Physics 1135 Homework for recitation 14: Gravitational potential energy

1. A comet of mass $m$ is in an elliptical orbit around a star of mass $M$. At the closest point of its orbit, point P, the comet is a distance $D$ from the center of the star, and at the farthest point, point A, the distance is $3D$. You may disregard the presence of all other celestial bodies. Derive an expression for the change in the comet’s kinetic energy as it moves from point P to point A, in terms of system parameters and constants. Does the comet have its greatest speed at point A or point P?

2. A spaceship of mass $m$ has its engines switched off and is moving in a circular orbit at height $R$ above the surface of a planet of mass $M$ and radius $R$.
   a) Derive an expression for total mechanical energy $E$ of the orbiting spaceship, in terms of $G, m, M$ and $R$.
   b) Derive an expression for the minimum speed $V$ the spaceship would need to escape from this orbit into deep space, in terms of system parameters. (The engines can’t fire for the whole trip; they can only give the spaceship one boost so it obtains this velocity. Ignore all other celestial objects.)

3. An asteroid of mass $5 \times 10^9 \text{kg}$ is heading straight towards Earth. When it is $4 \times 10^6 \text{km}$ from the surface of Earth, it is moving with a speed of 600m/s. Calculate the speed with which the asteroid strikes the Earth.

4. The kings of planet A (mass $4M$, radius $2R$) and planet B (mass $M$, radius $3R$) want to meet for negotiations. The planets are a distance $10R$ from one another, center to center. For absolute fairness, the kings (who possess no physics knowledge) decide that the meeting place $P$ is to be exactly halfway between the planets. A space capsule of mass $m$ is launched from point X on the surface of planet A by means of a giant cannon, which gives it a launch speed $V_L$. It travels directly along the line that connects the centers of both planets. Ignore the orbital motion of the planets.
   a) Derive an expression for the speed $V$ with which the capsule arrives at the meeting place $P$.
   b) Derive an expression for the net force (magnitude and direction) experienced by the capsule when it is at point $P$.
   c) At what distance from planet A is the net gravitational force zero?