Physics 1135: Homework for Recitation 12: Potential energy methods

1. a) The potential energy of a conservative force is given by $U(x, y) = A \left( \frac{1}{x} + \frac{1}{y} \right)$ where A is a constant. Find the corresponding force vector $\vec{F}(x, y)$ in unit vector notation.

b) A force is given as $\vec{F} = C \sin kx \hat{i}$. Derive an expression for a potential energy associated with this force, with the reference point at $x=0$ with a value $U(x)=0$.

2. A particle with total mechanical energy $E$ is moving along the x-axis under the influence of a conservative force whose potential energy is shown in the figure.

   a) At which of the points is the particle momentarily at rest?
   
   b) At which points is the force on the particle zero?
   
   c) At which point does the particle have its largest kinetic energy?
   
   d) At which points is the force directed to the left? To the right?

3. In a game, a box of mass $M$ is placed on a horizontal, rough table next to a spring of spring constant $k$. The table is a height $H$ above the ground. The coefficient of kinetic friction between the box and table is $\mu$. The box is pushed against the spring, compressing it a distance $s$. The box is now a distance $R$ from the left edge of the table and is launched from rest. It travels along the table and then flies off the edge, striking a target on the ground a horizontal distance $L$ from the left edge of the table, as shown in the figure. A fan creates a constant horizontal blowing force of magnitude $B$ that acts on the box throughout its entire motion from launch to hitting the ground.

   Derive an expression for the speed $V$ at which the box hits the ground in terms of system parameters.

A diatomic molecule consists of two atoms separated by a distance \( r \). The potential energy associated with the force between the atoms in a diatomic molecule can be modeled as

\[
U(r) = \frac{A}{r^{12}} - \frac{B}{r^6}
\]

where \( A \) and \( B \) are positive constants.

a) Qualitatively sketch the potential energy as a function of \( r \).

b) Derive an expression for the component of the force on each atom, \( F_r \).

c) Find the equilibrium separation \( r_{eq} \).

d) Calculate the dissociation energy of the molecule, i.e. the energy required to separate the two atoms from their state of lowest energy to \( r=\infty \).