Functionally Graded Materials (FGMs)

Thermal Buckling and Dynamic Post-Buckling Analysis of Piezoelectric FGM Hybrid Cylindrical Shells
Thermoelastic Buckling of Imperfect FGM Cylindrical Shells

Functionally Graded Materials and Structures

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Overview

We can learn from numerous examples found in nature maximizing the endurance and reliability of biological systems. One of remarkable lessons reflected in modern engineering science is a gradation of the properties and composition of such systems optimizing their performance.

An example of a biological system where the properties vary over a short distance is a tendon-to-bone insertion site. At this site, materials with a huge mismatch in the stiffness approaching two orders of magnitude (the modulus of bone is of an order of 20 GPa, while the longitudinal and transverse moduli of orthotropic tendon are close to 450 MPa and 45 MPa, respectively) are connected through a short insertion that is of an order of 1-mm long in humans and even shorter in animals, such as mice. The load is transferred through the insertion by collagen fibers with mineral inclusions whose concentration varies from predominantly mineral bone to tendon where mineral is absent. The optimum stress transfer minimizing stress concentration is achieved utilizing two gradation mechanisms, i.e., a variable mineral content from bone to tendon and a variable collagen fiber orientation [16], in addition to shaping the insertion site and interdigitation of tendon and bone. Considered from the biomimetic perspective, this representative example implies that by varying constituent material weight or volume fractions and the orientation and shape of constituent material inclusions, we can achieve desirable results in engineering systems.

Functionally graded materials (FGMs) are composite materials where the concentration, shape, and orientation of constituent phases vary in one or more directions optimizing the performance (an example of a functionally graded ceramic-metal material is shown in Fig. 1). Typical variables involved in grading are volume fractions and shapes of inclusions, angle of lamination, orientation, diameter, distribution of coating or chemical composition of fibers, variations in porosity, etc. Functional grading is often understood as grading of the material in one direction (e.g., grading through the thickness in thermal barrier coatings). However, grading can be implemented in several directions (e.g., in-plane grading of facings of sandwich panels investigated in [6]).

Manufacturing and Applications

The first mention of FGM was related to their potential application in thermal problems. In the
sintering, die pressing and pressureless sintering, thermal spray, electrodeposition, electrophoretic deposition, chemical vapor deposition, centrifugal casting, thermochemical diffusion, pulse laser deposition, self-propagating combustion synthesis, etc. While a complete list of manufacturing techniques and their description are outside the scope of this entry, mentioned here are the review by Kieback et al. [23] and a discussion in [5].

Miscellaneous objectives addressed in the course of optimization of FGM structures require the solution of relevant heat transfer, stress, stability and dynamics problems, and fracture and fatigue analyses. The target of optimization may be a minimum weight or a desired performance of the structure; the latter may include the static and dynamic response and temperature distribution. The designer has a variety of available tools, such as the choice of the spatial variation of the vol-

![Diagram](image.png)

**Functionally Graded Materials and Structures, Fig. 1** Functionally graded ceramic-metal material (From Aboudi et al. [1]). Ceramic phase dominates in the vicinity to the hot surface, while metallic phase is dominant in the section with lower temperature (Reprinted with permission from Elsevier)
element scale, and even nanoscopic grading). The paper of Yin et al. [32] represents an example of the study accounting for the local gradation effects. Furthermore, grading in FGM can be multidirectional (e.g., grading along in-surface coordinates of the shell or plate as well as through the thickness). Thus, FGMs have a uniquely large potential for the optimization of the structure.

An example of the optimization of a FGM component where a FGM cylinder was cooled from 1,000°K to 500°K was considered by Cho and Ha [9]. The cylinder consisted of a metallic section (Ni), a ceramic section (Al₂O₃), and an intermediate ceramic-metal graded section. By optimizing the distribution of the metal volume fraction through the intermediate section, the maximum (hoop) stresses were significantly reduced.

Another example of an optimization problem in FGM subject to thermal loading is the paper of Bobaru [7] who noted that the optimum solution depends on the micromechanical method adopted in the analysis. Being valid for all FGM, this observation is particularly relevant in the presence of temperature. The reason is that temperature has different effects on the properties of constituent materials. However, the local temperature is itself dependent on the solution of the heat transfer and micromechanics problems that are affected by these properties.

**Typical Applications**

Since their inception, FGMs have been considered for control of heat transfer and thermomechanical response in structures. For example, the concept of a ceramic-metal FGM is illustrated in Fig. 1. A nonuniform distribution of ceramic and metallic phases from hotter to colder surface may result in a reduced temperature of the colder surface as well as smaller residual and lifetime stresses and alleviated fracture trends. Ceramic-metal particulate graded composites were considered for the management of the heat transfer and thermal response problems (e.g., [15]) and in electronic packaging [12]. Ceramic-metal FGMs are also employed in armor applications where the hard ceramic frontal surface blunts the projectile, while the metallic back surface catches fragments and prevents penetration [8, 18]. Another application of ceramic-metal FGM is found in thermal barrier coatings (TBC). An example of materials constituting such TBC are partially stabilized zirconia (PSZ) and superalloy Inco 718 considered by Kawasaki et al. [22].

Other representative applications of FGM structures, some of them discussed in the review of Birman and Byrd [5], include:

- Functionally graded prosthetic joints increasing adhesive strength and reducing pain
- Polyester-calcium phosphate materials for bone replacement with a controllable in vitro polyester degradation rate
- Functionally graded layer between the Cr-MO shank and ceramic tip of a cutting tool improving thermal strength, (e.g., Fig. 2)
- Sandwich panels with graded facing-core interfaces
- Graded fibers or graded particulate medium around the opening reducing stress concentration
- Functionally graded piezoelectric actuators
- Thermal protection systems for spacecraft, hypersonic, and supersonic planes
- Functionally graded heated floor systems
- Functionally graded dental implants
- Solid oxide fuel cells
- Functionally graded blades, etc.

Among the applications listed above, graded fibers in the vicinity of the opening can involve a variable chemical composition and geometry of fibers as well as variations of their volume...
fraction or orientation. In addition to fibrous FGM, a desirable outcome, i.e., lower stresses, can be observed in a particulate material with a variable volume fraction of constituent phases in the vicinity to the hole. For example, the effect of variable material properties that can be attributed to a variable constituent material volume fraction was considered by Yang et al. [31], while a variable orientation of the fibers was analyzed by Cho and Rowlands [11]. In this noteworthy class of applications, FGMs have a profound advantage over conventional composites. Note, however, that while a variable stiffness of the material around the opening may result in a lower stress concentration, a comprehensive analysis should incorporate a variable strength associated with grading to ensure that the altered stress distribution does not cause failure in the graded region.

FGMs with graded porosity were considered for such diverse applications as hard tissue implants and electrodes of solid oxide fuel cells. For example, porosity grading of functionally graded electrodes was investigated and compared to the effectiveness of the particle size grading [26]. Both grading techniques increased the maximum power density compared to the conventional design.

**Micromechanics of FGM**

Micromechanical problems in conventional composites with a uniform distribution of constituent phases rely on standard homogenization techniques. The situation is more challenging in FGM where the volume fraction, shape, or orientation of constituent phases may vary with location, so that homogenization should be applied at numerous points throughout the domain occupied by the body. This can be done if the input parameters, e.g., the volume fractions, are represented by functions of coordinates that can be incorporated in such analytical procedures as the Mori-Tanaka or concentric cylinders model. A numerical homogenization using the finite element method or the method of cells is also broadly employed in relevant studies (e.g., [1]). The output includes coordinate-dependent tensors of stiffness, thermal expansion coefficients, and conductivities. The effect of temperature on material properties, if present, can be accommodated as long as the distribution of temperature is known in advance. This is indeed the case if temperature is uniform. Otherwise, it is necessary to solve coupled heat transfer and micromechanics problems as is explained below.

The homogenization techniques referred to above can be applicable to most FGMs where the gradient of material constituents is small. Then grading does not affect the validity of micromechanical models, being reflected only at the macromechanical level where the tensors of stiffness, coefficients of thermal expansion, and thermal conductivities are dependent on the coordinates. However, if the above-mentioned gradient is sharp, it may have to be reflected in the micromechanical model at the representative volume element level. For example, Yin et al. [32] accounted both for a pair-wise particle interaction using the Eshelby equivalent inclusion method and for the gradient effect at the microscopic scale yielding the model that was in good agreement with experimental data. Furthermore, it was demonstrated that if the particle interaction terms and the terms reflecting the microscopic gradient effect were dropped, the model converged to the Mori-Tanaka formulation.

**Methodology of the Analysis of Functionally Graded Material Structures**

Typical problems that have to be addressed in the analysis of FGM are outlined below. A more detailed discussion of some of these problems is presented in the article *Modeling and Analysis of Functionally Graded Materials and Structures*.

The analysis of FGM structures involving thermal effects often requires engineers to account for the influence of temperature on the properties of one or all constituent materials. If the effect of a nonuniform temperature on the properties is accounted for, the heat transfer and the equilibrium (or motion) differential equations involve temperature-dependent coefficients, usually necessitating a numerical analysis.
Heat Transfer Problem
In the presence of thermal loading, the solution of the heat transfer problem has to simultaneously account for the effect of temperature on thermal properties of each constituent of FGM as well as for the heat transfer through the material. The constituents being nonuniformly distributed throughout the domain occupied by the body, the development of a coordinate-dependent tensor of thermal conductivities as well as the specification of thermal boundary conditions may require combining the heat transfer problem with micromechanical modeling. Decoupling these two aspects of the analysis may only be possible if the properties of each constituent are assumed independent of temperature. The situation is further complicated by different effects of temperature on the properties of the constituent materials. For example, temperature that significantly affects the property of the metallic phase in a ceramic-metal FGM may have a negligible effect on the properties of ceramic.

As follows from the previous discussion, if temperature is sufficient to affect the properties of the most temperature-sensitive constituent material, the analysis of the heat transfer problem must account for thermal effects on the FGM properties throughout the domain occupied by the structure. If decoupling of the heat transfer and micromechanical problems is impossible, the analysis may have to be iterative. For example, the first step may be the heat transfer solution obtained by assumption that the material properties are unaffected by temperature. Subsequently, the properties are adjusted accounting for the temperature distribution obtained from the first iteration and the heat transfer analysis is repeated, etc. The final outcome of the heat transfer and micromechanical solutions is a distribution of temperature and a map of material properties throughout the structure.

Macromechanical and Fracture Problems
Once the distribution of temperature and properties throughout the domain occupied by FGM are known, the solution of such macromechanical problems as the stress analysis, stability, vibrations, impact response, fracture, and fatigue can be undertaken. In the case where grading is implemented as a function of the in-surface coordinates of a shell, plate, or beam, the stiffness tensor as well as other properties, such as the coefficients of thermal expansion, is dependent on these coordinates. In such a situation, the formulation of the macromechanical problem involves the equations of motion or equilibrium with variable stiffness coefficients. If these coefficients are slowly changing functions of in-surface coordinates, their derivatives may be disregarded. Otherwise, the solution should be numerical or it should rely on energy methods.

If grading is implemented in the thickness direction of the structure and temperature varies in the same direction, as is often the case, the equations of motion or equilibrium coincide with those for an asymmetrically laminated composite structure and an analytical solution may be available. The situation is even simpler in case of grading that is symmetric about the middle surface of the structure (e.g., a symmetric sandwich structure with graded facing-core interfaces) and temperature is uniform. Then the equations of motion or equilibrium coincide with those for a symmetrically laminated structure, i.e., coupling stiffness terms are eliminated. An interesting situation is encountered in a symmetric FGM subject to a nonuniform through the thickness temperature. While the structure is symmetric at the room temperature, asymmetry at elevated temperature is acquired due to the effect of temperature on the material.

Note that one aspect affecting the macromechanical analysis of FGM structures is their strength. Being dependent on the local composition of the material, the strength cannot readily be predicted from experiments (too many samples should be tested). Therefore, it is desirable to use available strength theories to predict the local strength of FGM and map it throughout the domain occupied by the structure. These theories are applicable to FGM as long as property gradients are sufficiently small to disregard the effect of microscale gradation on the local strength. In FGM with sharp composition gradients, the strength may be affected by grading at the
representative volume element scale. Such sophisticated strength analysis has not been published.

Some aspects of research representative of typical FGM fracture problems and methodologies of the analysis can be found in Paulino [28]. Considering thermal problems, one should realize that low fracture toughness in brittle ceramics can be significantly enhanced in ceramic-metal FGM. An important observation facilitating the fracture analysis of FGM is that as long as material properties are continuous or piece-wise continuous functions of coordinates, the stress field in the vicinity to the crack tip is identical to the counterpart in an isotropic material [19]. As a result, the stress intensity factor is employed in studies of fracture of FGM (e.g., thermal fracture problems considered by [17, 20]).

A typical problem that has to be addressed in FGM is the trajectory of the crack, i.e., the direction of its propagation in a heterogeneous medium where properties vary in the direction oriented at an angle to the crack axis (Fig. 3). Several criteria employed to address this problem have been outlined by Kim and Paulino [24], including:

- The maximum hoop stress criterion (crack propagating along the axis of the maximum hoop stress)
- The maximum strain energy release rate (crack propagating along the axis corresponding to the maximum strain energy release rate, provided this rate exceeds a critical value)
- The minimum strain energy density criterion identifying the direction of crack growth with the smallest strain energy density

The other typical fracture problem in FGM is that of a crack propagating along a graded interface between two dissimilar materials (e.g., a crack along the chip-substrate interface that is graded to increase fracture toughness and reduce stress concentration).

In general, the solutions of macromechanical strength, stability, and fracture problems in FGM rely on the appropriate micromechanical formulation. The approach to the analysis or design should begin with the assumption of the distribution, shape, and orientation of constituent phases, followed with a valid micromechanical analysis yielding the distribution of properties in the FGM. In the thermomechanical formulation, the heat transfer and micromechanical aspects of the solution may have to be combined (i.e., these problems are coupled as is the case where temperature affects the properties of the constituent materials), yielding both the temperature and properties throughout the domain occupied by the material. Once the heat transfer and micromechanical phases of the solution are accomplished, the macromechanical problems, such as the stress analysis, and fracture can be addressed. It is noted that although the stresses may affect the properties of the material [13], this effect is usually neglected and it has not been considered in FGM.

**Conclusion**

Functionally graded materials represent a class of composite materials that can serve as a useful tool available to engineers. By an appropriate grading of the constituent materials, it is possible to improve and/or control the process of heat transfer, reduce thermally induced lifetime and residual stresses, improve strength and stiffness of the structure, enhance fracture and fatigue behavior, and prevent or alleviate delamination.
The analysis of FGM, particularly in the presence of temperature, differs from that of typical laminated composites as discussed in more detail in the article *Modeling and Analysis of Functionally Graded Materials and Structures*.

**Cross-References**

- Functionally Graded Structures: Aerothermoelastic Interactions

**References**


**Functionally Graded Structures: Aerothermoelastic Interactions**

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**Synonyms**

Advanced composite materials; Aerothermoelasticity; Fluid–thermal–structure interaction; Flutter; Functionally graded materials and structures; High supersonic flight vehicles structures; Limit cycle oscillations; Thermal divergence; Thermal protection system

**Definitions**

*Functionally graded material (FGM)* is a two-component composite characterized by a compositional gradient from one component to the other. FGMs are advanced composites, and their mechanical properties vary smoothly from one surface to the other. Unlike fiber-matrix composites which have a mismatch of mechanical properties across an interface of two discrete materials bonded together and may result in rebounding at high temperature, gradually varying the material properties of FGMs can avoid or remove this problem. An example of FGM and its microstructure is provided in Fig. 1. This new generation of engineered composite materials has different phases of material constituents; therefore, microstructural details are spatially varied through nonuniform distribution of the reinforcement phase(s), Fig. 2. Examples include mixture of metal and ceramic and combination of different metals, like the one indicated in Fig. 3. While ceramics are heat resistant, have good antioxidant properties, and low thermal conductivity, metals have mechanical strength, high thermal conductivity, and high fracture toughness. The transition zone made by ceramics and metals has effective thermal stress and relaxation throughout properties.

*Aerothermoelasticity (ATE)* is the study of the response of elastic structures to the combined effect of aerodynamic heating and loading. The interdisciplinary nature of this field can be best illustrated by Fig. 4, representing the interaction of the four aerodynamics (A), dynamics (D), elasticity (E), and thermal (T) disciplines. Due to the mutual interaction of the aerodynamic, inertia, elastic, and thermal forces acting on a body, static and dynamic instabilities might occur.

**Overview**

The concept of FGM was first considered in Japan in 1984 during a space plane project and was proposed as the key technology of TPS (thermal protection system). Functionally graded materials have attracted significant interest as heat-shielding materials for space vehicle, skin substrutures, gas turbine blades technologies, and many other high-temperature industrial applications. The aircraft and aerospace industry and the computer circuit industry are very interested in the possibility of materials that can withstand very high thermal gradients. FGM structures are also considered in RLV (reusable launch vehicle) system as they could improve