### Thermomechanical Wrinkling and Strength of Functionally Graded Sandwich Panels with Nanoscale or Microscale Randomly Reinforced Core

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Abstract

Wrinkling represents one of the failure modes in sandwich structures, though in practical designs the loss of strength and global buckling often occur at lower compressive loads. However, the properties of both polymeric matrix in the facings as well as the polymeric core degrade under an elevated temperature. As a consequence, wrinkling that does not present a problem at the room temperature may become a dominant mode of failure at elevated temperatures. In this paper, we suggest that a reinforcement of the core material with stiff random nanoscale or microscale reinforcements may alleviate wrinkling. The solution accounts for the thermal loading history and the effect of temperature on the stiffness of the materials of the core and facings. While nano or microscale reinforcements increase the capacity of the structure to resist wrinkling, the strength of the core may be compromised due to the presence of such inclusions in the core material. Accordingly, the residual stresses in the reinforced core are evaluated using a finite element method and accounting for the effect of temperature on the properties and stresses. It is demonstrated that both wrinkling and the core strength analyses should account for the effect of temperature on the material properties.
Introduction

Wrinkling has been recognized as a local failure mode in sandwich structures since pioneering studies [1], [2], [3] and [4]. The early developments in wrinkling studies have been outlined in the review paper [5]. Both static and dynamic problems of wrinkling, in sandwich panels have been investigated in [6-13]. Modeling the core using high-order theories has been employed in wrinkling studies, such as [7, 14-17]. Examples of recent studies outlining the progress in the wrinkling analysis and demonstrating the effect of functionally graded cores on wrinkling are found in [18, 19]. Recent papers [20], [21] and [22] represent an example of a nonlinear analysis of wrinkling employing an extended high-order core model.

The effect of temperature on wrinkling in sandwich structures has been investigated in [18, 23-27]. While the adjustment of the properties affected by temperature has been included in these papers, the effect of the history of thermal loading on the thermally-induced stress in the facing has not been considered. In the present paper we account for the incremental thermal stress development in the facing tracing the continuous changes of the properties. The thermal stress in the facing is determined following the solution for the residual stress in a thin film originally developed in [28] and modified to account for the history of thermal loading [29].

Reinforcing the core or facing material with stiffer inclusions increases the wrinkling load in a sandwich structure. This was demonstrated on the examples of functionally graded cores (e.g., [18-20]) and follows directly from the Hoff and Plantema wrinkling stress formulas for the case of a homogeneous core where the wrinkling stress is proportional to a cubic root of a product of the elastic moduli of the facing and core and the shear modulus of the core. In this paper we develop a closed-form solution for the wrinkling stress accounting for the history of thermal loading and the effect of temperature on the material properties for the case of a nanotube or short fiber randomly reinforced polymeric core. It is demonstrated that neglecting the effect of temperature on the material properties as well as disregarding the history of thermal loading while evaluating wrinkling stability may result in an error.

While adding nano and microinclusions to the core is beneficial for the wrinkling stability, a potential problem may arise due to the stress concentration at the inclusion-core material interface and in the pristine core material in the vicinity to the stiffer and stronger inclusion. Therefore, it is necessary to evaluate the local stresses as was done in numerous studies in the
case of spherical inclusions (e.g., [30, 31]). In this paper we apply a finite element residual thermal stress analysis to evaluate residual stresses around microscopic inclusions and accounting for the history of thermal loading. We aim at confirming that the qualitative distribution of thermal stresses (either residual or during lifetime) is similar to the results obtained under mechanical loading. We also consider the effect of the size of a representative volume element on the accuracy of the results.

The shear-lag problem has been extensively studied for the case of an applied mechanical load transferred from the matrix to a fiber aligned with the applied stress. Mentioned here are representative papers outlining the analysis based on the shear-lag methodology [32-39]. Typically, thermal aspects of shear-lag problem at the fiber-matrix interface are not discussed in detail, even though some studies combine thermal and mechanical loading without a specific analysis of thermal residual or life-time stresses. However, the problem was considered for the fiber pull-out test, both analytically and numerically, in the paper [40], particularly concentrating the residual thermal stresses. As was shown in both analytical and FEA solution, interfacial shear stresses abruptly increased near the ends of the fiber and decreased toward its middle. Fiber axial and interface normal stresses increased from the ends of the fiber and remained nearly constant in its middle section. While these results were generated without accounting for the effect of temperature on the properties of the fiber and matrix as well as neglecting the history of thermal loading, i.e. a continuous change of properties with temperature, the qualitative aspect of the solution [40] is definitely valuable, with obvious limitations related to a difference between the fiber pullout problem and the stress problem around a nano or microinclusion.

The present paper is organized as follows. First, the analysis of wrinkling in sandwich structures subject to a simultaneous mechanical compression and thermal loading is presented employing an extension of the Hoff method. The solution may account for the effect of temperature on the material properties of the facing and polymeric core as well as for the history of thermal loading that affects the magnitude of the thermal stress. Numerical examples are presented in the particular case where the stiffness of the pristine core material and embedded nanotubes are cubic functions of temperature as was reported in literature for representative materials. It is demonstrated that embedding short stiff fibers or nanotubes within the polymeric core may significantly increase the wrinkling stress.
While embedding short fibers or nanotubes within the core material is beneficial in the wrinkling problem, the strength of the reinforced core, particularly the local stress concentration at the inclusion-core interface may be detrimental to the strength of such reinforced core. Accordingly, numerical examples present a finite element analysis of the residual thermal stress problem for a dilute microfiber encompassed with the “matrix” that represents a pristine core material. As is shown in the analysis, neglecting the continuous changes of the properties of the core with temperature can cause noticeable mistakes in the matrix, fiber and interface stresses. Using the properties corresponding to either the highest or lowest temperature under the manufacturing or lifetime thermal loading results in significant inaccuracy even in the limited range of temperatures considered in the examples.

Finally, two representative volume elements (RVE) are compared. It is shown that using a model where the boundaries of the element are too close to the fiber introduces unwarranted stress levels and incorrectly predicts a compressive axial stress zone in the matrix. The problems can be avoided if the distance between the fiber and the boundaries is sufficiently large as is demonstrated in the paper.

**Analysis**

1. Modification of the Hoff method in the thermomechanical wrinkling problem

The compressive stress applied to the facing in the case of a combined thermal and mechanical loading consists of the contributions of the applied mechanical stress that is assumed prescribed and the thermal stress generated as a result of a mismatch of the axial coefficient of thermal expansion of the facing and the coefficient of thermal expansion of the isotropic core. The thermally-induced stress in the facing can be determined as suggested in [28], i.e.

\[
\sigma_I(T) = \frac{E_{\text{face}}(T)\left(\alpha_{\text{face}}(T) - \alpha_{\text{core}}(T)\right)\Delta T}{1 - \nu_{\text{face}}(T)}
\]  

(1)

where \(E_{\text{face}}(T)\) and \(\nu_{\text{face}}(T)\) are the elastic modulus and the Poisson ratio of the facing, respectively, \(\alpha_{\text{face}}(T)\) and \(\alpha_{\text{core}}(T)\) are the coefficients of thermal expansion of the facing and core and \(\Delta T\) is the change of temperature from the reference value. The properties of the facing and core are evaluated at the current temperature \(T\).
If the variations of the properties of the facing and core with temperature can be presented in the form of analytical functions or alternatively, these properties are known at several temperature values within the temperature range considered in the analysis, equation (1) can be replaced with

\[
\sigma_f(T) = \int_{T_0}^{T} E_{\text{face}}(\tilde{T}) \left[ \alpha_{\text{face}}(\tilde{T}) - \alpha_{\text{core}}(\tilde{T}) \right] d\tilde{T}
\]

(2)

where the stress is accumulated in the range of temperatures from \( T_0 \) to \( T \).

Equations (1) and (2) were developed for a thin film supported with a substrate. In the case where the substrate is replaced with the core having a negligible axial stiffness, as is assumed in the first-order shear deformation theory, the coefficient of thermal expansion of the core should be taken equal to zero.

In the presence of a thermal compressive loads combined with mechanical compression the wrinkling stress is

\[
\sigma_f(T) + \sigma_{\text{mech}} = \sigma_{\text{wr}}(T)
\]

(3)

where \( \sigma_{\text{mech}} \) is a mechanically applied compressive stress.

The wrinkling stress obtained by the Hoff method has been shown in a good agreement with experimental data in case of mechanical loading (e.g., [7], [41]). For a sandwich large aspect ratio plate compressed along the long edges or for a sandwich beam the relationship between the applied mechanical stress and temperature is available from [26]

\[
\sigma_{\text{mech}} = 0.91 \sqrt{E_{\text{face}}(T) E_{\text{core}}(T) G_{\text{core}}(T)} - \frac{E_{\text{face}}(T) \alpha_{\text{face}}(T) T}{1 - \nu_{\text{face}}(T)}
\]

(4)

where the first term in the right side represents the wrinkling stress.

Note that the effect of the facing Poisson ratio was neglected in [26] as is proper in the case of a narrow sandwich beam. Accounting for the history of thermal loading, equation (4) is replaced with
\[ \sigma_{\text{mech}} = 0.91 \sqrt{E_{\text{face}}(T)E_{\text{core}}(T)G_{\text{core}}(T)} - \frac{\int_0^T E_{\text{face}}(\tilde{T})\alpha_{\text{face}}(\tilde{T})d\tilde{T}}{1 - \nu_{\text{face}}(\tilde{T})} \]  

Engineering constants of the facing and core in the first term in the right sides of equations (4) and (5), i.e. the wrinkling stress, do not depend on the history of thermal loading but are functions of the current temperature. This enables us to estimate the effectiveness of the reinforcement of the core using the factor representing the ratio of the wrinkling stresses of the structure with a reinforced core to that of the structure with the unreinforced core and facings:

\[ F(T) = \frac{\sigma_{\text{wr}}^{\text{rein}}(T)}{\sigma_{\text{wr}}^{\text{unrein}}(T)} = \sqrt{\frac{E_{\text{core}}^{\text{rein}}G_{\text{core}}^{\text{rein}}}{E_{\text{core}}^{\text{unrein}}G_{\text{core}}^{\text{unrein}}}} \]  

Naturally, it is advantageous to increase the factor \( F(T) \) for the entire range of the operational temperatures as much as possible.

2. Properties of nanotube or short fiber reinforced core material

A practical approach to reinforcing the core material is using randomly distributed stiff inclusions. Accordingly, the methods presented here are confined to a 3-D random reinforcement case. In particular, the Christensen-Waals solution developed for a 3-D random fiber distribution [41] provides an estimate for the modulus of elasticity of the material that is accurate for the fiber volume fraction below 20%:

\[ E_{\text{solid}}(T) = \frac{V_f}{6} E_f(T) + \left[ 1 + (1 + \nu_m(T))V_f \right] E_m(T) \]  

where the subscripts “f” and “m” refer to properties of the reinforcements and pristine core materials, respectively and \( V_f \) is the reinforcement volume fraction. Hereafter, the term “solid” refers to a solid reinforced material. If the core is designed using a foam, its properties can be specified in terms of those of the solid material as is shown below.

The Poisson ratio of a material with 3-D random fibers can be determined by the rule of mixtures as was demonstrated in [41]. Equation (7) is an asymptotic approximation of a more complete formula for the stiffness of a 3-D random fiber-matrix material that was developed in [41].
The coefficient of thermal expansion of a foam core with 3-D random isotropic fibers or nanotubes is [42, 43]:

\[
\alpha_{\text{solid}}(T) = \alpha_{\text{solid}}(T) + \frac{\alpha_m(T) - \alpha_f(T)}{1/K_m(T) - 1/K_f(T)} \left[ \frac{1}{K_{\text{solid}}(T)} - \frac{1}{K(T)} \right]
\]

(8)

where \( \alpha_{\text{solid}}(T) \) is the coefficient of thermal expansion obtained by the rule of mixtures.

The bulk modulus of a randomly reinforced core should be within the bounds [44]:

\[
K_{\text{solid}}(T) \leq K_{\text{solid}}(T) \leq K_{\text{solid}}^{(2)}(T)
\]

(9)

\[
K_{\text{solid}}^{(1)}(T) = K_m(T) + \frac{V_f}{1 - \frac{3V_f}{K_f(T) - K_m(T)} + \frac{3V_m}{3K_m(T) + 4G_m(T)}}
\]

\[
K_{\text{solid}}^{(2)}(T) = K_f(T) + \frac{V_m}{1 - \frac{3V_f}{K_m(T) - K_f(T)} + \frac{3V_f}{3K_f(T) + 4G_f(T)}}
\]

where \( V_j, (j = m, f) \) are the volume fractions of the pristine core material and reinforcements and \( K_j(T) \) are the bulk moduli of the pristine material and reinforcements, respectively.

The last term in equation (8) is

\[
\frac{1}{K(T)} = \frac{V_m}{K_m(T)} + \frac{V_f}{K_f(T)}
\]

(10)

In the case of spherical inclusions embedded in the core, the lower bound of the bulk modulus represents the exact solution that is extended here to account for temperature dependent properties of matrix and inclusions in the core:

\[
K_{\text{solid}}^{(1)}(T) = K_m(T) + \frac{V_s(K_s(T) - K_m(T))}{K_s(T) - K_m(T)} \left[ \frac{1}{K_m(T)} + V_m K_m(T) + 1.333G_s(T) \right]
\]

(11)

where the subscript “s” refers to spherical inclusions.
Knowing the elastic modulus and an estimated value of the bulk modulus, the Poisson ratio and shear modulus of an isotropic randomly reinforced material can be obtained from the relationships

\[
K_{\text{solid}} = \frac{E_{\text{solid}}}{3(1-2\nu_{\text{solid}})} = \frac{E_{\text{solid}} G_{\text{solid}}}{3(3G_{\text{solid}} - E_{\text{solid}})}
\]  

(12)

3. Properties of nanoreinforced foam core material

Closed cell and open cell foams can be employed as core materials in a sandwich structure. The effective properties of a randomly reinforced foam material depend on the properties of the solid reinforced foam material specified in the previous section, its volume fraction in the core and geometry of the foam. In particular, Gibson and Ashby suggested the following simple formulae for the effective elastic and shear moduli of polymeric foams at room temperature [45, 46]:

\[
\frac{E_{\text{core}}}{E_{\text{solid}}} = k_1 \left( \frac{\rho_{\text{core}}}{\rho_{\text{solid}}} \right)^{n_1}, \quad \frac{G_{\text{core}}}{G_{\text{solid}}} = k_2 \left( \frac{\rho_{\text{core}}}{\rho_{\text{solid}}} \right)^{n_2}
\]  

(13)

Using equations (13) in the present problem, the subscript “core” refers to the properties of the reinforced foam material. The ratio in the right side of equations (13) is a relative density, while $k_j$ ($j = 1, 2$) and $n_j$ are empirical constants. For open-cell foams, the values of these constants were found equal to $k_1 = 1.0, k_2 = 0.4, n_1 = n_2 = 2.0$ [45]. In closed-cell foams, the coefficients in the first equation (12) were $k_1 = 1.13, n_1 = 1.71$ [47]. The latter value is in agreement with the experimental findings in [48] where $n_1 = 1.70$. An alternative expression for the engineering constants of a closed-cell foam accounting for the separate effects of the foam material contained in the struts and in the walls is presented in [46].

The mass density of the foam (core) material is evaluated by

\[
\rho_{\text{core}} = V_{\text{solid}} \rho_{\text{solid}} = V_{\text{solid}} \left( V_m \rho_m + V_f \rho_f \right)
\]  

(14)

where $V_{\text{solid}}$ is the volume fraction of the solid material in the reinforced foam core. The mass density of gas (air) within the foam is neglected.
Approximate formulae for the bulk modulus of the foam have also been suggested. In particular, using the Kelvin foam model and ABAQUS, the bulk modulus of the foam with the solid material having the Poisson ratio equal to 0.49 was obtained in [49]. This result was generalized to account for a possible pressure inside the foam $p_0$ [50]:

$$K_{\text{core}} = \frac{0.222E_{\text{solid}} \rho_{\text{core}}}{1 - \nu_{\text{solid}}} \rho_{\text{solid}} + p_0$$  \hspace{1cm} (15)

The coefficient of thermal expansion of the foam core is obtained following the derivation in [50]:

$$\alpha_{\text{core}} = \alpha_{\text{solid}} + \left(\alpha_{\text{air}} - \alpha_{\text{solid}}\right) \frac{p_0}{K_{\text{solid}}}$$  \hspace{1cm} (16)

where $\alpha_{\text{air}}$ is the coefficient of thermal expansion of air or gas inside the foam and $p_0$ is the pressure inside the foam.

4. Effect of temperature on the reinforced core material properties

As follows from the first section, the wrinkling stress can be predicted if the properties of the facing and core at the current temperature are known. In particular, in case of aluminum or other metallic facings and a polymeric core, the effect of temperature on the facing properties within the anticipated range is small. Accordingly, we concentrate on the effect of temperature on the properties of the core.

There are numerous methods presenting the properties of materials as functions of temperature. They include representing material engineering constants as polynomial or tanh functions of temperature, see for example review [51]. In particular, a cubic polynomial relationship was found for the moduli of elasticity and shear of a typical foam material, Divinycell H100, in [1]:

$$\frac{\tilde{E}_{\text{core}}(T)}{\tilde{E}_{\text{core}}(25^\circ C)} = \frac{\tilde{G}_{\text{core}}(T)}{\tilde{G}_{\text{core}}(25^\circ C)} = -3.1943 \times 10^{-6} T^3 + 4.2436 \times 10^{-4} T^2 - 2.2653 \times 10^{-2} T + 1.3626$$  \hspace{1cm} (17)

where $\tilde{E}(T), \tilde{G}(T)$ refer to the moduli of a pristine foam without reinforcements.
Furthermore, molecular mechanics yielded a cubic polynomial elastic modulus-temperature relationship for single-walled nanotubes [52]. In particular, for zigzag nanotubes,

\[ E_{nano}(T) = 1.15 + 3.74 \times 10^{-11} T^3 - 3.98 \times 10^{-8} T^2 - 2.8 \times 10^{-4} T \ (GPa) \]  

(18)

For armchair nanotubes,

\[ E_{nano}(T) = 1.14 + 4.02 \times 10^{-11} T^3 - 4.67 \times 10^{-8} T^2 - 2.7 \times 10^{-4} T \ (GPa) \]  

(19)

As could be anticipated and as follows from the analysis of equations (17-19), the effect of temperature on the properties of pristine Divinycell H100 is much larger than that on the properties of carbon nanotubes. Accordingly, neglecting the effect of temperature on the material of the aluminum facings, the effect of both temperature as well as the contribution of reinforcements of the core to wrinkling stress of a sandwich can be represented in a closed form using equation (6).

**Numerical Examples and Discussion**

1. Effect of core reinforcements and temperature on the wrinkling stress

In the following examples we demonstrate the efficiency of reinforced cores in preventing wrinkling of the facings as well as the effect of temperature dependence of the core properties on the wrinkling stress. Two reinforced cores are considered:

1. Divinycell H-100 foam reinforced with carbon nanotubes;
2. QY89II-IV resin reinforced with E-glass epoxy fibers.

The effect of temperature within the ranges considered in the examples on the properties of nanotubes and E-glass fibers is neglected. The effect of temperature on the elastic modulus of Divinycell H-100 is calculated using equation (17), while the properties of QY89II-IV are evaluated using data in [53]. The results are collected in Table 1.
Table 1. Effectiveness of random reinforcements in two types of sandwich core for prevention of wrinkling at room and elevated temperatures.

<table>
<thead>
<tr>
<th>Materials of pristine core and reinforcements</th>
<th>Volume fraction of reinforcements</th>
<th>Temperature (°C)</th>
<th>Ratio of wrinkling stress with reinforced core to that of with pristine core</th>
</tr>
</thead>
<tbody>
<tr>
<td>Divinycell H 100 and zigzag nanotubes</td>
<td>0.05</td>
<td>25</td>
<td>1.16</td>
</tr>
<tr>
<td>Divinycell H 100 and zigzag nanotubes</td>
<td>0.10</td>
<td>25</td>
<td>1.25</td>
</tr>
<tr>
<td>Divinycell H 100 and zigzag nanotubes</td>
<td>0.05</td>
<td>100</td>
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</tr>
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<td>100</td>
<td>1.78</td>
</tr>
<tr>
<td>QY89II-IV and E-glass fibers</td>
<td>0.05</td>
<td>25</td>
<td>1.10</td>
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<td>0.10</td>
<td>100</td>
<td>1.19</td>
</tr>
</tbody>
</table>

As is evident from the results in Table 1, increasing the volume fraction of randomly distributed nanotubes or short stiff fibers in the core of a sandwich structure results in a significant enhancement to its wrinkling stability. This enhancement is particularly pronounced at elevated temperatures that detrimentally affects the stiffness of the pristine core material. Observing that wrinkling at room temperature is seldom the mode of failure of the sandwich structure, while it may become more dangerous at higher temperatures due to a degradation of the core properties,
such enhancement of wrinkling stability achieved with a relatively small volume fraction of core reinforcements is particularly important.

2. Thermal stresses in and around a short fiber embedded in the matrix

This analysis is necessary to address the problems of both thermal residual stresses during the manufacture of the reinforced core of the sandwich structure as well as to monitor thermal stresses during the lifetime of the structure. The goal is to assess the qualitative stress distribution throughout the fiber, fiber-matrix interface and the matrix as well as to specify the requirements to RVE used in the micromechanical analysis. While the residual temperature in shear-lag problems has been mentioned in literature, a detailed numerical analysis of such stresses is not known to the authors. A somewhat similar subject was specifically addressed in the fiber pullout problem in [40], without accounting for the effect of temperature on the material properties. Additionally, being concerned with the fiber pullout, this paper did not consider the stresses in the matrix along the axis of the fiber that as is shown below can be quite considerable.

Detailed numerical analysis of the residual thermal shear-lag problem was conducted to both verify the qualitative distribution of stresses in a fiber-matrix cell as well as to assess the effect of accounting for the temperature dependence of the properties. In the following analysis, we consider the case where temperature decreases from a higher to lower value, but the conclusions drawn from this solution may be modified and applied to the case of an increasing temperature as well.

The material of the fibers considered in the analysis was E-glass with the room temperature properties $E_f = 72.345 GPa$, $\alpha_f = 5.2 \times 10^{-6} \left( \frac{1}{C_0} \right)$, $\nu_f = 0.22$. QY89II-IV matrix had the following room temperature properties $E_m = 7.00 GPa$, $\alpha_m = 60.0 \times 10^{-6} \left( \frac{1}{C_0} \right)$, $\nu_m = 0.38$. As is evident from the comparison of the coefficients of thermal expansion, if temperature decreases from a higher to a lower value, the fiber tends to contract less than the matrix. Accordingly, the fiber is compressed and the matrix experiences tension. In the contrary, if temperature increases, it results in tensile stresses in the fibers and compression in the matrix.

Two models are analyzed. In Model 1, the fiber and RVE are very short, while in Model 2 the fiber longer and the gap between the fiber ends and the boundaries of RVE is larger.
Model 1: A short fiber.

The length and radius of the fibers were chosen equal to 0.5 mm and 0.05 mm, respectively. The distance between the axes of the fibers forming a hexagonal pattern around the central fiber and the axis of the latter fiber was equal to 1 mm (Fig. 1). Such relatively sparse fiber distribution may be associated with a dilute fiber model. The length and radius of the model were 1.5 mm and 2.0 mm, respectively.

Model 2: A longer fiber.

The length and diameter of the fiber were 5.0 mm and 0.05 mm, respectively. The fiber density, i.e. the distance between the axes of the central and hexagonally arranged other fibers was unchanged from Model 1, so the analysis still refers to a dilute model. The length and radius of the model were 15.0 mm and 2.0 mm, respectively. Accordingly, the effect of the boundary conditions at the flat ends of the model that could distort the results and conclusions for Model 1 was reduced.

The stresses were compared for a temperature range from 200°C to 120°C using the material properties at 120°C, at 200°C and incremental properties varying with temperature from 200°C to 120°C. The properties of the fibers are practically unaffected by temperature variations in such a narrow range. The stiffness of the matrix was obtained as a function of temperature using data presented in [53]. A micromechanical finite element model was created using Patran/Nastran Finite Element Software Package. The models shown in Figs. 1 and 2 have a finely meshed center cylindrical fiber surrounded by six outer fibers, all embedded within a cylindrical matrix. Note that only the center fiber’s stresses are analyzed; the surrounding fibers are coarsely meshed to simulate the effect of their stiffness.
Figure 1: FEA model used in the numerical analysis (Model 1). Six fibers surround the central fiber. The stresses were evaluated in and around the central fiber.

Figure 2: FEA model used in the numerical analysis (Model 2). Six fibers surround the central fiber. The stresses were evaluated in and around the central fiber.
The position of the central fiber considered to analyze the stresses in Model 1 is shown in Fig. 3. As is shown in the figure, the fiber is sufficiently remote from the unconstrained cylindrical boundary of the model to prevent these boundary conditions from interfering with local stresses. On the other hand, the flat boundaries prevented axial displacements, resulting in a more rigid model than would be anticipated in a larger RVE adopted in Model 2 that is not shown here for brevity.

![Figure 3. Position of the fiber relative to the model (Model 1).](image)

All finite elements employed in the model are HEX8 three-dimensional elements adopted since the stresses vary in both the axial and radial directions. Single Point Constraints (SPCs) in the translation X, translation Y, translation Z (axial fiber direction), rotation X, and rotation Y directions were applied at the matrix cylinder ends, so that the matrix cylinder with the embedded fibers was similar to a circular cross section composite beam with two fixed ends. While the qualitative effect of fixed flat boundaries on the stresses in the vicinity of the central fiber was limited, the stresses in the immediate vicinity to the supports did not accurately model the actual distribution in a bulk of the material. This limitation is discussed in more detail with respect to the axial stresses in the matrix in Fig. 5 below. The model was unconstrained against rotations in the circumferential direction reflecting its axisymmetry. Thermal loading was
applied at every node in the model. Mechanically applied stresses have not been considered in this analysis.

In the cases where the properties are specified at a constant temperature (200°C or 120°C), the results are generated without the need in an incremental analysis. However, in the more accurate solution where the properties of the matrix vary with temperature, the analysis is incremental. In such case, the temperature range is divided into 5°C intervals and the stresses generated within each temperature range using the corresponding matrix properties are summed. For example, for the 200°C-195°C interval the stresses found using 200°C matrix properties stress are added to the stresses in the 195°C-190°C interval using 195°C properties, etc.

An example of the fiber axial stresses in Model 1 is shown in Figure 4. Similar to the classical mechanical shear lag problem, the absolute value of the axial stresses increases from the ends of the fiber toward its mid-length. At the ends of the fiber, the axial stresses are relatively small, but present, reflecting the contact with the matrix.

![Figure 4. Fiber axial stresses (Pa) developed as a result of the change of temperature from 200°C to 120°C using the properties of the matrix at 200°C (Model 1).](image-url)
A sample of matrix axial stresses within the same range of temperatures and with the same matrix properties as those in Fig. 4 is presented in Fig. 5. Predictably, while the stresses in the fiber are compressive, the axial matrix stresses are tensile and much smaller than their fiber counterparts. Tensile stresses in the regions of the matrix encompassing the fiber are larger adjacent to the fiber surface and diminish at a larger distance from the fiber. A redistribution of axial stresses in the cross sections perpendicular to the fiber axis close to rigid boundaries of the model results in a small region of high compressive stresses probably associated with constrained warping of the cross sections. These stresses would diminish or disappear in a larger model including a “train” of co-axial fibers; however, their presence does not affect the qualitative results relevant to the stresses in and around the fiber and at the interface available from the analysis. Note that as is shown below for Model 2, a larger distance between the flat boundaries and the ends of the fiber results in an elimination of the perplexing zone of the axial matrix compressive stresses.
Figure 5: Matrix axial stress (Pa) developed as a result of the change of temperature from 200°C to 120°C using the properties of the matrix at 200°C (Model 1).

Axial stresses in the longer fiber (Model 2) are shown in Fig. 6. The qualitative stress distribution remains unchanged from that in Model 1 resembling the counterpart in a mechanical shear-lag problem, with a larger compression close to the middle of the fiber and a much smaller stresses at the ends. However, the magnitude of the stresses is reduced by an order of magnitude in longer fibers compared to that in Model 1. One of the possible explanations is a diminished effect of the boundary conditions at the ends of Model 2, as compared to Model 1 where the boundaries are very close to the ends of the fiber. A distribution of the matrix axial stresses for Model 2 is demonstrated in Fig. 7. In Model 2, the stresses in the matrix are tensile and a region of high compressive stresses that was perplexing in Model 1 is absent. This confirms our observation regarding the limitation of Model 1 and a necessity to use longer RVE models minimizing the boundary effects in shear-lag problems.

Figure 6. Fiber axial stresses (Pa) developed as a result of the change of temperature from 200°C to 120°C using the properties of the matrix at 200°C (Model 2).
Figure 7: Matrix axial stress (Pa) developed as a result of the change of temperature from $200^\circ C$ to $120^\circ C$ using the properties of the matrix at $200^\circ C$ (Model 2). The regions adjacent to the boundaries are excluded to avoid the boundary effect.

Interfacial shear stresses are of a particular interest concerning the integrity of the fiber-matrix bond. The evaluation of these stresses is complicated since Patran does not have a fringe option for shear stresses. Instead, a free body cut was made at the point of interest with all adjacent elements contributing to the shear force that is subsequently divided by the cross sectional area of the element producing the shear stress. A typical incremental shear stress distribution along the interface in the five-degree temperature interval is shown in Fig. 8. The total interfacial shear stress is obtained by the summation of the contributions at each five-degree temperature interval.
Figure 8. A distribution of the interfacial shear stress (Pa) along the fiber-matrix interface in the temperature interval from 125°C to 120°C using the matrix properties at 120°C. Position 0 and 10 are the end nodes of the fiber, while position 5 is the fiber midspan. Model 1.

The total interfacial shear stress for Models 1 and 2 is shown in Fig. 9 and 10 for the case of continuously incremental temperature adjusted matrix properties. As is shown in these figures, the qualitative distribution of the interfacial shear stress remains without a change for longer (Model 2) and shorter (Model 1) fibers. However, the stresses at the longer fiber interface are almost two orders of magnitude smaller than in short fibers.
Figure 9. A distribution of the interfacial shear stress (Pa) along the fiber-matrix interface in the temperature interval from 200°C to 120°C using the incremental temperature adjusted matrix properties. Position 0 and 10 are the end nodes of the fiber, while position 5 is the fiber midspan. Model 1.

Figure 10. A distribution of the interfacial shear stress (Pa) along the fiber-matrix interface in the temperature interval from 200°C to 120°C using the incremental temperature adjusted matrix properties. Model 2.
A qualitative distribution of fiber axial and interfacial shear stresses demonstrated above was in agreement with the results generated for thermal residual stresses using FEA in the fiber pullout problem [40].

The maximum axial fiber, the axial matrix stress and the interfacial shear stress generated as a result of lowering temperature from 200°C to 120°C are outlined in Table 2 for Model 1. The left and central columns of results utilized the matrix properties at 200°C and 120°C, respectively, for the entire range of temperature. The right column was obtained continuously adjusting the matrix properties at a current temperature value. As could be anticipated, the absolute value of the stresses is smaller with the matrix properties corresponding to 200°C since this is the case of a more compliant matrix. In the contrary, the absolute values of the stresses are largest with the least compliant matrix whose stiffness is found at 120°C. The intermediate case of a continuously adjusted pristine matrix stiffness falls between these two bounding cases.

Table 2. Maximum residual thermal stresses in and around E-glass fiber embedded in QY89II-IV matrix as a result of a reduction of temperature from 200°C to 120°C. Model 1.

<table>
<thead>
<tr>
<th></th>
<th>Properties at 200°C</th>
<th>Properties at (120°C)</th>
<th>Incremental Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Interfacial Shear Stress (Pa)</td>
<td>3.99E+07</td>
<td>4.48E+07</td>
<td>4.27E+07</td>
</tr>
<tr>
<td>Fiber-Maximum Axial Stress (Pa)</td>
<td>-2.01E+08</td>
<td>-2.39E+08</td>
<td>-2.22E+08</td>
</tr>
<tr>
<td>Matrix-Maximum Axial Stress (Pa)</td>
<td>1.31E+07</td>
<td>1.55E+07</td>
<td>1.44E+07</td>
</tr>
<tr>
<td>Matrix-Minimum Axial Stress (Pa)</td>
<td>-2.81E+07</td>
<td>-3.37E+07</td>
<td>-3.12E+07</td>
</tr>
</tbody>
</table>

The localized minimum axial matrix stress in Table 2 is due to closely spaced rigid boundaries of the model that prevents warping of the matrix cross sections parallel to the boundary. A
percentage difference of the stresses generated using the properties of the material at 120°C and at 200°C from the stresses obtained using continuously adjusted incremental pristine matrix properties are shown for Model 2 in Table 3. 

Table 3. Maximum residual thermal stresses in and around E-glass fiber embedded in QY89II-IV matrix as a result of a reduction of temperature from 200°C to 120°C. Model 2. Percentage difference from the stresses obtained using incremental properties.

<table>
<thead>
<tr>
<th></th>
<th>Properties at 200°C</th>
<th>Properties at (120°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Interfacial</td>
<td>-13%</td>
<td>11%</td>
</tr>
<tr>
<td>Shear Stress (Pa)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fiber-Maximum Axial</td>
<td>-5%</td>
<td>4%</td>
</tr>
<tr>
<td>Stress (Pa)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Matrix-Maximum Axial</td>
<td>-15%</td>
<td>13%</td>
</tr>
<tr>
<td>Stress (Pa)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Matrix-Minimum Axial</td>
<td>-16%</td>
<td>14%</td>
</tr>
<tr>
<td>Stress (Pa)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As shown in Table 2, maximum axial stresses obtained using incremental properties for the matrix are 10% higher than assuming the properties at 200°C and 8% lower than those obtained with 120°C properties. Interfacial shear stresses obtained using incremental properties are 7% higher than those calculated with the matrix properties corresponding to 200°C and 5% lower than those generated with 120°C properties. Therefore, neglecting the history of thermal loading in the stress analysis causes noticeable inaccuracies, even in the narrow range of temperatures. The qualitative distribution of fiber, matrix and interfacial shear stresses in this thermal problem is similar to that observed in the shear-lag problems in case of mechanical loading. It is noted that the stresses in E-glass fiber are of an order of 20 to 25% of its compressive strength. Considering a limited temperature range considered in the examples, these stresses are not inconsiderable, but they may be attributed to a deficiency of RVE that is too short. The strength
of the pristine matrix material is estimated in the ranges from 50MPa to 80MPa at room temperature and from 22MPa to 25MPa at 200\degree C [53]. Thus, the axial stresses in the matrix according to Model 1 may approach the strength value if temperature is lowered to the room value.

Although the residual stresses evaluated for a Model 2 where the effect of the boundaries is less pronounced than in Model 1 remain high, they are much smaller than their counterparts in Model 1. One of the observations from the comparison of these stresses is relevant to the finite element models employed in the analysis. As emphasized above, exceedingly high stresses in Model 1 may be related to a close proximity of the fiber to the boundaries of the model. Thus, it is recommended to employ sufficiently long models where the boundary effects can be minimized.

Based on the previous discussion, it is evident that while embedding stiff inclusions in the core of a sandwich structure is beneficial to its wrinkling stability, residual thermal stresses acquired during the manufacturing process may compromise its strength. A comprehensive strength analysis of nano or micro-reinforced cores, including residual thermal stresses is warranted.

Conclusions

Enhancement of the wrinkling stability in sandwich structures using functionally graded and reinforced cores has been verified as an efficient design tool. In this paper, we considered the effectiveness of using randomly nano and short fiber reinforcements core to prevent thermomechanical wrinkling. Besides the effect of core reinforcements on wrinkling, the effect of temperature on the material properties is investigated and shown significant. Moreover, the history of thermal loading, i.e. a continuous change of properties with temperature was shown to be a significant factor affecting the stresses. While acknowledging the beneficial effect of the reinforcements in the sandwich core, the need for the strength analysis, including the strength of the fiber- or nanotube-matrix interface, the strength of the matrix and even the strength of the fiber is emphasized. The study resulted in the following conclusions:

1. Adding random reinforcements to the sandwich core results in an increase in the wrinkling stress both at the room, but particularly at an elevated temperature. The beneficial effect of reinforcements at an elevated temperature is particularly important
since a reduction of the stiffness of typical polymeric cores at a higher temperature may drastically increase the vulnerability to wrinkling.

2. The analysis of residual thermal stresses at the fiber and pristine core representative volume element demonstrates that it is desirable to monitor incremental changes of properties with temperature. Using the properties evaluated at the highest or the lowest temperature of the temperature interval provides a kind of “bounds” to all residual stresses. The absolute values of the stresses are maximum using pristine matrix properties evaluated at the lowest temperature and minimum when evaluated with the properties at the highest temperature. A more accurate stress estimate is obtained monitoring variations of material properties with temperature. As demonstrated in the examples, the spread can be quite significant, even in a relatively narrow temperature range.

3. The quantitative distribution of the stresses in the matrix, nanotube or fiber and along the interface closely resembles the classical shear-lag distribution under mechanical loading. A model of fibers encompassed in the matrix used to analyze thermal stresses should be sufficiently long to minimize the influence of the boundary conditions on the results. This is evident from the consideration of axial matrix stresses in Model 1 that predict an unlikely region of compressive stresses between the fiber and boundary. Such consideration should dictate a selection of the representative volume element in the analysis of mechanical or thermal stresses.

4. While residual thermal stresses are unlikely to cause damage in a reinforced sandwich core, they may be sufficiently large to warrant the analysis, particularly as the core is subject to lifetime loads.

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FEA model used in the numerical analysis (Model 1). Six fibers surround the central fiber. The stresses were evaluated in and around the central fiber.

226x199mm (96 x 96 DPI)
FEA model used in the numerical analysis (Model 2). Six fibers surround the central fiber. The stresses were evaluated in and around the central fiber.

317x151mm (96 x 96 DPI)
Position of the fiber relative to the model (Model 1).

260x194mm (96 x 96 DPI)
Fiber axial stresses (Pa) developed as a result of the change of temperature from to using the properties of the matrix at (Model 1).

341x184mm (96 x 96 DPI)
Matrix axial stress (Pa) developed as a result of the change of temperature from \( T_1 \) to \( T_2 \) using the properties of the matrix (Model 1).

323x201mm (96 x 96 DPI)
Fiber axial stresses (Pa) developed as a result of the change of temperature from $T_1$ to $T_2$ using the properties of the matrix at $T_1$ (Model 2).

$319\times 154\text{mm (96 x 96 DPI)}$
Matrix axial stress (Pa) developed as a result of the change of temperature from $T_1$ to $T_2$ using the properties of the matrix at $T_3$ (Model 2). The regions adjacent to the boundaries are excluded to avoid the boundary effect.
A distribution of the interfacial shear stress (Pa) along the fiber-matrix interface in the temperature interval from 125°C to 120°C using the matrix properties at 120°C. Position 0 and 10 are the end nodes of the fiber, while position 5 is the fiber midspan. Model 1.

138x78mm (150 x 150 DPI)
A distribution of the interfacial shear stress (Pa) along the fiber-matrix interface in the temperature interval from 200°C to 120°C using the incremental temperature adjusted matrix properties. Position 0 and 10 are the end nodes of the fiber, while position 5 is the fiber midspan. Model 1.
A distribution of the interfacial shear stress (Pa) along the fiber-matrix interface in the temperature interval from 200°C to 120°C using the incremental temperature adjusted matrix properties. Model 2.