Effect of z-Pins on Fracture in Composite Cocured Double Cantilever Beams

Victor Birman and Larry W. Byrd

Abstract: The paper illustrates a new approach to the evaluation of the effect of z-pins on deformations and the strain energy release rate in composite double cantilever beams (DCB) subject to a standard fracture toughness test. The effect of z-pins is modeled by an elastic foundation, based on previously published work. The approach to the solution is based on a separate analysis of the intact and delaminated parts of DCB. The rotational stiffness of the intact part is obtained from the Rayleigh-Ritz solution for this part subjected to a force couple, rather than modeling the rotational restraint by introducing an elastic foundation, as has been done in the previous studies. Subsequently, the deformation of the delaminated part of DCB is analyzed exactly by solving the equation of equilibrium with the appropriate boundary conditions. Based on this solution, the compliance, the rate of change of compliance, and the strain energy of the specimen can be evaluated. The results illustrate the beneficial effect of z-pins on the resistance of DCB to delamination cracking.

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Introduction

Delamination cracks originating from the edge are recognized as the principal cause of damage and failure in bonded adhesive and cocured joints. Z-pins, i.e., small-diameter cylindrical rods embedded in the composite material and oriented perpendicular to the layer interface, represent a possible method of arresting these cracks. Extensive studies illustrating the beneficial effects of z-pins on various aspects of the behavior of composite structures have been published by Freitas et al. (1994), Barrett (1996), Lin and Chen (1999), Palazotto et al. (1999), Vaidya et al. (1999), and Mabson and Deobald (2000). In particular, this method may be effective in enhancing fracture and fatigue resistance of cocured joints between composite skin and stiffeners similar to that depicted in Fig. 1. In the present paper, the effectiveness of z-pins is estimated based on a standard double cantilever beam (DCB) test prescribed for composite adhesive joints (though an adhesive layer is not used in cocured joints).

Previous solutions employed in the analysis of DCB were based on modeling the effect of a limited rotational restraint of the intact part of the beam (Fig. 2; x > 0) through the introduction of an elastic foundation. Such an approach was originally proposed by Kanninen (1973, 1974) and Gehlen et al. (1979). It was further extended to transversely isotropic materials by Williams (1989) and to angle-ply laminates by Ozdil and Carlson (1999) Penado (1993) used the same approach to incorporate the effect of an adhesive layer between the two halves of the beam. In a recent paper, the writers approached the subject of the analysis of cocured z-pinned DCB using the same approach, i.e., modeling the rotational restraint of DCB via an elastic foundation (Byrd and Birman 2004). In the present paper, a new method to the solution of the problem is developed where the elastic rotational restraint at the tip of the crack is evaluated through the analysis of the intact section of DCB. Subsequently, the analysis of the delaminated section (Section 1 in Fig. 2) can be carried out without difficulties. Similar to the previous work, the effect of z-pins is introduced through an equivalent elastic foundation (Fig. 2).

The solution for the strain energy release rate for Mode I fracture toughness of DCB specimens that remain within the elastic range is given by

\[ G_I = \frac{P^2}{2b} \frac{dC}{da} \]  

(1)

where \( P \) = applied load; \( b \) = width of the specimen; and \( dC/da \) = rate of change of the compliance per unit crack growth. The critical strain energy release rate, i.e., fracture toughness, is given by Eq. (1), where \( P = P_c \) is the fracture load.

Analysis

Stiffness of Beam Material in Sections 1 and 2
Accounting for Contribution of z-Pins

Consider the response of Sections 1 and 2 of the beam shown in Fig. 2. The present solution is limited to the case where the length of the crack remains small, i.e., \( a < c \). Accordingly, the separation between the delaminated parts of the beam in Section 1 is sufficiently small to assume that the pins in this section remain partially embedded within the joint material (i.e., there is no complete pullout of z-pins from Section 1). The nonlinear pullout force-displacement response of the pins includes the initial linear relationship corresponding to the limited motion of the pins rela-

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Fig. 1. Cocured z-pinned joint between skin and stiffener.

Fig. 2. DCB with z-pins loaded in Mode I and model used in analysis based on modeling rotational stiffness of Section 2 through introduction of an elastic foundation; note that z-pins are pulled out of only one of delaminated sections [from Byrd and Birman (2004)].

Fig. 3. Computational scheme of Section 2 (half-thickness of intact part of DCB): applied moment and undeformed section shown on left; couple of forces equivalent to applied moment and deformed section shown on right.

Eq. (2) corresponds to the case where z-pins are pulled out of one Section 1, while they remain fully embedded in the second Section 1, as shown in Fig. 2. However, Eq. (2) is also valid in the situation where z-pins are simultaneously pulled from both Sections 1.

In Section 2, the pins do not experience partial pullout. This section is analyzed as an orthotropic beam with different stiffness in the x- and z-directions (Fig. 3). The beam is in the state of plain strain; i.e., it experiences no deformation in the direction perpendicular to the xz plane. The presence of z-pins changes the effective modulus in the x-direction in the same manner as fibers embedded in the matrix affect the transverse modulus of a laminate. The laminate and z-pins work in series in the axial beam direction. Accordingly, for the jth layer of Section 2:

$$\frac{1}{E_{xj}} = \frac{1 - V_p}{E_{1xj}} + \frac{V_p}{E_p}$$

where $E_{xj}$=modulus of the layer without the z-pins in the axial (x) direction; and $E_p$=modulus of the pin material. A similar equation can be employed to determine the shear modulus of the layer ($G_{xzj}$) in terms of the shear modulus of the pristine layer material (without z-pins) and the shear modulus of the pin material.

The modulus of Section 2 in the z-direction can be obtained by the rule of mixtures, i.e.

$$E_{zj} = (1 - V_p)E_{zj} + V_pE_p$$

where $E_{zj}$=modulus of the composite material in the thickness direction.

The Poisson's ratios necessary to evaluate reduced stiffness coefficients can be evaluated from (notations are self-evident)

$$v_{xzj} = (1 - V_p)v_{x3j} + V_pv_p$$

$$v_{zxj} = \frac{E_{zxj}}{E_{zj}}$$

As follows from this discussion, the presence of z-pins in Section 2 is treated as if these pins were "fibers" embedded within an orthotropic "matrix" represented by the composite material of the section. Accordingly, Eqs. (4) and (5) are similar to the corresponding equations in the mechanics of materials approach to the evaluation of the properties of a composite material.

Alternative micromechanical formulations are available to evaluate material constants of the layers with embedded z-pins. For example, a "simplified" micromechanical model can be more accurate for the transverse and shear moduli (Chamis 1984):

$$v_{xzj} = (1 - V_p)v_{x3j} + V_pv_p$$

$$v_{zxj} = \frac{E_{zxj}}{E_{zj}}$$
$$E_{xj} = \frac{E_{1j}}{1 - \sqrt{V_p} \left( 1 - \frac{E_{1j}}{E_p} \right)}$$

(6)

$$G_{xzj} = \frac{G_{13j}}{1 - \sqrt{V_p} \left( 1 - \frac{G_{13j}}{G_p} \right)}$$

where $G_{13j}$ is the transverse shear modulus of the pristine composite material of the $j$th layer.

Another convenient semiempirical model was developed by Halpin and Tsai (1969):

$$E_{xj} = \frac{1 + \xi \eta V_p}{1 - \eta V_p}$$

(7)

$$\eta = \frac{E_p/E_{1j} - 1}{E_p/E_{xj} + \xi}$$

where the curve-fitting parameter may be taken equal to $\xi = 2$ (Gibson, 1994).

The corresponding Halpin-Tsai formula for the shear modulus is

$$G_{xzj} = \frac{1 + \xi' \eta' V_p}{1 - \eta' V_p}$$

(8)

$$\eta' = \frac{G_p/G_{13j} - 1}{G_p/G_{13j} + \xi'}$$

where $\xi' = 1$.

Note that, while the equations presented previously can characterize material constants in Section 2, where z-pins do not experience pullout, these equations have to be adjusted in Section 1 to account for a partial pullout of z-pins from the specimen. Accordingly, the stiffness of the affected section decreases, beginning from the outer surface. However, parametric analysis illustrates that, even at relatively high volume fractions of z-pins, the variations in stiffness between the material with fully embedded z-pins and the material with partially pulled out z-pins remain fairly small. For example, if the volume fraction of z-pins is equal to 1%, variations in the stiffness of plain-woven carbon/epoxy with fully embedded and fully pulled out titanium z-pins is less than 12%. The effect on the effective stiffness of the specimen is even smaller, because the case of complete pullout is of little interest, corresponding to immediate failure of the specimen. Therefore, a sufficiently accurate analysis can be conducted by using uniform properties of the material obtained by the equations presented previously.

### Analysis of Section 2

The analysis of Section 2 aims at determining the rotational stiffness of this section. The section is modeled as a semi-infinite beam that is subject to a moment applied at the left end (Fig. 3). In the first approximation, deformations of the beam in the thickness $(z)$ direction can be neglected, i.e., it is assumed that the beam does not change its thickness under the action of the moment. However, it will experience shearing deformations due to displacements in the axial $(x)$ direction. A more accurate approach taking into account displacements $w(x,z)$ in the $z$-direction requires a complicated analysis. As is shown subsequently, the accuracy of the present solution is satisfactory as long as the ratio $h/a$ remains relatively small.

The applied moment can be replaced with a couple, as shown in Fig. 3:

$$N = \frac{M}{h}$$

(9)

Note that the width of Section 2 can be assumed equal to unity, in which case $M$ and $N$ are the stress couple and stress resultant, respectively.

The analysis of Section 2 is performed by the Rayleigh-Ritz method. The total potential energy is evaluated as

$$\Pi = \frac{1}{2} \int_{x=0}^{x=h/2} \int_{z=0}^{z=h/2} (Q_{11}\varepsilon^2_x + Q_{55}\varepsilon^2_z) dx dz - 2Nu(x=0, z=h/2)$$

(10)

where $Q_{ij}$ are the corresponding reduced stiffnesses; and $u$ is the displacement in the $x$-direction.

The reduced stiffnesses are defined as

$$Q_{11} = \frac{E_{xj}}{1 - \nu_{xzj}v_{xzj}}, \quad Q_{55} = G_{xzj}$$

(11)

The strains in Eq. (10) are the following functions of the axial displacements ($w=0$):

$$\varepsilon_x = u_{xx}, \quad \gamma_{xz} = u_{xz}$$

(12)

The Rayleigh-Ritz method can be applied if the assumed displacements satisfy the kinematic boundary conditions, while the static conditions can be violated. In this study, it is assumed that axial displacements can be represented by

$$u = Ue^{-\omega t} \sin \frac{\pi z}{2h}$$

(13)

where the amplitude value $U$ and the value of the displacement decay parameter $\omega$ are unknown.

Eq. (13) satisfies the kinematic conditions of the problem, i.e.,

$$u(z=0) = 0, \quad u(x \to \infty) = 0$$

(14)

The Rayleigh-Ritz method implies that

$$\frac{\partial \Pi}{\partial U} = \frac{\partial \Pi}{\partial \omega} = 0$$

(15)

Once the unknown amplitude and the displacement decay parameter have been determined from Eq. (15), the average rotation of Section 2 at $x=0$ can be evaluated from

$$\gamma_{xz}(x=0) = 2u(x=0, z=h/2)$$

(16)

Subsequently, the rotational spring coefficient of the boundary of the intact section of DCB (Section 2) is found as

$$K = \frac{Nh}{\gamma_{xz}(x=0)}$$

(17)

Substitution of Eq. (13) into Eqs. (12) and (10) yields the expression for the potential energy of Section 2. For example, if all layers are unidirectional and identical or if the material is plain-woven, this energy becomes
The computational scheme of Section 1 is shown in Fig. 4. Consider unidirectional or plain-woven materials, so that the delaminated end corresponds to a positive coordinate \(a\) with the solution.

The rotational stiffness of Section 2 is given by Eq. (28).

A particular case where the DCB does not incorporate z-pins is considered here to compare the present approach to available solutions. Consider unidirectional or plain-woven materials, so that the delaminated end corresponds to a positive coordinate \(x = a\). The analysis shown here is limited to the case of a “thin” Section 1 where transverse shear strains are negligible. The case of a “shear deformable” Section 1, which is relevant if the length of the crack is small, is also illustrated subsequently.

In the absence of an elastic foundation, the equation of equilibrium of Section 1 is of course

\[
\frac{d^4w}{dx^4} = 0
\]

with the solution

\[
w = A_0 + A_1 x + A_2 x^2 + A_3 x^3
\]

The constants of integration \(A_i\) can be determined from the boundary conditions:

\[
w(0) = 0, \quad M(0) = \frac{P}{b} w = K \frac{dw(0)}{dx}
\]

The constants of integration \(c\) can be determined from the boundary conditions:

\[
U = \frac{4N}{\pi \sqrt{Q_{11} Q_{55}}}
\]

Accordingly, the rotational stiffness of Section 2 obtained from Eq. (17) is

\[
K = \frac{\pi h^2}{8 \sqrt{Q_{11} Q_{55}}}
\]

The units of the rotational stiffness are Newtons; i.e., this is a stiffness of a unit-width Section 2.

**Analysis of “Thin” Section 1: Case of Double Cantilever Beam without z-Pins**

A particular case where the DCB does not incorporate z-pins is considered here to compare the present approach to available solutions. Consider unidirectional or plain-woven materials, so that the rotational stiffness of Section 2 is given by Eq. (21). The computational scheme of Section 1 is shown in Fig. 4 (the elastic foundation that represents the effect of z-pins should be disregarded). Note that the Section was “rotated” for convenience, so that the delaminated end corresponds to a positive coordinate \(x = a\). The analysis shown here is limited to the case of a “thin” Section 1 where transverse shear strains are negligible. The case of a “shear deformable” Section 1, which is relevant if the length of the crack is small, is also illustrated subsequently.

In the absence of an elastic foundation, the equation of equilibrium of Section 1 is of course

\[
\frac{d^4w}{dx^4} = 0
\]

with the solution

\[
w = A_0 + A_1 x + A_2 x^2 + A_3 x^3
\]

The constants of integration \(A_i\) can be determined from the boundary conditions:

\[
w(0) = 0, \quad M(0) = \frac{P}{b} w = K \frac{dw(0)}{dx}
\]

The accuracy of the approach considered in the present report can be checked by comparing the results available from Eq. (27) to available numerical and experimental data. For example, Kanninen (1973) derived the following formula for the compliance based on modeling the rotational stiffness of Section 2 by an elastic foundation approach:

\[
C_K = \frac{8}{E b h} \left( a \right)^3 \left[ 1 + 1.92 \frac{h^2}{a} + 1.22 \left( \frac{h}{a} \right)^2 + 0.39 \left( \frac{h}{a} \right)^3 \right]
\]

Eq. (28) was obtained for an isotropic material with the modulus of elasticity \(E\) neglecting Poisson’s effect.

According to the paper of Kanninen (1973) as well as the analysis of Ozdil and Carlsson (1999), who extended Eq. (28) to the case of composite DCB, this formula yields results that are in good agreement with experimental data. Note that Eq. (28) was derived for the case where the length of Section 2 is infinite, i.e., the same situation as that considered in the present solution. The effect of shear deformations, which becomes essential if the ratio \(h/a\) is large, was disregarded; i.e., Eq. (28) was obtained for “thin” beams.

The comparison between the compliance given by Eq. (28) and the solution obtained in the present report is presented hereafter. Eqs. (27) and (21) yield

\[
C = \frac{8}{E b h} \left( a \right)^3 \left[ 1 + \frac{2}{\pi} \sqrt{\frac{E h}{G a}} \right]
\]

If \(E = 2.6G\), this equation becomes

\[
C = \frac{8}{E b h} \left( a \right)^3 \left[ 1 + 1.03 \frac{h}{a} \right]
\]

The ratio between the compliance values obtained by two methods, i.e., \(R_1 = C_K/C\), is shown in Table 1.
The ratio between the strain energy release rate values obtained modeling the rotational stiffness by an elastic foundation and by the present approach is \((E=2.6G)\):

\[
R_2 = \frac{3 + 3.84\frac{h}{a} + 1.22\left(\frac{h}{a}\right)^2}{3 + 2.06\frac{h}{a}}
\]

(31)

This ratio is also shown in Table 1.

As follows from Table 1, the present method yields results that are in good agreement with the approach based on modeling the rotational stiffness by an elastic foundation. The difference between the two methods is significant only if Section 1 is relatively thick, in which case both solutions compared in Table 1 become irrelevant. The agreement between the strain energy release values is better than that between the compliance values.

In conclusion, it is obvious that the present solution yields close agreement with the solutions based on a less logical approach modeling rotational stiffness of the beam by an elastic foundation. The difference between the two methods increases if Section 1 is relatively thick. However, even if the ratio \(h/a\) \(\rightarrow 0.15\), the difference remains less than 15% for the compliance and less than 9% for the strain energy release rate and fracture toughness. The present approach yields results that are lower than those calculated by the elastic foundation method.

Analysis of “Thin” Section 1: Case of Double Cantilever Beam with z-Pins

In the presence of z-pins, the equation of equilibrium of Section 1 becomes (Poisson’s effect is neglected):

\[
d^4w \over dx^4 + \frac{12}{E,h^3}(-K_1w + K_0) = 0
\]

(32)

where \(E_0=\text{effective bending modulus.}\)

The solution of Eq. (32) is

\[w = A_1 \cos \lambda_1 x + B_1 \sin \lambda_1 x + C_1 \cosh \lambda_1 x + D_1 \sinh \lambda_1 x + \frac{K_0}{K_1}
\]

(33)

where \(\lambda_1 = \sqrt[4]{12K_1/(E,h^3)}\).

The following boundary conditions should be employed to specify the constants of integration:

\[w(0) = 0, \quad D_{11}w_{xx}(0) = K_{11} w_{xx}(0)
\]

\[w_{xx}(a) = 0, \quad w_{xx}(a) = -\frac{12P}{E_xh^3}
\]

(34)

Substitution of Eq. (33) into the boundary conditions of Eq. (34) yields a system of algebraic equations for the constants of integration:

\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{22} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix}
\begin{bmatrix}
A_1 \\
B_1 \\
C_1 \\
D_1
\end{bmatrix}
= \begin{bmatrix}
b_1 \\
b_2 \\
b_3 \\
b_4
\end{bmatrix}
\]

(35)

where

\[a_{11} = a_{13} = 1, \quad a_{12} = a_{14} = 0
\]

\[a_{31} = -a_{32} = \cos \lambda_1 a, \quad a_{32} = a_{34} = \sin \lambda_1 a
\]

\[a_{33} = -a_{44} = -\cosh \lambda_1 a, \quad a_{34} = -a_{43} = -\sinh \lambda_1 a
\]

and

\[b_1 = -\frac{K_0}{K_1}, \quad b_2 = b_3 = 0
\]

\[b_4 = -\frac{12P}{E_xh^3}\lambda_1^3
\]

The deflection of the loaded end of Section 1 is now evaluated as

\[w(a) = \frac{K_0}{K_1} + A_1 \cos(\lambda_1 a) + B_1 \sin(\lambda_1 a) + C_1 \cosh(\lambda_1 a)
\]

\[+ D_1 \sinh(\lambda_1 a)
\]

(38)

The rate of change of compliance, i.e.

\[
\frac{dC}{da} = \frac{2}{P} \frac{dw(a)}{da}
\]

(39)

cannot be easily evaluated, because the constants of integration depend on the length of the crack. However, it is possible to derive this rate from a graphical representation of the deflection of the delaminated end of the specimen shown as a function of the length of the crack.

The strain energy release rate can now be evaluated from Eq. (1) using the value of the applied force \(P\). In the case where fracture toughness is evaluated, the effect of z-pins on the fracture force can be found according to the observations from the writers previous paper (Byrd and Birman 2004). As was shown in this paper, the increase in fracture force due to the introduction of z-pins is proportional to the increase in strength of the material in the thickness \((z)\) direction. Therefore, adjusting the force \(P\), to account for the presence of the z-pins and using the rate of change of compliance corresponding to the fracture force, it is possible to predict fracture toughness. However, in the presence of z-pins, the strain energy release rate and fracture toughness become complicated functions of the length of the crack. This was also noted by Rugg et al. (2002), who did not recommend fracture toughness for engineering characterization of z-pinned joints.

Based on the observation in the previous paragraph, it is evident that there is need of a better characteristic of the efficiency of the z-pins for the prevention of delamination than fracture toughness. The obvious candidate is the rate of change of deflections with the length of the crack. If the deflections increase as the crack propagates, the structure eventually fails. However, if the deflections begin to decrease, this implies closing of the crack due to the effect of z-pins. In reality, decreasing deflections do not make physical sense, because z-pins cannot “push” the delaminated sections toward each other. However, if the deflections reach a maximum and start to decrease, as the crack length increases, this maximum indicates the arrest of the crack.
The analysis using a shear-deformable beam theory may be justified based on the results from the present and previous papers (Byrd and Birman 2004), where it was shown that a typical DCB with z-pins fails when the delamination crack is relatively short. In this case, the section of Section 1 at failure is small, i.e., this section can be treated as a “deep beam.” In the following analysis, the sections are assumed symmetrically laminated, i.e., coupling stiffness coefficients are absent.

Section 1 can be analyzed by integrating the equations of equilibrium or by using an energy formulation. In the present paper, the solution is obtained by the Rayleigh-Ritz method.

The potential energy of a unit-width Section 1 is composed of the following contributions:

\[ \Pi = U_b + U_k + U_f - W_p \]  

where the strain energy of the section is

\[ U_b = \frac{1}{2} \int_{a}^{b} \left( Q_{11} e_1^2 + Q_{55} \gamma_5^2 \right) dx dz \]  

The strain energy of the rotational spring representing the effect of Section 2 is

\[ U_k = \frac{1}{2} K \left[ \psi(0) + \psi_\alpha(0) \right]^2 \]  

The strain energy of the elastic foundation provided by z-pins is

\[ U_f = \frac{1}{2} \int_{x=0}^{a} (K_0 - K_i w) w^2 dx \]  

Finally, the work of the applied force is

\[ W_p = P w(a) \]  

The solution can be sought in power series. Accordingly,

\[ \psi = B_0 + B_1 x + B_2 x^2 + \cdots + B_n x^n \]

\[ w = C_0 + C_1 x + C_2 x^2 + \cdots + C_n x^n \]  

The boundary conditions that have to be satisfied are

\[ M(x = a) = 0, \quad Q(x = a) = \frac{P}{b} \]

\[ M(x = 0) = -K \left[ \psi(0) + \psi_\alpha(0) \right], \quad w(0) = 0 \]  

where

\[ M = D \psi_\alpha, \quad Q = \kappa A_{55} \left( \psi + \psi_\alpha \right) \]  

are the stress couple and transverse shear stress resultant, respectively, defined according to the first-order shear deformation theory; and \( \kappa \) is a shear correction factor.

The last boundary condition of Eq. (46) yields \( C_0 = 0 \). The remaining three boundary conditions enable us to reduce the number of independent constants in series Eq. (45) by three. Substitution of Eq. (45) into the first three parts of Eq. (46) results in the following system of algebraic equations:

\[ D_1 B_1 = -K B_0 - K C_1 \]  

Three of the coefficients can now be expressed in terms of other coefficients. For example, if Eq. (45) is limited to \( n = 2 \), Eq. (48) yields

\[ \begin{align*}
B_1 &= f_1 B_0 + f_2 C_1, \\
B_2 &= f_3 B_0 + f_4 C_1, \\
C_2 &= f_5 B_0 + f_6 C_1
\end{align*} \]  

where numerical values of \( f_i \) can easily be found.

Accordingly, the potential energy can be expressed in terms of only two independent coefficients, i.e., \( \Pi = \Pi(B_0, C_1) \). Subsequently, the Rayleigh-Ritz procedure, i.e.,

\[ \frac{\partial \Pi}{\partial B_0} = \frac{\partial \Pi}{\partial C_1} = 0 \]

produces a set of two equations for the unknown coefficients.

Substitution of Eq. (49) into Eq. (45) where \( n = 2 \) and the subsequent use of these series in the strain-displacement relations

\[ e_x = \gamma_5 \psi_\alpha, \quad \gamma_5 = \psi + \psi_\alpha \]  

results in the following expressions for the component of potential energy:

1. Strain energy of Section 1:

\[ \begin{align*}
U_b &= \frac{1}{2} \left[ (D_{11} g_1 + A_{55 g_2}) B_0^2 + (D_{11} g_2 + A_{55 g_3}) B_0 C_1 \\
&\quad + (D_{11} g_3 + A_{55 g_4}) C_1^2 \right]
\end{align*} \]  

Where the coefficients in Eq. (52) are given by

\[ g_1 = f_1^2 a + 2 f_1 f_2 a^2 + \frac{4}{3} f_3^2 a^3 \]  

\[ g_2 = 2 a \left[ f_1 f_2 + f_1 f_4 + f_3 f_3 a + \frac{4}{3} f_3 f_4 a^2 \right] \]  

\[ g_3 = f_3^2 a + 4 f_3 f_4 a^2 + \frac{4}{3} f_5^2 a^3 \]  

\[ g_4 = a + (f_1 + 2 f_3) a^2 + \left( (f_1 + 2 f_3)^2 + 2 f_3 \right)^{\frac{a^3}{3}} + \frac{(f_1 + 2 f_5) f_6 a^4}{2} \]

\[ + f_3^2 a^5 \]

\[ g_5 = a + (f_1 + f_2 + 2 f_4 + 2 f_6) a^2 + (f_3 + f_4 + f_1 + 2 f_2 + 2 f_5) a^2 \]

\[ + 2 f_5 f_6 + f_3 f_6 a^3 + (f_1 + f_2 + 2 f_4 + 2 f_6) a^4 + \frac{2 f_3 f_6 a^5}{3} \]  

\[ + f_3 f_6 a^6 \]
Table 2. Materials Used in Analysis of Cocured z-Pinned Composite Double Cantilever Beam

<table>
<thead>
<tr>
<th>Composite material</th>
<th>$E_1$ (GPa)</th>
<th>$E_z$ (GPa)</th>
<th>z-pin material</th>
<th>$E_p$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plain-woven AS4/3501-6</td>
<td>57.2</td>
<td>9.53</td>
<td>Titanium</td>
<td>115.0</td>
</tr>
<tr>
<td>(carbon/epoxy)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SiC/CAS</td>
<td>140.0</td>
<td>130.0</td>
<td>Carbon</td>
<td>190.0</td>
</tr>
</tbody>
</table>

\[ g_6 = a + (f_2 + 2f_4)a^2 + [(f_2 + 2f_4)^2 + 2f_4]\frac{a^3}{3} + \frac{(f_2 + 2f_4)f_4a^4}{2} + f_4^2\frac{a^5}{5} \]

2. Strain energy of rotational spring representing the effect of Section 2:

\[ U_K = \frac{K}{2}(B_0^2 + 2B_0C_1 + C_1^2) \]  

(54)

3. Strain energy of the elastic foundation representing z-pins can be represented in the form:

\[ U_F = g_7B_0^2 + g_8B_0C_1 + g_9C_1^2 + g_{10}B_0^3 + g_{11}B_0^2C_1 + g_{12}B_0C_1^2 + g_{13}C_1^3 \]

(55)

where the coefficients $g_i$ are omitted for brevity.

4. Work of the applied force:

\[ W_p = P[f_3a^2 + (1 + f_4)aC_1] \]  

(56)

Now the potential energy can be represented as:

\[ \Pi = n_1B_0^2 + n_2B_0C_1 + n_3C_1^2 + g_{10}B_0^3 + g_{11}B_0^2C_1 + g_{12}B_0C_1^2 + g_{13}C_1^3 - P[f_3a + (1 + f_4)aC_1] \]  

(57)

where

\[ n_1 = \frac{1}{2}(D_{11}g_1 + A_{55}g_4) + g_7 + \frac{K}{2} \]

\[ n_2 = \frac{1}{2}(D_{11}g_2 + A_{55}g_5) + g_8 + \frac{K}{2} \]

\[ n_3 = \frac{1}{2}(D_{11}g_3 + A_{55}g_6) + g_9 + \frac{K}{2} \]

Minimization of the potential energy according to Eq. (50) yields the following system of nonlinear algebraic equations for the coefficients $B_0$ and $C_1$:

\[
2n_1B_0 + n_2C_1 + 3g_{10}B_0^2 + 2g_{11}B_0C_1 + g_{13}C_1^2 = 0
\]

\[
n_3B_0 + n_3C_1 + g_{11}B_0^2 + 2g_{12}B_0C_1 + 3g_{13}C_1^2 - P(1 + f_4)a = 0
\]  

(59)

Solution of Eq. (59) and use of Eq. (49) yield all coefficients in Eq. (45) for the case where $n=2$.

Numerical Examples

The illustration of the effectiveness of z-pins in a cocured composite DCB was presented for two different material systems, whose properties are outlined in Table 2. One of the materials in Table 2 is plain-woven carbon/epoxy (AS4/3501-6), which was employed by Rugg et al. (2002) in their investigation of the effect of z-fibers on delamination resistance of composite mixed-mode bending laminates. The other material is a unidirectional CMC (SiC/CAS) considered by Domergue et al. (1995, 1996) in their research.

The thickness of each delaminated section of DCB was taken as $h=2.19$ mm in all cases, following the work of Ozdil and Carlsson (1999). The width of the specimens was $b=20$ mm. The radius of the carbon and titanium z-pins equalled 0.6 and 0.47 mm, respectively. The latter value corresponds to the pins considered by Rugg et al. (2002).

The interfacial shear strength between the z-pins and composite material is usually unknown; therefore, this strength was assumed equal to 20 MPa in all cases. Note that this value is also close to the fiber push-out sliding resistance for SiC/CAS material (Domergue et al. 1995).

The results shown in this paper were generated using the solution for a “thin” Section 1; i.e., transverse shear deformability was neglected. The analysis accounting for transverse shear effects illustrated that a difference between the “thin” and shear deformable Sections 1 becomes prominent only for very short cracks (Birman 2003).

The comparison between the present solution and the previously considered solution, which employs an elastic foundation to model the rotational stiffness of the intact section of DCB (Byrd and Birman 2004), is presented in Fig. 5. As follows from this figure, the two solutions yield results for the deflection of the loaded (delaminated) end and the compliance of DCB that are in close agreement. The advantage of the present solution is its relative simplicity and more logical approach to the formulation of the problem. Predictably, a higher volume fraction of z-pins results in a decrease in deflections and compliance. This illustrates the effectiveness of z-pins for the enhancement of fracture resistance of joints working in Mode 1 conditions.

It should be noted that in all examples the deflections of the delaminated end of the DCB were closely monitored. If the volume fraction of z-pins is too high, the solution yields negative deflections $w(a)$. Physically, this means that z-pins arrest the crack and its propagation stops. This is in agreement with the results reported for z-pinned laminates by Rugg et al. (2002), who noticed a change from the delamination mode of failure to microbuckling as a result of the introduction of z-pins. On the other hand, if crack arrest did not occur and the length of the crack increased, the deflections of the delaminated end exceeded the...
thickness \( h \). This implies that the z-pins are pulled out of the delaminated section of DCB and the analysis has to be modified accordingly. The analysis indicated that the deflection of the delaminated end abruptly increases in the vicinity of the z-pin pullout value. Therefore, it is reasonable to assume that the specimen fails as \( w(a) \rightarrow h \); i.e., the onset of the process of pulling z-pins out of the specimen may be identified with failure.

The effect of the length of the crack on the response of SiC/CAS DCB is shown in Figs. 6 and 7 for various z-pin volume fractions. The deflections of DCB with z-pin volume fractions equal to 0.8 and 1.0% abruptly increased at the crack lengths exceeding 30 mm. This means that the z-pins began to be pulled out of the delaminated end of the DCB and it failed. On the other hand, DCB with z-pin volume fractions equal to 1.2% exhibited a reduction of deflections at \( a > 25 \text{ mm} \). Therefore, the crack propagation was arrested in the specimen with this z-pin volume fraction at the crack length equal to 25 mm. The same conclusion was obtained for carbon/epoxy, AS4/3501-6 DCB, as shown in Fig. 8.

The effect of the magnitude of the applied force on the deflections and compliance is illustrated for both materials considered in the paper in Fig. 9. Similar tendencies were observed for both materials. The deflection and compliance increased with a larger force applied to DCB. Note that the force for carbon/epoxy DCB was monitored only to 450 N, because at large forces the deflections exceed \( h \); i.e., z-pins are pulled out of DCB and the specimen fails. This phenomenon does not occur for CMC DCB, which is apparently capable of resisting a much larger force.

It is necessary to emphasize that the applicability of the present solution should be considered, keeping in mind a possible loss of strength of the cantilever section of DCB. A quick and conservative estimate of the strength may be conducted by analyzing this section as a cantilever subject to the end lateral force, as shown in the previous paper (Byrd and Birman 2004). In addition, it is necessary to account for a possible matrix cracking of brittle matrix CMC under tensile in-plane stresses [see discussion in the paper by Byrd and Birman (2004)]. As follows from the analysis, neither phenomenon is likely to affect the results shown in the present paper.

Conclusions

The present paper outlines a new approach to the estimate of the effect of z-pins on Mode I fracture of cocured composite joints. The approach to the analysis is illustrated in the example of a double cantilever beam, i.e., a standard test employed to predict fracture toughness of joints.

The solution consists of two phases. In the first phase, the rotational restraint of the intact section of a partially delaminated DCB is evaluated based on the energy solution for this section subject to the action of a bending couple applied at the end. Subsequently, the deflections and compliance of the delaminated section of DCB are determined. The results compare favorably to available data for DCB without z-pins published in literature.

The analysis illustrates the effectiveness of z-pins as an enhancement of the delamination resistance of cocured composite
DCB. Even a relatively small volume fraction of z-pins was sufficient to completely arrest delamination cracks in the representative materials. However, if the volume fraction of the z-pins was too small, the specimens failed. Failure was identified with a “critical deflection” of the free end of the delaminated section of DCB corresponding to the onset of the process of pullout of z-pins. This failure criterion is justified by an abrupt increase in the deflections and compliance observed when deflections approached the critical value.

In conclusion, z-pins have been shown to be an effective tool for the enhancement of the delamination resistance of composite joints. Further numerical and experimental studies are recommended to better understand the mechanism of the process involving gradual pullout of z-pins as the deformations of joined components increase. Such studies will eventually lead to a reliable design tool, enabling engineers to confidently introduce z-pins in the composite joint design.

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References


