15}S_5 + k_{16}S_7 + k_{17}
\]
\[
R_4 = k_{15}S_6 + k_{16}S_8
\]

(30)

The solution of eqn (29) consists of static and dynamic parts:

\[
\dot{W}_{1n} = \dot{W}_{1n}^0 + \dot{W}_{1n}^i \sin \omega t
\]
\[
\dot{W}_{2n} = \dot{W}_{2n}^0 + \dot{W}_{2n}^i \sin \omega t
\]

(31)

where \(\omega\) is the frequency of harmonic vibrations of the fluttering panel.

The substitution of eqn (31) into eqn (29) yields two independent sets of algebraic equations. The solution of the set including the static deflections is

\[
\dot{W}_{1n}^0 = \frac{(R_2 + k_{14}R_4/R_3) \tilde{M}}{(k_{14}k_{18}/R_3) \tilde{M}^2 - R_1}
\]
\[
\dot{W}_{2n}^0 = (k_{18} \tilde{M} / R_3) \dot{W}_{1n}^0 - R_4/R_3
\]

(32)

The set including the dynamic deformations can be written in the form:

\[
(R_1 - \tilde{\omega}^2) \ddot{W}_{1n}^i - k_{14} \tilde{M} \dot{W}_{2n}^i = 0
\]
\[
- k_{18} \tilde{M} \dot{W}_{1n}^i + (R_3 - \tilde{\omega}^2) \ddot{W}_{2n}^i = 0
\]

(33)

where

\[
\tilde{\omega}^2 = \rho h \omega^2 / E_T
\]

(34)

The Mach number parameter \(\tilde{M}\) corresponding to the flutter oscillations of the panel is obtained from eqn (32) as

\[
\tilde{M} = \sqrt{(R_1 - \tilde{\omega}^2)(R_3 - \tilde{\omega}^2) / (k_{14}k_{18})}
\]

(35)

Note that \((k_{14}k_{18}) < 0\). Therefore, the frequency of motion of the fluttering panel is in the interval

\[
\min(R_1, R_3) < \tilde{\omega}_2 < \max(R_1, R_3)
\]

(36)
The flutter velocity parameter corresponding to the minimum value of flight velocity at which flutter becomes unavoidable is given by

\[ M_{cr} = \frac{|R_1 - R_3|}{2k_{14}} \]  

(37)

It can be concluded that the panel may have two types of deformations. Static deformations given by eqn (32) exist if the distribution of temperature is nonuniform. If the temperature is uniformly distributed over the platform of the panel, these deformations vanish within the Galerkin-type analysis. Panel flutter can be superimposed on static deformations at the Mach number given by eqn (35). In the numerical example, attention is concentrated on static deflections of the panel at the center (see eqn (32)):

\[ \bar{W} = \sum_n \bar{W}_{1n} \sin n\pi/2 \]  

(38)

which is the largest deflection.

3 NUMERICAL EXAMPLES

The cylindrical panel considered in the numerical examples is assumed to be made from boron fiber/aluminum 7178-T6 matrix composite, with room temperature properties: \( E_L = 214 \) GPa, \( E_T = 131 \) GPa, \( G_{LT} = 44 \) GPa, \( \nu_{LT} = 0.255 \), \( \alpha_L = 9.2 \mu m/m K \) and \( \alpha_T = 18.2 \mu m/m K \).

The calculations were carried out by assuming that \( T_0 = 371^\circ C \) and the effect of high temperature on mechanical properties is uniform throughout the plate. This assumption imposes limits on the magnitude of \( a_1a, a_2a, b_1b, b_2b^2 \), which cannot be too large. Degradation of mechanical properties due to high temperature was calculated, based on data on the mechanical properties of boron/aluminum at \( T_0 = 0^\circ \) and 371°C in Ref. 14: \( E_L = 154 \) GPa, \( E_T = 35.9 \) GPa and \( G_{LT} = 11.7 \) GPa. The Poisson ratios and the coefficients of thermal expansion were assumed to be unaffected by temperature. Circumferential load \( N_2 \) was zero in all examples. The panel was formed of 30 layers symmetrically oriented with the lamination scheme \( 0^\circ \pm 45^\circ \) (two layers adjacent to the middle surface have the lamination angle \( 0^\circ \)).

The effect of the Mach number on static deflections of the panel appeared to be negligible. This can be explained by the analysis of the coefficient \( \rho_s c^2/E_T \) in \( k_{14} \) and \( k_{18} \) which has an order of \( 10^{-6} \). There-
fore, static deformations are due to aerodynamic heating and only to a negligible degree to aerodynamic pressure. Note that in geometrically nonlinear problems this conclusion may be invalid. In the following examples, the Mach number was always taken as \( M = 4 \).

As can be seen from the solution, temperature change causes transverse deflections only if it is nonuniformly distributed over the platform. In the examples, it was supposed that \( a_0 = b_0 = 1 \), \( b_1 b = b_2 b^2 = 1 \), i.e. temperature was nonuniform in the circumferential direction. The values of \( a_1 a \) and \( a_2 a \) were taken equal to either zero or 0.1.

The effect of the panel central \((\text{included})\) angle on static deflections of cylindrical panels is shown in Figs 2 and 3. Deflections are always larger in narrower panels; they decrease at larger panel angles (except for the case of very thick panels as illustrated by curve 3 in Fig. 2) and converge to a constant value. This indicates that wide panels behave as closed circular shells of the same geometry. Similar conclusions were obtained by Sobel and coworkers in problems of static buckling of isotropic panels.\(^{15,16}\) Physically, the decrease of the deflection in wider panels can be explained by the fact that the temperature is 'more uniformly' distributed in such panels if \( b_1 b = b_2 b^2 = \text{a constant} \). Apparently, this affects the magnitude of deflections more than the tendency of wider plates to have large deformations due to the lesser effect of the boundary conditions along the straight edges.

The influence of the length-to-radius ratio is illustrated in Figs 4 and 5. Deflections of the panels tend to approach zero if their radius increases \((\text{small} \ \bar{a})\). The explanation for this fact is that if the radius increases, the panel approaches a flat plate, which has a very small deflection due to nonuniform temperature distribution.

The effect of axial compression is shown in Fig. 6. The deflection curve in all cases considered consists of two branches. At certain values \( N_1/N_{cr} < 1 \), the deflection approaches infinity, i.e. the panel becomes unstable. The right branch can be reached only if the temperature is applied after the application of compressive load, provided that instability in the transient regime is avoided.

The estimation of the influence of temperature on deflections is shown in Figs 7 and 8. The calculations for these figures were carried out by assuming that the mechanical properties of the material remain constant within the range of temperatures considered. The effect of \( T_0 \) on deflections of relatively thick panels is not very large. However, deflections of thin panels increase at a much higher rate with increase in temperature \((\text{curve 1 in Figs 7 and 8})\).

Finally, the influence of spatial distribution of temperature is shown in Fig. 9. The nonuniform distribution of temperature along the straight
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Fig. 2. Effect of panel included angle on static deflection ($\tilde{a} = 0.5$, $a_1 a = a_2 a^2 = 1$; $N_1 = 0$). Curves 1, 2 and 3 correspond to dimensionless thicknesses $\tilde{h}$ of 0.01, 0.02 and 0.05, respectively.

Fig. 3. Effect of panel included angle on static deflection ($\tilde{a} = 0.5$, $a_1 a = a_2 a^2 = 0$; $N_1 = 0$). Curves 1 and 2 correspond to dimensionless thicknesses $\tilde{h}$ of 0.01 and 0.02, respectively.

Fig. 4. Effect of length-to-radius ratio on static deflection ($\tilde{h} = 0.01$, $\tilde{a} = 2$, $N_1 = 0$). Curves 1 and 2 correspond to values of $a_1 a = a_2 a^2$ of 0 and 0.1, respectively.
Fig. 5. Effect of length-to-radius ratio on static deflection ($\hat{h} = 0.02$, $\lambda = 2$, $N_1 = 0$). Curves 1 and 2 correspond to values of $a_1, a = a_2 a^2$ of 0 and 0.1, respectively.

Fig. 6. Effect of dimensionless axial compression on static deflection ($\lambda = 2$). Curves 1 and 2 correspond to $a_1, a = a_2 a^2$ values of 0 and 0.1, with $\hat{h} = 0.01$ for both; curves 3 and 4 correspond to $a_1, a = a_2 a^2$ values of 0 and 0.1, with $\hat{h} = 0.02$ for both.

Fig. 7. Effect of temperature on static deflection ($\beta = 0.5$, $\lambda = 2$; $a_1, a = a_2 a^2 = 0$; $N_1 = 0$). Curves 1, 2 and 3 correspond to dimensionless thicknesses of 0.010, 0.015 and 0.020, respectively.
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edges does not change the character of the deflection versus relative thickness \( \bar{h} \) relationship. However, if \( b_1 b = b_2 b^2 = 0 \) while \( a_1 a = a_2 a^2 = 0.1 \), the magnitude of \( \bar{W} \) is of the order of \( 10^{-3} \) only (this curve is not shown in Fig. 9). This means that significant static deformations can exist only if the temperature is nonuniformly distributed along the curved edges.

4 CONCLUSIONS

Deformations of nonuniformly heated composite cylindrical panels in a supersonic gas flow have been considered. It was found that static deformations can be considerable if temperature is nonuniformly distributed over the planform. The effect of aerodynamic pressure on static deformations is negligible compared with the effect of nonuniform aerodynamic heating at high velocities of flight. Significant static deformations can be reached only if temperature is nonuniformly distributed along the curved edges.
As the panel angle is increased, the static deformations approach a constant value, which is equal to deformation of a circular cylindrical shell of the same geometry. The deformations are much larger in curved panels than in the shallower panels. There exists, on heating, a critical value of the axial compressive load, which is usually much smaller than the classical buckling load calculated in the absence of heating. The deflection of the panel increases as the temperature is increased. This increase is more pronounced in thinner panels.

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