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Wrinkling of Composite-facing Sandwich Panels Under Biaxial Loading

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ABSTRACT: The problem of face wrinkling in sandwich structures was identified as an important failure mode and first analyzed in 1940 by Gough et al. [1], who used a Winkler-type elastic foundation to model the core. This work was followed by experiments and further analysis by many others. Until 1966 all of the research was devoted to uniaxial compression loading. However, in many applications, such as naval and aircraft structures, panels are subjected to biaxial loading. Such loadings were first analyzed by Plantema in 1966 [2] for the case of sandwich constructed of isotropic materials. Sullins et al. [3] suggested an interaction equation that can be used as a criterion of wrinkling under compression in the principal directions. This equation is formulated in terms of the ratios of the principal compressive stresses to the corresponding wrinkling stresses. More recently Fagerberg [4, 5] and Vonach and Rammerstorfer [6] considered the case of sandwich with orthotropic facings.

The present work attacks the subject problem using three different models for the core. One model uses a simple Winkler elastic foundation approach. The second model, following Hoff and Mautner [Hoff, N.J. and Mautner, S.E. (1945). The Buckling of Sandwich-Type Panels, *Journal of the Aeronautical Sciences*, **12**: 285–297.], uses a linear model for the decay in deformation from the core-facing interface, while the third model, following Plantema [Plantema, F.J. (1966). *Sandwich Construction*, John Wiley & Sons, Inc., New York.], uses an exponential decay.

KEY WORDS: sandwich panels, biaxial loading, wrinkling instability.

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INTRODUCTION

BY THE VERY geometry of sandwich structures, they experience some instability and failure modes not found in conventional monolithic structures or even compact laminates. One of these is known as face wrinkling because the faces experience buckling of much shorter wavelength than those associated with general instability of the panel. The use of sandwich structures in the 1930s and 40s stimulated the early investigations of face wrinkling. Gough et al. [1], Hoff and Mautner [7], and Plantema [2] represented the supporting action of the core by a simple Winkler elastic foundation model. Numerous studies elucidating various aspects of wrinkling under uniaxial compression include the monograph of Allen [8] and the papers [9–15]. For additional references relating to analysis of sandwich structures, see the extensive survey paper by Noor et al. [16] and the review concerned with wrinkling failure published by Ley et al. [17].

In many practical applications of sandwich panels, they are subjected to biaxial in-plane loading. To the best of the present investigators' knowledge, the only previous analyses of face wrinkling under biaxial loading are due to Plantema [2, pp. 44–48] and Sullins et al. [3], for isotropic facings and Fagerberg [4,5] and Vonach and Rammerstorfer [6], for orthotropic facings. In particular, the approach of Fagerberg is based on using the uniaxial wrinkling theory and calculating the wrinkling load as a function of the angle of the wrinkling wave relative to the direction of applied force. Unfortunately, it is impossible to compare the experimental and numerical results obtained by this approach to the solution shown in the present paper since the results shown in [4,5] were obtained for the case of an orthotropic facing oriented at an angle to the applied load. Accordingly, such facing becomes anisotropic, i.e. bending stiffnesses D_{i6} $(i = 1, 2)$ have to be accounted for. In the present analysis, these stiffness terms are assumed equal to zero, as is the case where the facing is composed of a large number of balanced symmetrically laminated layers. In the paper of Vonach and Rammerstorfer [6], the solution was developed that can be applied to the case of biaxial compression of the orthotropic facings. The feature of this solution is that it accounts for the effect of the opposite facing on wrinkling of the facing under consideration. The modulus of the elastic foundation provided by the core is evaluated from the solution of the elasticity problem for the core subject to the appropriate boundary conditions along the interfaces with the facings. The solution provides an alternative to FEA, although it requires a rather long procedure to evaluate the stiffness of the foundation formed by the core and opposite facing. It is noted that the normal stress in the thickness direction in the opposite facing (the facing that does not wrinkle) is evaluated through minimizing the strain energy of the foundation formed by the core and this facing. This is not necessarily an accurate approach since the energy should be calculated for the entire system formed by both facings and the core.

It is noted that a recent paper by Vonach and Rammerstorfer [15] presents the solution accounting for in-plane deformations of an anisotropic (honeycomb) core. The approach is based on the theory of elasticity solution of the elastic plane stress problem in the core. The solution is obtained for the case of a unidirectional compression of the sandwich structure, but an extension to the case of composite facings and biaxial loading could be considered as well. However, as was shown in [15], if the core is isotropic, the solution converges to the solution of Plantema [2]. The present paper is concerned with the case of a polymeric isotropic core, i.e. a more complicated model based on the approach shown in [15] is not needed here and thus is not considered.

Analysis

In the present paper, we consider two possible modes of wrinkling. The first mode is represented by a long wave that may be perpendicular to the direction of one of the applied loads or inclined relative to the load direction (Figure 1). Such wrinkling was observed by Fagerberg [4]. The second possible mode of failure in symmetrically laminated facings is represented by rectangular wrinkles that are oriented along the load direction (Figure 2). This mode was analyzed for isotropic facings by Plantema [2]. Three models reflecting the reaction of the core that are considered in the paper include a Winkler elastic foundation, and the Hoff and Plantema methods. In all cases considered in the paper, the facing is assumed infinite. This is justified by a small size of wrinkles (small width of a wrinkling wave and small length and width of a rectangular wrinkle) compared to the planform dimensions of a typical facing. Furthermore, following standard design practice, the facing is composed of a large number of symmetrically laminated layers. In this case, even if the layers are generally orthotropic relative to the coordinate axes of the sandwich panel, the facing bending stiffnesses D_{16} and D_{26} are negligible compared to other bending stiffnesses. It is emphasized that all models of the core considered in this paper are limited to the case where the core is relatively thick. In the case where the core is thin, global buckling is dominant and the validity of the assumed modes of core deformations according to Hoff's and Plantema's models is questionable.

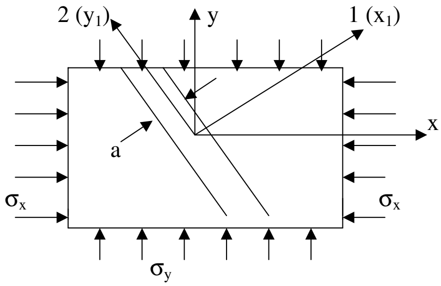

Figure 1. A facing subject to biaxial compression and experiencing wrinkling instability characterized by long waves. Two coordinate systems are shown: x–y coordinate system is aligned with the load direction and 1–2 (or x_1–y_1) coordinate system is aligned with the wrinkle wave. The angle between the axes x and 1 (x_1) is θ.

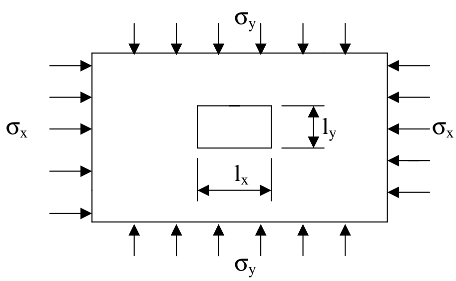

Figure 2. A facing subject to biaxial compression and experiencing wrinkling instability characterized by rectangular wrinkles.

MODEL 1: FACING SUPPORTED BY AN ELASTIC FOUNDATION (WINKLER FOUNDATION)

Wrinkling Waves in an Orthotropic Facing Supported by an Elastic Foundation

The equation of equilibrium of a wrinkle shown in Figure 1 is

$$Dw,_{1111} + \sigma_1 t w,_{11} + kw = 0 \tag{1}$$

where D is a generalized bending stiffness, t is the thickness of the facing, w is a buckling displacement, and k is the stiffness of the elastic foundation that can be defined in terms of the modulus and stiffness of the core, dependent on the mode of wrinkling in the opposite facing. The commas indicate the derivatives with respect to the corresponding direction. For example, if the opposite facing wrinkles forming a symmetric pattern relative to the middle plane of the facing, $k = 2E_c/h_c$ where E_c and h_c are the modulus and thickness of the core, respectively. On the other hand, if the sandwich experiences bending and the opposite facing remains straight, the factor 2 should be omitted from the formula for the stiffness of the foundation. It is worth mentioning that the antisymmetric pattern of wrinkling does not result in vertical stretching or compression in the core and accordingly, the elastic foundation model is not applicable in this case. Therefore, the problems considered here can include wrinkling as a result of biaxial compression applied to the panel that results in a simultaneous instability of both facings forming a symmetric deformation pattern. The second class of problems is related to bending of the panel, in which case one facing is subject to tension and remains stable, while the other facing is compressed and wrinkles. The problem considered in the paper is linear, i.e. the analysis is confined to determining the condition under which wrinkling initiates, rather than the postbuckling deformations. Accordingly, such phenomena as coupling between global and wrinkling buckling or interaction between bending deformations of a panel subject to transverse loading and local buckling are not analyzed here.

The bending stiffness of the facing is calculated as a function of the facing stiffnesses in the x–y coordinate system:

$$D = D_{11}\cos^4\theta + D_{22}\sin^4\theta + 2(D_{12} + 2D_{66})\sin^2\theta\cos^2\theta \tag{2}$$

where D_{11} and D_{22} are flexural stiffnesses in the 1 and 2 directions, D_{12} is the Poisson coupling stiffness and D_{66} is the twisting stiffness.

The stress σ_1 that is perpendicular to the axis of the wrinkling wave is obtained in terms of the applied stresses and the wrinkle angle relative to the x-direction:

$$\sigma_1 = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta \tag{3}$$

The assumed shape of the wrinkling wave is

$$w = W\sin\frac{\pi x_1}{a} \tag{4}$$

where x_1 is a coordinate along the 1-axis, as shown in Figure 1, and the length of the wave a is still unknown.

The substitution of (4) into (1) and the minimization of the resulting wrinkling stress with respect to the length of the wrinkling wave yield the known result for the length of the wave and for the critical stress:

$$a = \pi\sqrt[4]{D/k}$$

$$\sigma_{1,\,cr} = \frac{2}{t}\sqrt{Dk} \tag{5}$$

If the ratio between the applied compressive stresses is known, say $\sigma_y = r\sigma_x$, one can obtain the critical value of the applied stress from (3) and (5):

$$\sigma_{x,\,cr} = \frac{2\sqrt{Dk}}{t(\cos^2\theta + r\sin^2\theta)} \tag{6}$$

The minimization of the critical stress given by (6) with respect to the wrinkling wave angle θ yields a relationship between the stress ratio r and the wrinkle angle:

$$r = \frac{2D\sin 2\theta + g(\theta)\cos^2\theta}{2D\sin 2\theta - g(\theta)\sin^2\theta} \tag{7}$$

where

$$g(\theta) = 2\left(D_{22}\sin^2\theta - D_{11}\cos^2\theta\right)\sin 2\theta + H\sin 4\theta \tag{8}$$

In (8), $H = D_{12} + 2D_{66}$.

The straightforward computational procedure is to prescribe the wrinkle wave angle and calculate the corresponding stress ratio from (7). Subsequently, the critical stress can be determined from (5) and (6). Note that if $r = 0$, i.e. $\sigma_y = 0$, $\theta = \pi/2$ and we have the case of a unidirectional loading in the x-direction. On the other hand, if $r = \infty$, i.e. $\sigma_x = 0$, and the applied stress is acting in the y-direction, the possible solution is $\theta = 0$ indicating that the wrinkling wave is perpendicular to the direction of the applied stress.

Rectangular Wrinkles in an Orthotropic Facing Supported by an Elastic Foundation

Similar to the case of an isotropic facing considered by Plantema [2], it is assumed that the wrinkle is oriented along the applied stress directions. However, even in the case where the applied stresses in the x and y directions are equal, it is not correct to assume that the wrinkle in an orthotropic facing is square.

The equilibrium equation for the facing is

$$D_{11}w,_{xxxx} + 2(D_{12} + 2D_{66})w,_{xxyy} + D_{22}w,_{yyyy} + \sigma_x t\big(w,_{xx} + rw,_{yy}\big) + kw = 0 \tag{9}$$

The mode shape of wrinkling is given by

$$w = W \sin \alpha x \sin \beta y \tag{10}$$

where $\alpha = \pi/l_x$ and $\beta = \pi/l_y$, l_x and l_y being the dimensions of the wrinkle, as shown in Figure 2.

The substitution of (10) into (9) yields

$$\sigma_x = \frac{D_{11}\alpha^4 + 2H\alpha^2\beta^2 + D_{22}\beta^4 + k}{t(\alpha^2 + r\beta^2)} \tag{11}$$

Minimizing the stress given by (11) with respect to α^2 and β^2 results in a set of equations

$$\begin{aligned} &2\big(D_{11}\alpha^2 + H\beta^2\big)\big(\alpha^2 + r\beta^2\big) - \big(D_{11}\alpha^4 + 2H\alpha^2\beta^2 + D_{22}\beta^4 + k\big) = 0 \\ &2\big(H\alpha^2 + D_{22}\beta^2\big)\big(\alpha^2 + r\beta^2\big) - r\big(D_{11}\alpha^4 + 2H\alpha^2\beta^2 + D_{22}\beta^4 + k\big) = 0 \end{aligned} \tag{12}$$

Multiplying the first Equation (12) by r and subtracting from it the second equation yields the ratio that characterizes the shape of the wrinkle

$$\lambda = \frac{\alpha}{\beta} = \sqrt{\frac{D_{22} - Hr}{D_{11}r - H}} \tag{13}$$

Clearly, the ratio defined by (13) must be a real positive number. This results in the necessary conditions that must be satisfied to ensure that rectangular wrinkles are feasible. These conditions are:

$$D_{11} > H/r \qquad D_{22} > Hr \tag{14a}$$

or

$$D_{11} < H/r \qquad D_{22} < Hr \tag{14b}$$

If conditions (14a) or (14b) are satisfied, the substitution of λ into one of Equations (12) yields the wrinkle mode shape characteristic parameters, i.e. α and β. For example, α can be obtained as the following function of λ:

$$\alpha = \sqrt[4]{\frac{k}{D_{11} + 2D_{11}r\lambda^{-2} + (2Hr - D_{22})\lambda^{-4}}} \tag{15}$$

Note that in most practical situations, if a real and positive value of λ exists, α obtained from (15) is also real. Therefore, conditions (14) are usually sufficient for the feasibility of rectangular wrinkles. The buckling stress can be determined from (11) substituting the values of α and β determined as shown above.

Model 2: Facing Supported by Hoff's Core

According to the approach of Hoff, the core deformations in the thickness z-direction vary linearly with a distance from the facing–core interface. In-plane displacements of the core are neglected, implying that the waves of facing buckling deformation remain shallow. It is easy to see that this approach is justified in a linear formulation. In the case where the opposite facing remains stable or undergoes symmetric wrinkling and the size of the wrinkles is small, the transverse deformation in the core corresponding to a long wrinkling wave that is shown in Figure 1 is

$$w_c = W\frac{h - z}{h}\sin\frac{\pi x_1}{a} \tag{16}$$

where the coordinate z is counted from the facing–core interface and the depth of the zone within the core that is affected by wrinkling, i.e. h, is still undefined. In the case where the core is thin, i.e. $h > h_c/2$, and the opposite facing wrinkles are symmetric with respect to the middle plane, the value of h in (16) is replaced with half-thickness of the core, i.e. $h_c/2$.

Long Wrinkling Waves in an Orthotropic Facing

The solution of Hoff for a wrinkling wave oriented in the 1-direction has to be modified to account for a two-dimensional facing panel, rather than a one-dimensional facing in a sandwich beam. This solution is based on a minimization of the potential energy per unit width of the wrinkling wave that is composed of the strain energy of the core U_c and facings U_f and the work of the applied stress U_p, i.e.,

$$\Pi = U_c + U_p + U_f \tag{17}$$

where the energy and work components per unit width of the wrinkling wave are

$$U_c = \frac{1}{2E_c}\int_0^h \int_0^a \sigma_{cz}^2 \, dx_1 \, dz + \frac{1}{2G_c}\int_0^h \int_0^a \tau_{cx1z}^2 \, dx_1 \, dz \tag{18}$$

$$U_P = -\frac{\sigma_1 t}{2}\int_0^a \left(\frac{\partial w_f}{\partial x_1}\right)^2 dx_1 \tag{19}$$

$$U_f = \frac{D}{2}\int_0^a \left(\frac{\partial^2 w_f}{\partial x_1^2}\right)^2 dx_1 \tag{20}$$

The stresses in the isotropic core are

$$\sigma_{cz} = E_c w_{c,\,z} \qquad \tau_{cx1z} = G_c w_{c,\,x1} \tag{21}$$

The substitution of (16) and (18)–(21) into (17) and the subsequent minimization of the potential energy with respect to the amplitude value W yields the critical stress resulting in wrinkling:

$$\sigma_{x,\,cr} = \frac{(E_c a^2/\pi^2 h) + (G_c h/3) + (\pi^2 D/a^2)}{t\left(\cos^2\theta + r\sin^2\theta\right)} = \frac{\sigma_{1,\,cr}}{\cos^2\theta + r\sin^2\theta} \tag{22}$$

The minimization of this expression with respect to the wrinkling wavelength (a) and the depth of the affected zone in the core (h) yields

$$h = \sqrt[3]{\frac{9DE_c}{G_c^2}} \qquad a = \pi\sqrt[6]{\frac{3D^2}{E_c G_c}} \tag{23}$$

The substitution of (23) into (22) yields the critical value of the stress in the 1-direction:

$$\sigma_{1,\,cr} = \sqrt[3]{9DE_c G_c}/t = \frac{2.08}{t}\sqrt[3]{DE_c G_c} \tag{24}$$

Note that in the case of a sandwich beam with isotropic facings, $D = E_f t^3/12$ and (24) reduces to the result obtained for the corresponding case using the Hoff model:

$$\sigma_{1,\,cr} = \sqrt[3]{0.75 E_f E_c G_c} \tag{25}$$

If the depth of the core affected by the wrinkling deformation of the facing exceeds the core's half-thickness, i.e. Equation (23) yields $h > h_c/2$, and the wrinkling of the opposite facing is simultaneous and symmetric relative to the middle plane of the sandwich structure, this depth is assumed equal to half-thickness of the core, $h = h_c/2$. Accordingly, in this case the solution yields

$$\sigma_{1,\,cr} t = 2\sqrt{\frac{2DE_c}{h_c}} + \frac{G_c h_c}{6} \tag{26}$$

It is easy to show that in the case of a sandwich beam with isotropic facings, Equation (26) reduces to the corresponding solution.

If the panel is subject to compression applied to one facing and tension applied to the opposite facing, as is the case where the load is represented by lateral pressure, the opposite facing remains stable. In this case, if $h_c > h > h_c/2$, the solution is still given by (24).

The wrinkling stress given by (24) or (26) has to be minimized with respect to θ. This yields the following relationship between the stress ratio and the wrinkle angle for the case where the wrinkling stress is given by (24):

$$r = \frac{D \sin 2\theta + (\cos^2 \theta/3) g(\theta)}{D \sin 2\theta - (\sin^2 \theta/3) g(\theta)} \tag{27}$$

If the wrinkling stress is given by (26),

$$r = \frac{\sigma_1 \sin 2\theta + (1.414/t)\sqrt{(E_c/Dh_c)}\, g(\theta) \cos^2 \theta}{\sigma_1 \sin 2\theta - (1.414/t)\sqrt{(E_c/Dh_c)}\, g(\theta) \sin^2 \theta} \tag{28}$$

where $\sigma_1 = \sigma_1(\theta)$ is defined by (26).

Note that if the stress ratio $r = 0$ or $r = \infty$, the angle is equal to $\theta = \pi/2$ or $\theta = 0$, respectively. Of course, this corresponds to the cases of wrinkling under a uniaxial compressive stress.

Rectangular Wrinkles in an Orthotropic Facing

The following solution is derived by assumption that $h < h_c/2$. If this assumption is violated, the corresponding modifications in the solution are straightforward.

The wrinkle is assumed oriented along the load directions. The mode shape of the wrinkle is given by (10). Equation (16) that characterizes transverse deformations in the core has to be replaced with

$$w_c = W\frac{h-z}{h}\sin\alpha x\sin\beta y \tag{29}$$

Now the process of derivation of the buckling load suggested by Hoff is expanded to the case of a two-dimensional facing. The strain energy of the core is

$$U_c' = \frac{1}{2E_c}\int_0^{l_x}\int_0^{l_y}\int_0^h \sigma_{cz}^2\,dz\,dy\,dx + \frac{1}{2G_c}\int_0^{l_x}\int_0^{l_y}\int_0^h \left(\tau_{cxz}^2 + \tau_{cyz}^2\right)dz\,dy\,dx \tag{30}$$

Substituting into (30) the stresses in the core given by

$$\sigma_{cz} = E_c w_{c,z} \quad \tau_{cxz} = G_c w_{c,x} \quad \tau_{cyz} = G_c w_{c,y} \tag{31}$$

we obtain

$$U_c' = \frac{l_x l_y}{8}W^2\left[\frac{E_c}{h} + \frac{G_c h}{3}\left(\alpha^2+\beta^2\right)\right] \tag{32}$$

The work of applied stresses is given by

$$U_p' = -\frac{\sigma_x t}{2}\int_0^{l_x}\int_0^{l_y}\left(w,_x^2 + rw,_y^2\right)dy\,dx \tag{33}$$

Upon the substitution of the facing transverse displacement given by (10) or by (29) with $z=0$ we obtain

$$U_p' = -\frac{\sigma_x t l_x l_y W^2}{8}\left(\alpha^2 + r\beta^2\right) \tag{34}$$

The strain energy in the facing is

$$U_f' = \frac{1}{2}\int_0^{l_x}\int_0^{l_y}\left(D_{11}w,_{xx}^2 + 2D_{12}w,_{xx}w,_{yy} + 4D_{66}w,_{xy}^2 + D_{22}w,_{yy}^2\right)dy\,dx \tag{35}$$

The substitution of the facing transverse displacement yields

$$U_f' = \frac{l_x l_y W^2}{8}\left(D_{11}\alpha^4 + 2H\alpha^2\beta^2 + D_{22}\beta^4\right) \tag{36}$$

The potential energy is obtained as a sum of the three components evaluated above. The requirement $\partial\Pi/\partial W = 0$ yields the critical stress

$$\sigma_{x,\,cr} = \frac{D_{11}\alpha^4 + 2H\alpha^2\beta^2 + D_{22}\beta^4 + (E_c/h) + \left(G_c h\left(\alpha^2 + \beta^2\right)/3\right)}{t(\alpha^2 + r\beta^2)} \tag{37}$$

The stress given by (37) depends on the dimensions of the wrinkle in the x and y directions, i.e. α and β (or l_x and l_y) and on the depth of the affected zone of the core, i.e. h. The minimization of this stress with respect to h yields

$$h = \sqrt{\frac{3E_c}{G_c(\alpha^2 + \beta^2)}} \tag{38}$$

The substitution of (38) into (37) results in

$$\sigma_{x,\,cr} = \frac{D_{11}\alpha^4 + 2H\alpha^2\beta^2 + D_{22}\beta^4 + 2\sqrt{E_c G_c(\alpha^2 + \beta^2)/3}}{t(\alpha^2 + r\beta^2)} \tag{39}$$

The minimization of the critical stress given by (39) with respect to α and β requires unnecessarily long transformations. It is more convenient to introduce $\omega = \alpha/\beta$; in this case the critical stress given by (39) can be minimized with respect to α. The result is a relationship between α and ω corresponding to wrinkling instability:

$$\alpha = \sqrt[3]{\frac{E_c G_c(1 + \omega^2)}{3\left(D_{11} + 2H\omega^2 + D_{22}\omega^4\right)^2}} \tag{40}$$

The most convenient approach is to prescribe the wrinkle shape parameter ω. Then the corresponding value of α is available from (40). Subsequently the critical stress is obtained from (39). The process continues, i.e. the value of ω is varied, until the minimum (wrinkling) stress $\sigma_{x,\,cr}$ is found. Note that this process is based on the presumption that the stress ratio r is constant.

If the depth of the affected core zone, h, obtained from (38) exceeds half-thickness of the core and the opposite facings wrinkle symmetrically with respect to the middle plane, h should be replaced with $h_c/2$ and the procedure is simplified accordingly.

In a particular case of an isotropic facing subject to equal compressive stresses in both x- and y-directions, the solution is significantly simplified. Obviously, in this case $D_{11} = D_{22} = H = D$, $\alpha = \beta$, and $r = 1$. Accordingly,

$$\sigma_{x,\,cr} = \frac{2D\alpha^3 + \sqrt{2E_c G_c/3}}{t\alpha} \tag{41}$$

The minimization of this stress with respect to α yields

$$\alpha = \sqrt[6]{\frac{E_c G_c}{24D^2}} \tag{42}$$

The wrinkling stress becomes

$$\sigma_{x,\,cr} = \frac{2.08}{t}\sqrt[3]{DE_c G_c} \tag{43}$$

Note that the same result is given by Equations (22) and (24) if $r = 1$. Therefore, in the case of an isotropic facing and $h < h_c/2$, the wrinkling stress values corresponding to long wrinkling waves and to square wrinkles are equal to each other.

It is also interesting to compare the result in Equation (43) to the buckling stress for isotropic facings subjected to equal compression in two mutually perpendicular directions obtained by Plantema [2]. According Case IV in Table 2.1. of this monograph, the wrinkling value of the applied stresses is equal to

$$\sigma_{x,\,cr} = \frac{1.89}{t}\sqrt[3]{DE_c G_c} \tag{44}$$

Therefore, the Plantema method predicts a lower wrinkling stress, than the Hoff method, at least in the example considered here.

Model 3: Facing Supported by Plantema's core

The approach adopted by Plantema [2] was based on the assumption that the facing wrinkling deformation causes local deflections in the core that decay exponentially with the distance from the affected facing. As in the case of Hoff's approach, in-plane displacements of the core are neglected. Accordingly, in the case of a long wrinkling wave, the deflections in the core are given by

$$w_c = We^{-nz}\sin\frac{\pi x_1}{a} \tag{45}$$

where n is an unknown coefficient.

Long Wrinkling Waves in an Orthotropic Facing

The stresses in an isotropic core are still given by (21). The strain energy components of the core and the facing per unit width of the wrinkling wave and the work of the applied stress per unit wave width are given by (18)–(20). However, the upper limit of integration through the core thickness in (18) should be ∞, rather than h. The substitution of (21) into the expressions for the energy and work components and the minimization of the potential energy with respect to the amplitude of the wrinkling deflection, W, yields the critical stress:

$$\sigma_{x,\,cr} = \frac{(E_c n a^2/2\pi^2) + (G_c/2n) + (\pi^2 D/a^2)}{t\left(\cos^2\theta + r\sin^2\theta\right)} = \frac{\sigma_{1,\,cr}}{\cos^2\theta + r\sin^2\theta} \tag{46}$$

The minimization of the wrinkling stress with respect to the constant n results in

$$n = \frac{\pi}{a}\sqrt{\frac{G_c}{E_c}} \tag{47}$$

The critical stress can now be expressed in terms of the length of the wrinkling wave as

$$\sigma_{1,\,cr} t = \left[D\left(\frac{\pi}{a}\right)^2 + \frac{a}{\pi}\sqrt{E_c G_c}\right] \tag{48}$$

The minimization of the stress given by (48) with respect to a yields both the length of the wrinkling wave and wrinkling stress:

$$a = 1.26\pi\sqrt[6]{\frac{D^2}{E_c G_c}} \tag{49}$$

$$\sigma_{1,\,cr} = \frac{1.89}{t}\sqrt[3]{D E_c G_c} \tag{50}$$

Note that this expression coincides with the formula obtained by Plantema [2] for the wrinkling stress in an isotropic facing, except for the fact that in the present paper, $D = D(\theta)$, i.e. the magnitude of the critical stress acting perpendicular to the wrinkling wave is affected by the wave orientation relative to the x-axis.

If the stress ratio, r, is prescribed, the minimization of the stress $\sigma_{x,\,cr}$ with respect to the angle θ yields the same expression (27) as in the Hoff method. Therefore, the orientation of the wrinkling wave is the same in both the

Hoff and Plantema methods, as long as the core is thick and/or the opposite facing remains stable. This fact enables us to compare the magnitudes of the wrinkling stresses given by Equations (24) and (50) and to identify the Plantema method as more conservative. This is predictable since the same conclusion was obtained in the case of isotropic facings by Fagerberg [5].

Rectangular Wrinkles in an Orthotropic Facing

The core deformation is given by

$$w_c = We^{-nz} \sin \alpha x \sin \beta y \tag{51}$$

The stresses in the core can be evaluated by Equation (31). The strain energy of the core and the facings and the work of the applied stresses are calculated by Equations (30), (33), and (35) where the upper integration limit h in (30) is replaced with ∞, similar to the case of a wrinkling wave. The resulting expression for the potential energy is

$$\Pi = \frac{l_x l_y W^2}{8}\left[D_{11}\alpha^4 + 2H\alpha^2\beta^2 + D_{22}\beta^4 + \frac{E_c n}{2} + \frac{G_c}{2n}\left(\alpha^2+\beta^2\right) - \sigma_x t\left(\alpha^2 + r\beta^2\right)\right] \tag{52}$$

The critical stress is immediately available by minimizing the potential energy with respect to the amplitude of the deflection:

$$\sigma_{x,\,cr} = \frac{D_{11}\alpha^4 + 2H\alpha^2\beta^2 + D_{22}\beta^4 + (E_c n/2) + (G_c\left(\alpha^2+\beta^2\right)/2n)}{t(\alpha^2 + r\beta^2)} \tag{53}$$

The minimization of the critical stress with respect to n yields the value of this constant:

$$n = \sqrt{\frac{G_c}{E_c}(\alpha^2+\beta^2)} \tag{54}$$

Let $\beta = \varpi\alpha$ where ϖ is a parameter that characterizes the shape of the wrinkle. Then the substitution of (54) into (53) and transformations yield

$$\sigma_{x,\,cr} = \frac{\alpha^3(D_{11} + 2H\varpi^2 + D_{22}\varpi^4) + \sqrt{E_c G_c(1+\varpi^2)}}{t\alpha(1 + r\varpi^2)} \tag{55}$$

This stress can be minimized with respect to α yielding the value of this parameter as a function of the shape of the wrinkle:

$$\alpha = \sqrt[6]{\frac{E_c G_c (1 + \varpi^2)}{4(D_{11} + 2H\varpi^2 + D_{22}\varpi^4)^2}} \tag{56}$$

The solution can be obtained by varying the value of ϖ. Then α is calculated from (56) and the critical stress is found from (55). The wrinkling stress corresponds to the smallest value of $\sigma_{x,cr}$. As indicated above, if the facing is isotropic, the solution yields the wrinkling stress given by (44). The fact that both the wrinkling wave for a composite facing and a rectangular wrinkle in the case of an isotropic facing correspond to a smaller applied stress by the Plantema method than by the Hoff method encourages us to recommend the former method for the analysis of wrinkling.

Numerical Examples and Discussion

Sandwich panels considered in the following examples consisted of E-glass–vinyl ester facings (fiber volume fraction equal to 30%) and several different cores, including various grades of balsa and Divinylcell™ foam. The properties of composite laminae of the facings were [18]: $E_1 = 24.4$ GPa, $E_2 = 6.87$ GPa, $G_{12} = 2.89$ GPa, $\nu_{12} = 0.32$. The facings were constructed from 0.125 mm thick layers. The properties of the cores considered in the paper are outlined in Table 1.

Two wrinkling modes considered in the paper included a large-aspect-ratio wrinkling wave and rectangular wrinkles, as shown in Figures 1 and 2. The results of the computations have shown that the wrinkling waves and rectangular wrinkles are likely to appear at the same stress level in balanced cross-ply facings (as shown above, the same situation is encountered in the case of isotropic facings). However, if a composite facing lamination is different from cross-ply, the wrinkling wave appears at a lower level of stresses. All results were obtained for the case where the sandwich panel is subject to transverse pressure. Accordingly, the facing that is opposite to the compressed facing under consideration was assumed stable. Although the magnitude of compressive stresses in the facing of the panel subject to bending associated with transverse pressure varies throughout the planform of the facing, such variations are disregarded, considering a small size of wrinkles compared to the size of the facing.

The relationships between the applied stress ratio and the angle of orientation of the wrinkling wave are shown in Figure 3 for two different models of the core (the Hoff model of the core is not considered here since it yields consistently unconservative results, as compared to the Plantema

Table 1. Properties of cores considered in the paper [5,19].

Core Material	Mass Density (kg/m^3)	Shear Modulus (MPa)	Young's Modulus (MPa)
Balsa	96	108.0	280.8
Divinylcell H-30	36	13.0	20.0
H-45	48	18.0	40.0
H-100	100	40.0	105.0
H-160	160	66.0	170.0
H-250	250	108.0	300.0

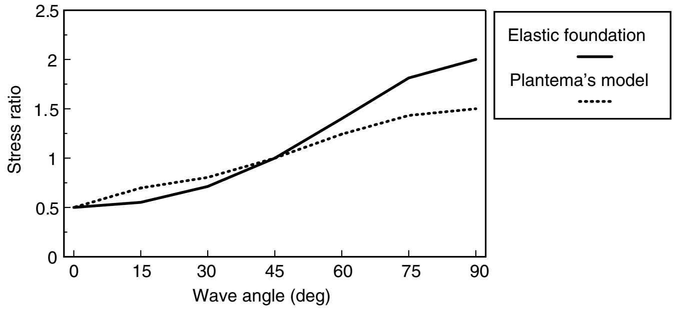

Figure 3. *Stress ratio $r = \sigma_y/\sigma_x$ corresponding to the orientation angle of the wrinkling wave θ. The thickness of the facing is 2.5 mm.*

model). The facings were balanced and cross-ply laminated. It is immediately evident from the solutions for all core models that the orientation of the wrinkling wave is independent of the core parameters, although it is affected by the core model. The orientation of the wrinkling wave is affected by the stress ratio and the facing layers lamination. In the example considered here, at the stress ratios that are lower than those shown in Figure 3, the wave is perpendicular to the larger stress σ_x, while at the stress ratios exceeding those shown in the figure, the wave is perpendicular to σ_y.

The critical value of the stress σ_x is shown in Figure 4 for cross-ply facings as a function of the stress ratio and the balsa core thickness. For completeness, the results are compared with the strength criterion (maximum principal stress). As follows from Figure 4, the failure of the facing may occur by the loss of strength, rather than wrinkling, if the core is relatively thin (20 mm). This is expected since a thinner core provides a firmer support to the facing, increasing its stability. The wrinkling solution based on the elastic foundation core model provided a more conservative result than those generated by either Plantema or Hoff models of the core.

The results shown in Figure 5 were obtained for the sandwich panels with Divinylcell 20 mm thick cores and cross-ply laminated facings. Similar to the

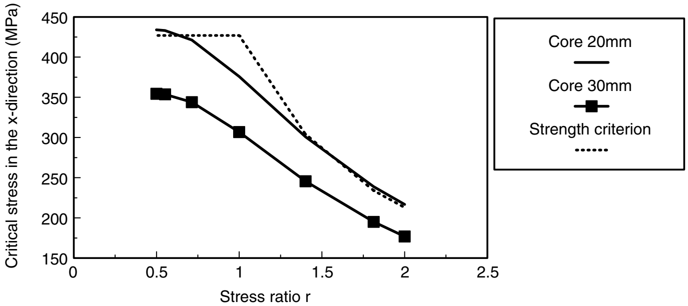

Figure 4. *Critical (wrinkling or loss of strength) value of the stress σ_x for cross-ply facings supported by balsa core (density 96 kg/m^3) as a function of the stress ratio and core thickness. The thickness of the facing is equal to 2.5 mm.*

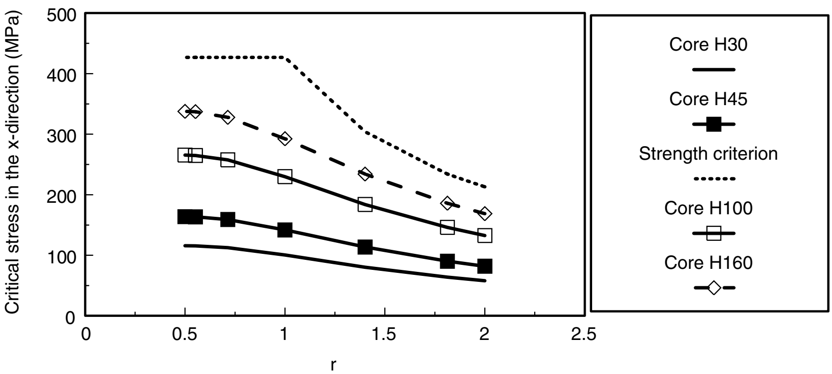

Figure 5. *Critical (wrinkling or loss of strength) value of the stress σ_x for cross-ply facings supported by various Divinylcell cores that are 20 mm thick as a function of the stress ratio r. The thickness of the facing is equal to 2.5 mm.*

previously discussed results for balsa core panels, the elastic foundation model provided more conservative estimates of the wrinkling stress shown in the figure. A relatively dense and stiff core may provide sufficient support to the facing to avoid wrinkling (the loss of strength was the mode of failure for H-250 core that is not shown in the figure). Similar results are shown in Figure 6 for a different facing lamination and thickness. In this case, the facings are thicker and the loss of strength becomes the mode of failure for lighter cores than in the case shown in the previous figure (H-100, H-160). The elastic foundation core model provided the wrinkling stress in the case considered in Figure 6, while the Plantema and Hoff models have proven unconservative.

Finally, the results in Figure 7 refer to panels with facings identical to those analyzed in Figure 6 but with a much thinner core that provides a firmer support to the facing. Accordingly, the loss of strength dominated the

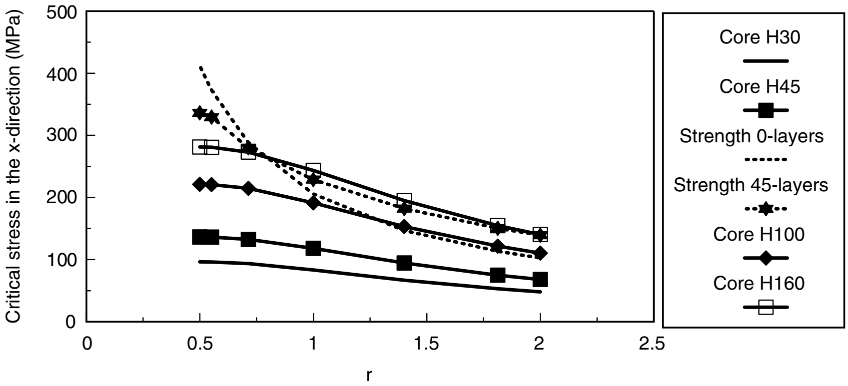

Figure 6. *Critical (wrinkling or loss of strength) value of the stress σ_x for $[0/+45°/-45°]_n$ facings supported by various Divinylcell cores that are 20 mm thick as a function of the stress ratio r. The thickness of the facing is equal to 3.0 mm.*

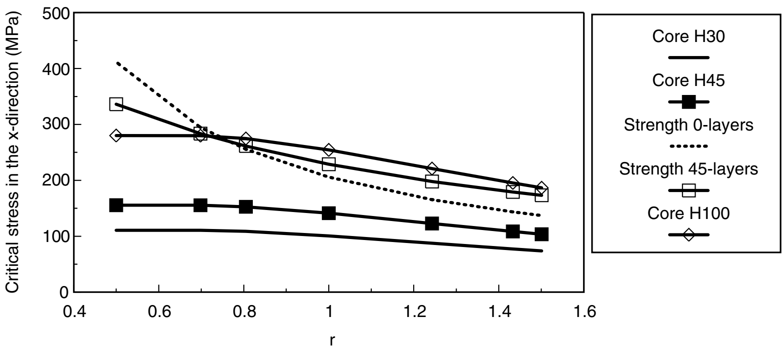

Figure 7. *Critical (wrinkling or loss of strength) value of the stress σ_x for $[0/+45°/-45°]_n$ facings supported by various Divinylcell cores that are 6 mm thick as a function of the stress ratio r. The thickness of the facing is equal to 3.0 mm.*

failure prediction for all cores, except for very light grades (H-30, H-45). Notably, in this case, the wrinkling analysis utilizing the Plantema core model provided a more conservative estimate for the wrinkling stress.

CONCLUSIONS

The paper illustrates straightforward closed-form techniques for the evaluation of the wrinkling stresses of composite facings in sandwich panels subject to biaxial compression. Three models of the core that supports the facing are employed, namely the Wrinkler elastic foundation, and the Hoff and Plantema models. The former model is suitable for the wrinkling instability modes characterized by relatively large size of buckles, in which case the effect of shearing stresses in the core is negligible. The second and third (Hoff's and Plantema's) models can be used for the analysis of instability characterized by a smaller size of wrinkles. Two wrinkling instability modes were considered, i.e. long wrinkling waves that can be perpendicular or oblique relative to the directions of the applied stresses and rectangular wrinkles. The following general conclusions are available from the analysis:

1. The Plantema model of the core predicts lower (conservative) combinations of wrinkling stresses than the Hoff method. Therefore, this model is recommended for the analysis, in conjunction with the Wrinkler elastic foundation model.
2. The orientation of the wrinkling wave depended on the ratio of applied compressive stresses in the x and y directions. If the stress in one of the directions dominates, the wave is perpendicular to this stress. In the intermediate range of the stress ratio, the wave is inclined relative to the x and y axes. The angle of wave orientation is dependent on the stress ratio and the facing material, thickness, and lamination.
3. The elastic foundation core model was appropriate for the wrinkling analysis in the case of a light (low stiffness) core of a relatively large thickness. If the core density and stiffness are higher and/or the thickness of the core is small, the core provides a firmer support to the facings and the size of buckles decreases. In this case, the Plantema model is more conservative than the elastic foundation model.
4. If the core provides an even firmer support, the mode of failure of the facing is the loss of strength, rather than wrinkling instability.
5. In considered examples, the wrinkling wave was the dominant facing instability mode for composite angle-ply facings. In the case of isotropic and balanced cross-ply facings, the wrinkling wave and rectangular wrinkle instability occurred at the same level of applied stresses.

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