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Properties and response of composite material with spheroidal superelastic shape memory alloy inclusions subject to three-dimensional stress state

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Abstract
This communication illustrates the method of evaluation of the stiffness tensor and the stress–strain relationship of a particulate composite material with randomly distributed spheroidal superelastic shape memory alloy (SMA) inclusions that is subject to three-dimensional stresses. The Mori–Tanaka homogenization technique in conjunction with the Tanaka phenomenological SMA theory is applied to develop relations between the properties of the composite material and the applied stresses. In addition, the composite stress–strain loop reflecting hysteresis within the SMA material is generated. It is shown that the energy dissipation within a metal matrix composite with spherical SMA inclusions is comparable to that observed in composites with SMA fibres. The solution is exact within the framework of the assumptions of the Mori–Tanaka and phenomenological SMA theories.

1. Introduction
Shape memory alloy (SMA) composites could be attractive in a number of applications utilizing their transformation strain recovery and a high loss factor of SMA inclusions. The difficulty involved in the characterization of SMA composites is related to the fact that the properties of SMA are affected by the local stress tensor that in turn depends on these properties. Numerous theories characterizing SMA materials as well as some of their applications have been described in a number of reviews [1–3]. Numerous phenomenological theories rely on assumptions regarding the relationship between the rates of change in the martensite volume fraction and the transformation strain tensor. The “inverse” approach to the analysis that estimates applied stresses as functions of the martensite fraction was proposed for fibrous SMA composites [4] and for particulate SMA composites subject to uniaxial loading [5]. This approach is applicable if the transformation kinetics law of the SMA material is prescribed as is the case in the phenomenological theories of Tanaka [6, 7], Liang–Rogers [8], Boyd–Lagoudas [9, 10] and Brinson [11]. This paper illustrates that the inverse method can be extended to the case of arbitrary SMA inclusions in a composite material subject to three-dimensional stresses.

The homogenization of a composite medium can be undertaken by one of the numerous micromechanical methods [12, 13]. Among these methods, the Mori–Tanaka technique based on averaging of strains within each constituent phase of the material has been proven accurate if the volume fraction of inclusions remains below 30% [14] as is anticipated in SMA composites. Tandon and Weng developed exact solutions for the stress field and properties of arbitrary spheroidal inclusions utilizing the Mori–Tanaka technique [15]. Using these solutions it is possible to generalize the inverse method of the analysis to the case where a SMA-particulate composite material is subject to a three-dimensional stress tensor.

2. Analysis
The tangent 3D constitutive relations of phenomenological theories based on assumed kinetics laws for the martensite volume fraction in superelastic SMA inclusions are

\[ d\sigma_s = L_s(\xi) d(\epsilon_s - \epsilon^e), \]  

(1)

where \( \sigma_s \) and \( \epsilon_s \) are the tensors of stress and strain, respectively (hereafter subscript ‘s’ identifies SMA inclusions), \( L_s \) is the stiffness tensor, \( \xi \) is the martensite fraction and \( \epsilon^e \) is the
transformation strain tensor that reflects the phase change between the martensite and the austenite. In phenomenological SMA theories the rate of change in the latter tensor is related to the rate of change in $\xi$ and an assumed transformation tensor.

The stiffness tensor of SMA is affected by the proportion of martensite and austenite fractions being routinely determined by the rule of mixtures:

$$L_s(\xi) = L^A + \xi(L^M - L^A)A,$$  \hspace{1cm} (2)

where the superscripts identify the austenitic (A) and martensitic (M) phases, while $A$ is a tensor of strain concentration factors. For a random distribution of dispersed phases of various shapes, the tensor $A$ is close to the identity tensor [10].

In both martensitic and reverse transformations the martensite fraction is a function of temperature $T$ and effective stress $\sigma_{\text{eff}}$ in the SMA material, i.e. $\xi = \xi(T, \sigma_{\text{eff}})$ [6–11]. The martensite and austenite start and finish temperatures as well as other material properties affect such kinetics laws. The effective stress in SMA inclusions is defined through the components of the deviatoric stress tensor as $\sigma_{\text{eff}} = \sqrt{1.5(\sigma_{\text{dev}})^2}$.

The stress tensor within spheroidal inclusions can be represented in terms of the applied stresses $\sigma_0$ [15]. In the case of SMA inclusions, these relationships have to be replaced with incremental equations, so that

$$d\sigma_s = d[F(\xi)\sigma_0],$$  \hspace{1cm} (3)

where $F(\xi)$ is the matrix of the sixth order that depends on the Eshelby tensor as well as on the volume fractions and shear and bulk moduli of the inclusions and the matrix. For example, if the composite material with spherical inclusions is subject to a uniaxial stress $\sigma_0^{(11)}$ applied along the 1-axis, the stresses in the inclusions are found from

$$d\sigma_s^{(11)} = \left[1 + \frac{1 - V_s}{(1 + V_m)(1 - 2V_m)}(b_1(\xi)p_1(\xi)$$
$$+ 2b_2(\xi)p_2(\xi))\right]d\sigma_0^{(11)},$$
$$d\sigma_s^{(22)} = d\sigma_s^{(33)} = \frac{1 - V_s}{(1 + V_m)(1 - 2V_m)}[b_3(\xi)p_1(\xi) + (b_4(\xi)$$
$$+ b_5(\xi)p_2(\xi))d\sigma_0^{(11)},$$ \hspace{1cm} (4)

$$d\sigma_s^{(kn)} = 0 \quad k \neq n,$$

where superscripts (11), (22) and (33) identify the axial stress components, $V_s$ is the SMA volume fraction and $V_m$ is the matrix Poisson ratio. The coefficients $b_i(\xi)$ and $p_j(\xi)$ depend on the properties of the matrix and SMA inclusions, the latter being affected by the martensite fraction. This fraction is in turn affected by the stresses within the inclusions. While the formulae for these coefficients are available [15], these expressions cannot be directly applied in the present problem since the properties of inclusions are unknown in advance. Accordingly, the nonlinearity of the problem that is evident from (1) is further reflected in (3) and (4).

It is possible to express the increment of the effective stress in terms of the increment of the applied composite stress tensor using (3), so that

$$d\sigma_{\text{eff}} = d\Psi(\xi, \sigma_0),$$  \hspace{1cm} (5)

where $\Psi(\xi, \sigma_0)$ is a nonlinear function of applied stresses and the martensite fraction. In particular, in the case of a uniaxial load,

$$d\sigma_{\text{eff}} = \left\{1 + \frac{1 - V_s}{(1 + V_m)(1 - 2V_m)}[(b_1(\xi) - b_5(\xi))p_1(\xi)$$
$$+ (2b_2(\xi) - b_4(\xi) - b_5(\xi))p_2(\xi)]\right\}d\sigma_0^{(11)}.$$ \hspace{1cm} (6)

The evaluation of the response of the composite material is possible only upon specifying the tensor of stiffness. As the inclusions exhibit a physically nonlinear behaviour, the Mori–Tanaka approach to the evaluation of the stiffness has to be modified, similarly to such methodology in elastoplastic problems. The incremental composite stress–strain relationship used in the analysis is

$$d\sigma_0 = L(\xi)d\epsilon_0,$$  \hspace{1cm} (7)

where the tangent stiffness tensor is given by

$$L = L_m + V_s(L_s(\xi) - L_m)T_s((1 - V_s)I + V_sT_s)^{-1}. \hspace{1cm} (8)$$

In (8), $L_m$ is the stiffness tensor of the matrix and $I$ is the fourth-order identity tensor.

The tensor

$$T_s = [I + S_sL_m^{-1}(L_s(\xi) - L_m)]^{-1}$$  \hspace{1cm} (9)

depends on the Eshelby tensor $S_s$ which is affected by the matrix Poisson ratio and the aspect ratio of the inclusions (however, this tensor is not affected by the phase transformation within the inclusions).

The inverse method of the solution includes the following steps:

1. The martensite volume fraction is assumed and the corresponding tensors of stiffness of the SMA inclusions and the composite material are determined from (2) and (8). The effective stress corresponding to this martensite fraction is found from the kinetics law $\xi = \xi(T, \sigma_{\text{eff}})$. Then the surface in the six-dimensional stress space representing the components of the applied stress tensor can be specified from (5).

2. For a particular combination of the components of the applied stress tensor the corresponding composite strain tensor is determined using the inverse of (7). If the applied stresses are expressed in proportion to a ‘characteristic’ stress (such as the axial stress $\sigma_0^{(11)}$), the results can be presented as a family of curves relating the martensite volume fraction, the composite material properties and the stress–strain response of the composite matrix to this stress.

The approach to the analysis described above can be applied to materials with multiple inclusion classes (e.g. carbon fibres and SMA particles embedded within a metallic matrix). In such cases the expression for the tangent stiffness tensor (8) should be generalized following [14].
Composite material analysed in the examples consisted of an aluminium matrix and embedded spherical nitinol inclusions with the volume faction equal to 20% and the properties specified by Lee and Taya [16]. The case of biaxial loading was considered with nonzero elements of the applied stress tensor $\sigma_0^{(11)}$ and $\sigma_0^{(22)}$ and the stress ratio $R = \sigma_0^{(22)} / \sigma_0^{(11)}$. The results that refer to the stress-induced phase transformation at 40°C were generated using the Tanaka SMA theory. However, the solution could also utilize kinetics laws of other phenomenological theories referred to above.

Relationships between the applied axial stress $\sigma_0^{(11)}$ and the composite moduli for the case of the stress-induced martensitic transformation within SMA inclusions are shown in figure 1 for three different ratios $R$. The stresses necessary for the onset and completion of the martensitic transformation correspond to $\xi = 0$ and $\xi = 1$, respectively (in the Tanaka theory, the latter number is taken equal to $\xi = 0.99$). The moduli of the composite material significantly decrease as a result of the transformation that occurs in SMA inclusions, even though these inclusions constitute only 20% of the volume. The effect of the applied stress on the stiffness is relatively little affected by the stress ratio. The reason is understood if one considers a SMA material without the matrix. In such a case, the same martensite volume fraction corresponds to a prescribed value of $\sigma_0^{(11)}$ for $R = 0$ as well as for $R = 1$. In other words, in the absence of the matrix, the corresponding curves would coincide, while in SMA composites they are close but not identical.

The stress–strain curves reflecting the martensitic transformation are shown in figure 2. These curves illustrate that the complete transformation can be accomplished only at high applied stresses exceeding 500 MPa that may cause plastic flow in the matrix. Even in the case of epoxy matrix the stresses producing the complete transformation are quite high [5]. Accordingly, it may be difficult to design composites with epoxy matrices utilizing the phase transformation in SMA particles (or fibres). However, SMA composites could be designed with metallic matrices capable of withstanding high stresses. Only one curve for the transverse composite stresses and strains is shown in figure 2 since such stresses are absent if $R = 0$, while if $R = 1$ the stress–strain relationships in both the 1- and 2-directions are identical (the latter observation is applicable only in the case of spherical inclusions).

The effect of the volume fraction of SMA inclusions is evident from the comparison of figures 2 and 3 and figures 1 and 4. The range of applied composite stresses necessary for the complete transformation is little affected by the SMA volume fraction, but if this fraction is low, the transformation is accomplished at much smaller strains (see figure 3 versus figure 2). Note that this observation may become inaccurate depending on the choice of the materials of the inclusions and the matrix. The variations in the elastic modulus (as well as other moduli of the composite material) that occur during the transformation are affected by the volume occupied by SMA inclusions. A smaller SMA volume fraction results in a smaller sensitivity of the composite stiffness to the applied stresses (compare elastic moduli in figure 4 versus those in figure 1). This smaller sensitivity is explained by the higher stiffness of the aluminium matrix compared with that of SMA (in particular, the modulus of elasticity of martensite considered in

![Figure 1](image1.png)  
**Figure 1.** Variation of the elastic, shear and bulk moduli due to the applied composite stress tensor during the martensitic transformation (curve 1: $R = 0$, curve 2: $R = 0.5$, curve 3: $R = 1.0$). SMA volume fraction is equal to 20%.
Figure 2. Composite stress–strain curves during the martensitic transformation (curve 1: $R = 0$, curve 2: $R = 0.5$, curve 3: $R = 1.0$). SMA volume fraction is equal to 20%.

Figure 3. Composite stress–strain curves during the martensitic transformation (curve 1: $R = 0$, curve 2: $R = 0.5$, curve 3: $R = 1.0$). SMA volume fraction is equal to 5%.

Figure 4. Variation of the elastic modulus due to the applied composite stress tensor during the martensitic transformation (curve 1: $R = 0$, curve 2: $R = 0.5$, curve 3: $R = 1.0$). SMA volume fraction is equal to 5%.

Figure 5. The stress–strain loop for a uniaxially loaded SMA composite ($R = 0$). SMA volume fraction is equal to 20%. The arrows identify the loading and unloading branches of the loop.

Finally, an example of the hysteresis in the composite material is shown in figure 5. The shape of the hysteresis loop for a particulate SMA composite material is similar to those in SMA fibre-reinforced composites or in isotropic SMA materials. It is anticipated that the energy dissipation in particulate SMA composites is smaller than that in SMA fibre-reinforced materials loaded in the fibre direction. However, particulate SMA composites can dissipate more energy than SMA fibre composites loaded in the transverse direction. A detailed numerical analysis is needed to assess potential advantages of particulate SMA composites over SMA fibre-reinforced laminated composites in the case of multiaxial loading. Factors that may be accounted for in estimating the energy dissipation in superelastic SMA include the rate of loading [17]. The area enclosed within the loop, i.e. the energy dissipation during the cycle, tends to decrease...
with the number of cycles. It is also noted that Tanaka’s phenomenological theory employed in this study has been experimentally validated for uniaxial loading. The phase diagram and strain evolution may be more complicated in the case of a multiaxial load reflecting changes in the martensitic variants affected by the load as well as the microstructural texture of the parent material (e.g. [18]). These effects may be considered in future studies appropriately modifying the methodology suggested in this paper.

In conclusion, the proposed method can be applied to the analysis of properties, response and hysteresis in a composite material with superelastic SMA spheroidal inclusions subject to a 3D stress state. The solution shown in the paper is ‘exact,’ within the framework of the assumptions of the employed theories. Further studies are recommended to establish the limits of the usefulness and to quantify the advantages of these materials compared with SMA fibre-reinforced composites.

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References