Effect of temperature on stresses and delamination failure of z-pinned joints

Larry W. Byrd\textsuperscript{a}, Victor Birman\textsuperscript{b,\ast}

\textsuperscript{a}Air Force Research Laboratory, AFRL/VASM, Bldg. 65, 2790 D Street, Wright-Patterson AFB, OH 45433, USA
\textsuperscript{b}University of Missouri-Rolla, Engineering Education Center, One University Boulevard, St. Louis, MO 63121, USA

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Abstract

Although z-pins have been shown effective in preventing delaminations in adhesively bonded and co-cured joints, their applicability depends on a reliable assessment of the strength of a z-pin-composite assembly. In particular, high residual thermal stresses that have been found in experiments dictate the necessity in a local stress analysis. Elevated temperature applied to the joint during its lifetime may also affect its effectiveness in preventing delaminations. The present paper illustrates an approach to determining local residual stresses confirming the observations regarding a possible delamination and cracking in the composite structure due to high post-processing transverse stresses. The analysis of the effect of elevated temperature applied at one of the surfaces on the response of a z-pinned joint is conducted using the concept of a double cantilever beam with an “insulated” crack. In addition, it is illustrated that an elevated temperature may actually benefit the integrity of the joint if it causes an increase in the z-pin-composite interfacial strength.

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1. Introduction

Delamination cracks originating from the edge are recognized as the principal cause of damage and failure in bonded adhesive and co-cured joints. Z-pins, i.e. small-diameter cylindrical rods embedded in the composite material and oriented perpendicular or at an angle to the layer interface, represent a possible method of arresting these cracks. Extensive studies that illustrated beneficial effects of z-pins on various aspects of the behavior of composite structures have been published by Freitas et al. [1], Barrett [2], Lin and Chan [3], Palazotto et al. [4], Vaidya et al. [5] and Mabson and Deobald [6]. In particular, this method may be effective in enhancing fracture and fatigue resistance of co-cured joints between composite skin and stiffeners similar to the joint depicted in Fig. 1.

The advantages associated with using z-pins to reduce or prevent delamination in PMC have been documented in numerous studies. For example, Freitas et al. [1,7] illustrated that a 1.9\% volume fraction of carbon z-pins can increase Mode I fracture toughness of laminates by a factor of 18 with only a modest reduction in in-plane tensile strength. Z-pins inclined at 45\(^\circ\) have also been shown beneficial for lap shear specimens [8].

Recent studies of z-pinned co-cured joints conducted by the authors on the example of a double cantilever beam (DCB) shown in Fig. 2 have further illustrated the effectiveness of z-pins in preventing fracture and arresting existing delamination cracks [9,10]. Other recently published papers considering z-pin technology dealt with manufacturing aspects of z-pinned laminates [11], experimental evaluation of the z-pin bridging law [12], and thermal residual stresses around z-pin [13]. In particular, the results of a finite element analysis and experimental data presented in the latter paper illustrated the presence of large residual stresses in z-pinned laminates. These stresses could be sufficient to cause failure in a standard matrix resin of PMC resulting in post-processing cracking. Therefore, the first part of the present study justifies and illustrates a simple, yet accurate, computational procedure.
capable of predicting residual thermal stresses in a z-pinned composite laminate.

Thermal loading applied during the lifetime of a z-pinned component produces both local thermal stresses due to the thermal mismatch between z-pins and substrate material as well as a “global” deformation and associated stresses. An approximate approach to estimating the former effect is presented in the second part of the analysis.

The third part of the analysis deals with the effect of elevated temperature on fracture of z-pinned co-cured joints associated with their global thermally and mechanically induced deformations. The paper provides a methodology of the analysis of the effect of z-pins on the integrity of z-pinned co-cured joints for the case of a nonuniform temperature introducing a new concept of “insulated crack” in the DCB test setting. A number of conflicting tendencies may affect the integrity of the joint in the presence of temperature, including variations of the material properties, thermally induced stresses, and changes in the interfacial shear strength between z-pins and the composite material. A large effect of the interfacial shear strength on the integrity of joints undergoing Mode I loading observed in numerical examples leads to the recommendation to employ artificially uneven surfaces of z-pins to maximize their resistance to pullout, even at the room temperature.

2. Analysis

2.1. Part 1: Residual thermal stresses in a z-pinned laminated material

The model (representative cell) employed for the analysis is shown in Fig. 3. Typical z-pin joints contain a large number of z-pins but their volume fraction, i.e. the fraction of the unit volume of the composite material occupied by z-pins, is invariably small, never exceeding 3% (in actual applications, it is usually closer to 1%). Therefore, the model shown in Fig. 3 represents a single pin surrounded by the composite laminated material. The outer radius of the model shown in Fig. 3 is determined from the requirement that the areal density of the z-pin in this model corresponds to the density of z-pins in the actual joint (in case where z-pins are extended through the entire thickness of the composite laminate, the areal density of z-pins is equal to their volume fraction). Accordingly, if the areal
density of z-pins is denoted by \( A_p \) and the z-pin radius is \( r = a \), the outer radius should be \( b = a/\sqrt{A_p} \).

The analysis is based on the following assumptions:

1. The pin material is transversely isotropic, and the composite laminate consists of identical transversely isotropic layers. Note that a transversely isotropic pin reflects typical applications in CMC joints where such pin is manufactured from woven or straight ceramic fibers with the possible addition of a ceramic matrix. The properties of such pin are isotropic in the planes perpendicular to the axis of the pin. Z-pins employed in PMC joints include T300/Epoxy, T300/BMI, titanium, aluminum, S-Glass/Epoxy, etc. [11], all these materials are either transversely isotropic or isotropic.

2. Temperature changes during post-processing cooling are uniform. Transient effects during the cooling period are not considered.

3. The material constants of the laminate and pin are not affected by temperature.

4. The z-pin extends to the entire depth of the laminate.

5. The requirement of zero normal stresses in the \( \zeta \)-direction at each point of the pin surfaces \( z = 0, h \) is replaced with the requirement of zero net force acting in the \( \zeta \)-direction. This should result in a small inaccuracy of the solution in the immediate vicinity to the surfaces, but the effect of such simplification on the stress distribution should rapidly die out with the distance from the surfaces, according to the St. Venant principle.

6. It is further assumed that cross sections \( z = \) constant remain plane both during cooling and when the surface \( z = 0 \) is subject to a uniform heat flux or elevated temperature (as is the case in the second part of the analysis). Physically, such assumption is in agreement with the observation that the surface of the joint remains flat during lifetime. In the case of uniform temperature changes transverse shear strain and stress are assumed negligible implying that radial displacements are independent of the transverse \( (\zeta) \) coordinate.

Obviously, these assumptions render the problem considered here axisymmetric. Accordingly, circumferential stresses as well as all derivatives with respect to the circumferential coordinate are equal to zero. Axisymmetric equations of equilibrium in a transversely isotropic material of the z-pin or composite quasi-statically deforming without transverse shearing deformations due to a uniform change of temperature \( T \) are [14]:

\[
\frac{\partial \sigma_r^{(i)}}{\partial r} + \frac{\sigma_r^{(i)} - \sigma_{\theta}^{(i)}}{r} = 0, \quad \frac{\partial \sigma_{\theta}^{(i)}}{\partial \zeta} = 0, \quad (1)
\]

where the radial coordinate is counted from the axis of the z-pin and the index ‘‘\( i \)’’ refers to the z-pin \( (i = p) \) or composite material \( (i = c) \), and we use standard notations for the stresses.

The thermoelastic stress–strain relationships can be written in the form

\[
\begin{bmatrix}
\sigma_r^{(i)} \\
\sigma_{\theta}^{(i)} \\
\sigma_{\zeta}^{(i)}
\end{bmatrix} =
\begin{bmatrix}
C_{11}^{(i)} & C_{12}^{(i)} & C_{13}^{(i)} \\
C_{12}^{(i)} & C_{11}^{(i)} & C_{13}^{(i)} \\
C_{13}^{(i)} & C_{13}^{(i)} & C_{33}^{(i)}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_r^{(i)} \\
\varepsilon_{\theta}^{(i)} \\
\varepsilon_{\zeta}^{(i)}
\end{bmatrix} - \begin{bmatrix}
0 \\
0 \\
2 \zeta
\end{bmatrix} T. \quad (2)
\]

In these equations, the coefficient of thermal expansion in the isotropic plane is without a subscript, while its counterpart in the \( \zeta \)-direction is identified by the subscript ‘‘\( \zeta \)’’. The elements of the matrix of stiffnesses are the following functions of the engineering constants of the corresponding material (for simplicity, the superscript ‘‘\( s \)’’ is omitted in the following equations) [15]:

\[
\begin{align*}
C_{11} &= 1 - \nu_{\zeta} v_{\zeta}, & C_{12} &= \nu + \nu_{\zeta} v_{\zeta} \frac{E_{\zeta}}{E}, \\
C_{13} &= \nu_{\zeta}(1 + \nu), & C_{33} &= 1 - \nu^2, \\
\Delta &= 1 - \nu^2 - 2 \nu_{\zeta} v_{\zeta} - 2 \nu_{\zeta} v_{\zeta} v_{\zeta}. \quad (3)
\end{align*}
\]

The strains that appear in (2) are given by the following functions of the radial displacement \( u \) and a displacement in the \( \zeta \)-direction \( w \):

\[
\begin{align*}
\varepsilon_r &= u_r, & \varepsilon_{\theta} &= \frac{u}{r}, & \varepsilon_{\zeta} &= w. \quad (4)
\end{align*}
\]

The assumption that plane cross-sections \( z = \) constant do not warp during cooling implies that \( \varepsilon_{\zeta} = 0 \). Therefore, the first equation of equilibrium (1) yields

\[
\varepsilon_{rr} + \frac{u_r}{r} - \frac{u}{r} = 0. \quad (5)
\]

The solution of (5) is

\[
u^{(p)} = C_1 r + \frac{C_0}{r}, \quad u^{(c)} = C_3 r + \frac{C_2}{r}, \quad (6)
\]

where the constant of integration \( C_0 = 0 \) to avoid the singularity at the axis of the pin \( r = 0 \).

The boundary conditions in the planes \( z = \) constant that should be satisfied are identical to those in a standard concentric cylinders micromechanical model, except for the presence of temperature:

\[
\begin{align*}
\sigma_r^{(c)}(r = b) &= 0, \\
u^{(c)}(r = a) &= u^{(p)}(r = a), \\
\sigma_r^{(c)}(r = a) &= \sigma_r^{(p)}(r = a). \quad (7)
\end{align*}
\]

These conditions yield the following three algebraic equations:

\[
k_{jk} C_1 + k_{jk} C_2 + k_{jk} C_3 + k_{jk} w = k_{jk} T \quad (j = 1, 2, 3), \quad (8)
\]
where

\[ k_{11} = 0, \quad k_{12} = \frac{C_{12}^{(c)} - C_{11}^{(c)}}{b^2}, \quad k_{13} = C_{11}^{(c)} + C_{12}^{(c)}, \]
\[ k_{14} = C_{13}^{(c)}, \quad k_{15} = \left( \frac{C_{11}^{(c)} + C_{12}^{(c)}}{C_{12}^{(p)}} \right) x_z^{(c)} + C_{13}^{(c)} x_z^{(p)}, \]
\[ k_{21} = -k_{23} = a, \quad k_{22} = -\frac{1}{a}, \quad k_{24} = k_{25} = 0, \]
\[ k_{31} = -\left( \frac{C_{11}^{(p)} + C_{12}^{(p)}}{C_{12}^{(p)}} \right), \quad k_{32} = \frac{C_{12}^{(c)} - C_{11}^{(c)}}{a^2}, \quad k_{33} = C_{11}^{(c)} + C_{12}^{(c)}, \]
\[ k_{34} = C_{13}^{(c)} - C_{13}^{(p)}, \quad k_{35} = k_{15} - k_{13}^{(p)}. \] (9)

In the last formula (9),

\[ k_{15}^{(p)} = \left( \frac{C_{11}^{(p)} + C_{12}^{(p)}}{C_{12}^{(p)}} \right) x_z^{(p)} + C_{13}^{(p)} x_z^{(p)}. \] (10)

The additional condition necessary to satisfy the assumption of zero net force in the axial direction implies that

\[ \int_0^a 2\pi r \sigma_z^{(c)} dr = \int_0^b 2\pi r \sigma_z^{(c)} dr. \] (11)

This requirement yields

\[ k_{41} C_1 + k_{42} C_2 + k_{43} C_3 + k_{44} e_z = k_{45} T, \] (12)

where

\[ k_{41} = -k_{31} a^2, \quad k_{42} = \frac{\left( C_{12}^{(c)} - C_{11}^{(c)} \right) (a^2 - b^2)}{a^2 b^2}, \]
\[ k_{43} = -k_{33} \left( b^2 - a^2 \right), \quad k_{44} = \frac{C_{13}^{(p)} a^2}{b^2} - \frac{C_{13}^{(c)} b^2 - a^2}{2}, \]
\[ k_{45} = \frac{k_{13}^{(p)} a^2}{2} - \frac{k_{15} (b^2 - a^2)}{2}. \] (13)

The solution of the system of Eqs. (8) and (12) yield all unknowns, i.e. \( C_1, C_2, C_3 \) and \( e_z \), in terms of temperature changes \( T \). Note that the second equilibrium condition (1) is identically satisfied since all strains found in this solution are independent of the axial \( (z) \) coordinate.

2.2. Part 2: Transient thermal stresses due to an elevated temperature applied at one of the surfaces of the joint

The thermal load applied to the joint during its lifetime is dynamic. This load usually represents a heat flux or an elevated temperature applied at the exposed surface of the joint. In this section, the local thermal stresses around a z-pin are estimated. Notably, thermal stresses due to an elevated temperature are superimposed on the residual thermal stresses. The sign of these stresses is opposite, so that it is plausible that the total stresses are actually reduced due to an elevated temperature applied during the lifetime of the joint. Nevertheless, an accurate analysis requires the designer to monitor the stresses both after post-processing cooling as well as when the joint is subject to dynamically applied operational temperatures, since these temperatures are accompanied by cyclic or quasi-static mechanical stresses.

The qualitative analysis can be performed based on the following additional assumptions, in conjunction with those introduced in the previous part:

1. Inertial effects do not affect the stress analysis. This is justified by a large difference in the frequency of thermal load applied to the joint and the fundamental frequency of the joint. Accordingly, it is possible to account for variations in temperature with time, while neglecting inertial terms in the equations of equilibrium.
2. Temperature being a function of the \( z \)-coordinate, it is impossible to neglect variations of radial displacements with this coordinate. Therefore, transverse shear strains are present in this problem, contrary to the previous problem of residual stresses. However, the effect of transverse shear stresses on the equilibrium is neglected.

The previous assumption means that the terms dependent on the shear stress are neglected in the equilibrium equation in the axial direction

\[ \frac{\partial \sigma_z^{(c)}}{\partial z} + \frac{\partial \sigma_z^{(c)}}{\partial z} + \frac{\tau_{rz}}{r} = 0, \] (14)

so that this equation converges to the second equation (1).

The temperature distribution in a representative cell consisting of a quarter pin and the surrounding composite material resulting from exposure of the surface \( z = 0 \) to hot gas and an insulated surface \( z = h \) was obtained using the FEM code ABAQUS (Fig. 4) and compared to the well-known Heisler approximation [16]. The results of this comparison are shown in Fig. 5 where the temperature distribution is presented for a representative time instant. As follows from these results, the radial heat flow at each cross-section \( z = \) constant is negligible compared to the heat flow in the thickness direction, i.e. the heat transfer process is predominantly one-directional. Therefore, the Heisler approximation is adequate for the prediction of temperature distribution corresponding to such one-dimensional process. Accordingly, temperature is represented by the following function of time and the coordinate counted from the heated surface [16]:

\[ T = T(z', t) = T_{\infty z} + (T_0 - T_{\infty z}) e^{-z'/h} \cos \frac{\xi z'}{h}, \] (15)

where

\[ \xi \tan \xi = \frac{\hat{h} h}{k}, \quad \bar{C}_1 = \frac{4 \sin \xi}{2 \xi + \sin 2 \xi}. \] (16)

\( z' \) is a coordinate counted from the heated surface and \( \hat{h} \) and \( k \) are the convective heat transfer coefficient of the surface exposed to thermal loading, and the heat conduction coefficient in the \( z \)-direction, respectively.

In the subsequent solution, there is no need to consider the first term in the right side of (15) since it corresponds to a uniform reference temperature. The second term can be
conveniently written in the form

\[ T = T(z, t) = \bar{T} \cos \frac{\varepsilon \zeta'}{h}, \quad (17) \]

where \( \bar{T} = \bar{T}(t) \) is evident.

The assumptions that inertial terms and transverse shear stresses are negligible in the equilibrium equations imply that all displacements, strains and stresses are the same exponential functions of time as temperature that is given by (15). Accordingly, the solution for transient local thermal stresses can be obtained from the corresponding results generated in case of a uniform temperature by replacing constants of integration and the axial strain with the adjusted functions of time and axial coordinate, i.e.

\[ C_i = C'_i T \rightarrow C'_i \bar{T}(t) \cos \frac{\varepsilon \zeta'}{h}, \]

\[ e_z = e'_z T \rightarrow e'_z \bar{T}(t) \cos \frac{\varepsilon \zeta'}{h}, \quad (18) \]

where \( C'_i \) and \( e'_z \) are found from (8) and (12) using \( T = 1 \).

Note that any attempt to generate the solution neglecting this approach would make it impossible to satisfy the continuity conditions for the radial stresses at the pin–composite interface \( r = a \). The present solution satisfies all interface and boundary conditions introduced in Part 1. In addition, transverse shear stresses can be calculated as

\[ \tau_{rz}^{(p)} = G_{rz}^{(p)} u_z^{(p)} = -G_{rz}^{(p)} C'_i \bar{T}(t) \sin \frac{\varepsilon \zeta'}{h}, \]

\[ \tau_{rz}^{(c)} = G_{rz}^{(c)} u_z^{(c)} = -G_{rz}^{(c)} \left( C'_i r + \frac{C'_i}{r} \right) \bar{T}(t) \sin \frac{\varepsilon \zeta'}{h}. \quad (19) \]

These expressions for the shear stress satisfy the boundary condition on the surface \( z = 0 \), i.e. \( \tau_{rz}^{(0)}(z' = 0) = 0 \). The continuity of transverse shear strains along the interface
The only boundary condition for transverse shear stresses that is violated by this solution is on the colder analysis is closer to the heated surface solution. 

correspond to three different values of z-pin volume fraction. Rotational approach, as was indicated by Rugg et al. [17]. The reason be successfully carried out using a fracture toughness function of the crack length and its evaluation requires a complicated bridging analysis. A new criterion of failure of the former stress on the equilibrium equations is negligible. It is noted that by comparing the magnitude of transverse shear stress to the radial, circumferential and axial stresses one can assess the validity of the assumption that the effect of the former stress on the equilibrium equations is negligible.

2.3. Part 3: Methodology of the analysis of the effect of a nonuniform or uniform temperature on fracture

The analysis of the integrity of z-pinned joints may not be successfully carried out using a fracture toughness approach, as was indicated by Rugg et al. [17]. The reason is that the strain energy release rate in such joints is a function of the crack length and its evaluation requires a complicated bridging analysis. A new criterion of failure of z-pinned co-cured joints based on the analysis of deflections of the delaminated section was introduced by the authors [9,10]. This convenient criterion enables us to estimate the effectiveness of z-pins in the joints at both room and elevated temperatures. For example, deflections of the free end of a ceramic matrix DCB are shown in Fig. 6 as a function of the length of the delamination crack and the volume fraction of z-pins. As follows from this figure, if the volume fraction of z-pins is relatively small, deflections increase as the crack propagates. Furthermore, at a relatively small length of the crack, z-pins are pulled out of the DCB, beginning with the delaminated end. This is accompanied by a rapid growth of deflections and failure of the specimen. On the other hand, if the volume fraction is larger (1.2%), deflections stop growing as the crack propagates due to the reaction of z-pins. Moreover, deflections begin to decrease at a larger length of the crack. Of course, a decrease of deflections is physically impossible, i.e. once the deflections reach a maximum value (at \( a \approx 25 \) mm), the crack is arrested. An expansion of the existing solutions [9,10] to the case where an elevated temperature is applied at one of the surfaces of the joint based on the standard DCB test model is shown below.

Co-cured z-pinned CMC joints will likely be used in a high-temperature environment characterized by the range of temperature from 650°C to 1200°C (≈1200°F to 2200°F). An elevated temperature will often be applied at one of the surfaces, while the opposite surface of a joined component will be at a lower temperature. Even PMC z-pinned joints employed in aerospace applications will experience high temperature variations. Accordingly, the problem of a nonuniform temperature distribution through the thickness of the joint represents a major interest. While local stresses due to such temperature have been considered in the previous part, the present section deals with a global response of z-pinned joints in the presence of temperature. The approach to the analysis is illustrated for a DCB model that yields valuable conclusions regarding realistic joint geometries as was shown in [9,10].

An elevated temperature that is uniform over the surface of the component but varies in the thickness direction has the following effects on the joint:

1. Thermally induced stress couples and stress resultants associated with both a nonuniform temperature distribution as well as the corresponding nonuniform distribution of the properties of the constituent materials through the thickness of the joint. In a partially delaminated joint, such as that modeled by DCB, the axial stress resultants are equal to zero since there is no constraint against axial displacements. However, the stress couples are present and they cause bending of both the intact section of DCB as well as bending of the delaminated legs of the specimen.

2. Modification of mechanically induced deformations (due to the forces applied at the delaminated end of DCB) as a result of a degradation of the properties caused by an elevated temperature.

Obviously, both these effects have to be considered in the framework of a single solution. This solution should rely on a distribution of temperature that represents a complicated heat transfer problem by itself (note that the Heisler solution shown in the previous part was obtained by assumption that material properties are not affected by temperature). For example, it was shown by Birman [18] that if the conductivity is a linear function of temperature, i.e.,

\[
k = k_0 + k_1 T,
\]  

the quasi-static temperature varying in the thickness direction is distributed in a homogeneous material.
according to

\[ T = -\frac{k_0}{k_1} + \sqrt{B_1 z + B_0}, \tag{22} \]

where \( B_i \) are constants available from the thermal boundary conditions. In the case where a perfect thermal boundary is maintained between the layers of a laminate, the previous relationship would remain valid, even in a laminated joint.

If local heat transfer in the radial direction from individual \( z \)-pins can be neglected, the solution can be generated using an average conductivity in the \( z \)-direction obtained by the rule of mixtures, i.e.

\[ k = V_p k_p + (1 - V_p) k_l, \tag{23} \]

where \( V_p \) is the \( z \)-pin volume fraction and \( k_p \) and \( k_l \) denote the conductivities of the \( z \)-pin and layer materials, respectively.

The solution given by (21) and (22) could be useful for the analysis of static problems in DCB composed of transversely isotropic layers. The transient temperature distribution in dynamic problems could be estimated using the Heisler approximation discussed above. However, in a typical DCB subject to static or dynamic temperature or heat flux applied at one of the surfaces the assumption of a one-dimensional heat flow may become invalid due to the flow toward the side surfaces of the beam in the cross sections \( x = \) constant (the flow along the axis of the beam will be negligible, except for the region adjacent to the ends as long as thermal loading is independent of the \( x \)-coordinate). In such situations, the distribution of temperature can be obtained from the solutions of Chou and his colleagues referred to in the monograph [19]. The only limitation of these otherwise accurate solutions is related to the assumption that temperature does not affect material properties.

The analysis of a representative DCB can be conducted introducing the concept of “insulated crack”, i.e. assuming that there is no heat loss from the side surfaces of the beam and there is no temperature discontinuity between the faces of the crack (see Fig. 7). This implies that there is no heat transfer out of the crack to the left of the cross section \( x = -a \) of the specimen. This enables us to make the following observations.

The intact section (Section 2) will bend as a result of a nonuniform temperature, resulting in a rotation of the cross section \( x = 0 \). This rotation will cause the same rotation of both legs of Section 1, i.e. there are no associated relative displacements of the legs at the cross section \( x = -a \) as long as warping of the cross section \( x = 0 \) is negligible. However, even if \( T_2 = (T_1 + T_3)/2 \), the property degradation in two legs of Section 1 is different as the magnitude of temperature in two sections differs. Accordingly, thermally induced bending moments in these sections and the corresponding deformations are different as well. Moreover, a different degree of degradation of material constants in two legs causes a different correction to deformations produced by the applied forces. Note that a degradation of the material in Section 2 will affect the elastic clamping coefficient at the cross-section \( x = 0 \).

The approach to the solution extrapolates the method developed by Birman and Byrd [10] for a uniform temperature. The comprehensive list of assumptions employed in this analysis is outlined in [10]. It is noted here that the length of Section 2 is large compared to its thickness, i.e. the section can be treated as semi-infinite. It is also assumed that \( z \)-pins do not experience a pullout from Section 2.

First of all, it is necessary to determine elastic clamping provided by Section 2 to delaminated Section 1. As follows from the solution at the room temperature, the total energy of one half of Section 2 subject to a bending couple at the edge, as shown in Fig. 8, can be obtained neglecting deformations in the thickness direction, so that for a unit-width section

\[
II = \frac{1}{2} \int_{x=0}^{x=\infty} \int_{z=0}^{z=h} \left( Q_{11}(z)c_{11}^2 + Q_{55}(z)c_{55}^2 \right) dx dz - 2NU, \tag{24}
\]

where reduced stiffnesses \( Q_i(z) \) should be specified, accounting for a distribution of temperature and its effect on the material constants. The last term in (24) reflects the work of the couple of stress resultants \( N \) on the displacements of the points of Section 2 where these forces are applied (these are displacements in the \( x \)-direction). Note that (24) is written for a section subject to mechanical loading that causes shear and rotation of the cross section \( x = 0 \), while thermally induced deformations are not considered (these deformations result in a rotation of the
cross section $x = 0$, but they do not affect the elastic clamping.

The strains in (24) are
\[ \varepsilon_x = u_{xx} \quad \gamma_{xz} = u_{xz}, \]
where following the conclusion in [10], the contribution of displacements in the thickness ($z$) direction is neglected. The axial displacements are assumed in the form
\[ u = U e^{-\alpha x} \cos \frac{\pi z}{h}, \]
where $\alpha$ is a decay parameter that is determined below.

The displacements in the form (26) reflect the anticipated shape of a deformed section and satisfy the boundary conditions of the problem, i.e. $u \to 0$ as $x \to \infty$.

Substituting (25) and (26) into (24) and integrating one obtains
\[ \Pi = \frac{1}{4\omega} \left( A_{11} \omega^2 + \left( \frac{\pi}{h} \right)^2 A_{55} \right) U^2 - 2NU, \]
where
\[ A_{11} = \int_0^h Q_{11}(z) \cos^2 \frac{\pi z}{h} \, dz, \]
\[ A_{55} = \int_0^h Q_{55}(z) \sin^2 \frac{\pi z}{h} \, dz. \]

Minimization of the total energy with respect to the decay parameter $\omega$ and with respect to $U$ yields
\[ \omega = \frac{\pi}{h} \sqrt{\frac{A_{55}}{A_{11}}}, \]
\[ U = \frac{2N}{(\pi/h) \sqrt{A_{11} A_{55}}}. \]

Accordingly, the rotational elastic constraint (elastic clamping) coefficient for a unit-width section, accounting for a nonuniform temperature can be evaluated as
\[ K = \frac{\pi h \sqrt{A_{11} A_{55}}}{4}. \]

Note that the elastic clamping coefficients obtained from (31) are different for two legs of Section 1 (Figs. 2 and 7) since the temperature distribution in the corresponding halves of Section 2 differs affecting $A_{11}, A_{55}$. Accordingly, it may be convenient to operate with two different values for legs $a$ and $b$ (Fig. 2), i.e. $K_a$ and $K_b$.

The analysis of deformations of the delaminated legs of Section 1 can now be conducted. Note that even if the temperature gradient through the thickness of each leg is the same, thermally induced moments will differ since different absolute values of temperature result in different material properties for each leg. The moments, properties, and deformations of two legs in the following solution are distinguished assigning to them the subscripts “$a$” (upper leg) and “$b$” (lower leg).

Consider deformations of the legs of the delaminated Section 1 subject to a nonuniform temperature. Each leg is free at the left end and loaded by the force $P$, while at the right end it is elastically clamped with the clamping constant $K_i$ where $i = a, b$. The analysis is conducted by a slender beam theory that yields accurate results in practical configurations [20–23].

The reaction of z-pins to deformations of the legs is given by
\[ p = K_0 - K_1(w_a + w_b). \]

Let us assume that the effect of temperature on the interfacial shear strength can be averaged through the depth of DCB. In this case, following [6,24]:
\[ K_0 = 2V_p \tau_p/r, \quad K_1 = 2V_p \tau/r, \]
where $\tau$ is the interfacial shear strength of the pin-composite interface.

Using the technical theory of beams, we can write the following equations of equilibrium for DCB of width $b$:
\[ \frac{d^2 M_j}{dx^2} = pb, \]
where
\[ M_i = -D_j w_{i,xx} - M_i^T, \quad D_i = b \int_0^h E_i(T, z) z^2 \, dz, \]
\[ M_i^T = b \int_0^h E_i(T, z) \chi_i(T, z) T(z) \, dz. \]

In (35), $D_i$ is the stiffness of the $i$th leg that is affected by temperature (accordingly, the stiffness differs for each leg, even if the legs were identical prior to thermal loading), $E_i(T, z)$ is the modulus of elasticity, $\chi_i(T, z)$ is the coefficient of thermal expansion, and $M_i^T$ is a thermally induced bending moment acting on the corresponding leg. The latter moment is independent of the axial coordinate. Accordingly, coupled equations of equilibrium obtained by substituting (35) and (32) into (34) are
\[ D_a \frac{d^4 w_a}{dx^4} - K_1 (w_a + w_b) b + K_0 b = 0, \]
\[ D_b \frac{d^4 w_b}{dx^4} - K_1 (w_a + w_b) b + K_0 b = 0. \]

Eight constants of integration in the solution of (36) can be determined from the following boundary conditions (for convenience, we change the positive direction of the $x$-axis, so that $x = a$ corresponds to the free end of the delaminated DCB):
\[ w_a(0) = w_b(0) = 0, \quad K_i b w_{i,xx}(0) = -D_i w_{i,xxx}(0) - M_i^T, \]
\[ D_i w_{i,xxx}(a) + M_i^T = 0, \quad D_i w_{i,xxxx}(a) = P. \]

The solution of (36) can be obtained by reducing the system to one eight-order differential equation. For example, from the first equation,
\[ w_a = -w_a + \frac{D_a}{K_1} \frac{d^4 w_a}{dx^4} + \frac{K_0}{K_1}. \]
Then substituting (38) into the second equation (36) yields

$$\frac{D_a D_b}{K_1 b} \frac{d^4 w_a}{dx^4} - (D_a + D_b) \frac{d^4 w_a}{dx^4} = 0. \quad (39)$$

The solution of the system of Eqs. (36) becomes

$$w_a = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + A_4 \sin \lambda x + A_5 \cos \lambda x + A_6 \sinh \lambda x + A_7 \cosh \lambda x,$$

$$w_b = \frac{K_0}{K_1} \left( A_0 - A_1 x - A_2 x^2 - A_3 x^3 + f(A_4 \sin \lambda x + A_5 \cos \lambda x + A_6 \sin \lambda x + A_7 \cosh \lambda x), \right) \quad (40)$$

where $A_j (j = 0, 1, \ldots, 7)$ are eight constants of integration.

$$\lambda = \sqrt{\frac{(D_a + D_b)K_1 b}{D_a D_b}},$$

$$f = \frac{D_a A_4^2}{K_4 L_w} - 1. \quad (41)$$

An alternative solution could be obtained expanding the Kanninen approach that models the rotational stiffness of the beam through the introduction of an elastic foundation [20–23]. In this case, the equations of equilibrium for each leg and for the affiliated half-depth of the intact section of DCB can easily be written as an extension of this previous research [9]. The only explicit difference from the solution [9] is related to thermal terms that appear in these equations. Implicitly, these equations are also different as temperature affects the values of material constants and the stiffness terms.

If temperature is uniform, the properties of the material experience a uniform change throughout DCB. The solution for the coefficient of elastic clamping of the intact section of DCB given by (31) is valid. Thermally induced moments acting on the delaminated section of DCB are equal to zero. Accordingly, the only effect of a uniform temperature, besides the change of stiffness of the intact section, will be the changes of the stiffness of the material and the change in the strength $\tau$.

The stiffness of polymer–matrix and metal–matrix composites (PMC and MMC) invariably decreases with elevated temperature, implying larger deflections in DCB tests and an increased vulnerability to fracture and fatigue damage. The situation may be different for CMC [25]. In particular, Nicalon–fiber/SiC–matrix orthotropic composites manufactured by DuPont illustrate a noticeable increase in the tensile modulus until temperature reaches 1000 °C, followed by an abrupt reduction of modulus at higher temperatures. On the other hand, similar materials manufactured by another company (SEP) exhibit a monotonous decrease of the tensile modulus in the entire range of temperatures from 0 to 1400 °C. Carbon/SiC orthotropic and quasi-isotropic CMCs experience a moderate increase in the tensile modulus to about 1100 °C followed by a reduction of the modulus at higher temperatures (at 1600 °C). In SiC/SiC 2D composites, the stiffness is little affected by temperature, the tensile modulus increasing from 90 GPa at 23 °C to 100 GPa at 1000 °C and remaining nearly constant in the range from 1000 °C to 1400 °C.

As indicated above, temperature influences the interfacial strength between z-pins and the material of layers. This effect can be rather significant, the strength often increasing with temperature. In this case, the stiffness of the equivalent elastic foundation provided by z-pins increases. This implies that the effectiveness of z-pins may be enhanced as a result of a uniform temperature.

### 3. Numerical examples

Thermal residual stresses were found in a transversely isotropic SiC/SiC material where Tyranno SA fibers were embedded in a SiC matrix with the following properties (pristine material, without z-pins): $E_x = E_y = 41.4$ GPa, $E_z = 32.2$ GPa, $\nu_{xy} = 0.04$, $\nu_{xz} = 0.08$, $\nu_{yz} = 0.08$, $s_x = 3.96 \times 10^{-6} (1/\text{C})$, $s_y = 4.32 \times 10^{-6} (1/\text{C})$. The properties of the isotropic pin material were (SCS-6 silicon carbide) $E = 400.0$ GPa, $v = 0.3$, $z = 4.0 \times 10^{-6} (1/\text{C})$.

The residual stresses in the composite material found using the solution in Part 1 of the analysis are shown as functions of the radial coordinate in Figs. 9–11. The stresses in the z-pin are not shown since they are constant. The change in temperature is chosen equal to $–500$ °C since all stresses are proportional to temperature (as long as the material properties remain independent of temperature) and accordingly, the results for any other temperature values can be immediately calculated.

According to Figs. 9–11, the highest in-plane stresses in the composite material are observed at the interface with the z-pin. The axial stress in the pin ($\sigma_z$) is independent of

![Stresses in composite material, T=-500C](image)

Fig. 9. Stresses in the composite material as a result of cooling ($T = –500$ °C). Z-pin radius is 0.7 mm, z-pin volume fraction is 1%. The curves 1, 2 and 3 represent radial ($\sigma_r$), tangential ($\sigma_\theta$) and transverse ($\sigma_z$) stresses, respectively. The stresses in the z-pin are equal to $\sigma_r = \sigma_\theta = –0.88$ MPa, $\sigma_z = –57.3$ MPa.
the radius as a result of neglecting warping in the cross-sections \( z = \text{constant} \). If the volume fraction of z-pins is small (1%), all three stress components in the composite material at the interface with the z-pin are of the same order of magnitude.

The effect of the z-pin volume fraction on the stresses at the pin-composite interface is shown in Fig. 12. If the z-pin volume fraction increases, the transverse stress in the composite material increases as well (see Fig. 12). The radial and circumferential stresses in the z-pin are small (they are not shown) but the transverse stress can reach very high values. As is reflected in Fig. 12, the compressive stress \( \sigma_z \) in the z-pin decreases as a result of a higher z-pin volume fraction, while the transverse tensile stress in the composite material increases. In conclusion, it is noted that two possible mechanisms of post-processing failure associated with residual stresses are failure of the composite material due to tensile transverse stresses and microbuckling of the z-pin as a result of high compressive stresses. The latter mode of failure seems rather unlikely, though it may be of interest at small z-pin volume fraction (this mode of failure was also predicted in [13]).

The information on the effect of an elevated temperature on the properties of composite materials that are considered for a z-pin reinforcement is limited. It can be shown that if the properties are not affected by temperature, the relative displacement between delaminated sections of the structure (such as the “opening” between legs “a” and “b” considered above) is not affected by thermal effects. On the other hand, it is well known that the interfacial strength between fibers and matrices is significantly affected by temperature. Therefore, the following examples illustrate the influence of the magnitude of the interfacial shear strength between z-pins and composite material of DCB on the deflections of the delaminated end of DCB and accordingly, on its fracture.

The materials analyzed in the following examples include carbon/epoxy AS/3501 with \( E_x = 138 \text{ GPa}, E_z = 9 \text{ GPa}, G_{xz} = 6.9 \text{ GPa}, v_{xz} = 0.3 \) and a 1-D SiC/CAS ceramic matrix composite material with \( E_x = 140 \text{ GPa}, E_z = 130 \text{ GPa}, G_{xz} = 60 \text{ GPa}, v_{xz} = 0.3 \). Two types of z-pins were considered. Carbon z-pins of radius equal to 0.6 mm and the modulus of elasticity equal to 190 GPa were analyzed in SiC/CAS CMC DCB. Titanium Z-pins in carbon/epoxy DCB had radius equal to 0.47 mm and the
modulus of elasticity equal to 119 GPa. In all examples the DCB specimens had the width and half-thickness equal to 
\( b = 20 \text{ mm}, \ h = 2.19 \text{ mm} \), respectively. Similar to the approach adopted in [9,10], the arrest of the crack was associated with a negative rate of the change in the crack opening displacement of the delaminated ends of DCB with the length of the crack. This crack arrest condition can easily be observed in the graphs showing the deflection of the delaminated end as a function of the length of the crack.

The effect of the interfacial strength is illustrated for three z-pin volume fractions for a SiC/CAS CMC DCB in Fig. 13 (in this and subsequent figures \( w(a) \) denotes a deflection of the delaminated end). As is shown in this figure, even a small increase in the interfacial shear strength results in an abrupt reduction in the deflections of the delaminated end of DCB. This conclusion is further reinforced by results shown for CMC and carbon/epoxy DCB in Figs. 14 and 15 where the changes in deflections are depicted as functions of the crack propagation. The arrest and failure cases are clearly observed in these figures and it is evident that even a small increase in the interfacial shear strength can result in altering the response from delamination failure to the arrest of the crack.

As is shown above, a higher interfacial shear strength between z-pins and the material of the joint has a beneficial effect on the resistance to fracture and fatigue damage. Besides increased temperature, this goal might be achieved by using a rougher surface of z-pins. In particular, if z-pins for CMC joints are formed from ceramic fibers, these fibers may be interwoven producing a rough surface. This approach would increase the resistance to a relative slip between the z-pin and material providing a higher degree of interconnection and therefore, a higher resistance to pullout and fracture. A potential weakness of the proposed approach may be related to local microstress concentrations and microcracking. Obviously, experimental work in this area is warranted due to potential advantages of the suggested method.

4. Conclusions and recommendation

The present study showed that z-pins can be successfully employed for suppression of Mode I fracture in co-cured composite joints. The study outlined here dealt with the effect of temperature on z-pinned joints that is manifested in local residual thermal stresses, local thermal stresses applied during the lifetime of the joint and the influence of temperature on the global response of the joint. It was shown that local stresses at the level of a representative cell consisting of a z-pin and surrounding composite material can result in two principal modes of failure, namely cracking of the composite material and microbuckling of the z-pin. Both these modes are due to stresses acting in the direction perpendicular to the surface of the joint, i.e. along the pin axis. The transverse stresses in a typical z-pin are compressive, while the stresses in the composite material are tensile. Although microbuckling of z-pins is unlikely in practical configurations, it may become a problem if the
z-pin volume fraction is small, while the probability of cracking of composite material increases if this volume fraction increases. The solution for the effect of temperature on the global response of a DCB is also shown in the paper, though a comprehensive numerical analysis is not included due to the lack of information on the effect of temperature on the entire set of material properties necessary to implement this solution.

Elevated temperature often results in an increase of the interfacial strength between z-pins and composite layers. This increase may make z-pinned joints even more efficient in high-temperature applications than at room temperature. A practical recommendation from the conclusion regarding the positive effect of a higher interfacial strength is related to a possible design of the surface of z-pins. If a z-pin is manufactured from several fibers, a woven configuration resulting in a rough surface with a potentially better “grasp” between the z-pin and composite material may be considered. A drawback of such approach may be related to unavoidable microscopic stress concentrations and interfacial cracks. Nevertheless, experimental studies of such “rough” z-pin surfaces should be conducted since possible benefits could be very significant.

Besides the interfacial shear strength, temperature affects the properties of the composite material of the joint. This effect depends on the material. For example, temperature always results in a decrease of stiffness of polymer matrix and metal matrix composites. However, elevated temperature was shown to sometimes improve the stiffness of CMC. In the latter case, temperature might further benefit the integrity of a joint, particularly if it also results in an increased interfacial shear strength.

It should be noted that even if elevated temperature improves both the interfacial shear strength as well as the stiffness of the joint material, it might still be detrimental to a CMC joint. This is related to oxidation of fiber–matrix interfaces that occurs at an elevated temperature as a result of oxygen transmitted to the interface through cracks in a ceramic matrix. Such oxidation is accompanied by an abrupt embrittlement of the material.

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References