Physics 1135 Homework for recitation 13: Universal gravitational force

1. a) Uranus has a mass of $8.68 \times 10^{25}$ kg and a radius of $2.33 \times 10^{7}$ m. Find the free-fall acceleration on the surface of Uranus.
b) Jupiter’s moon Io has a mass of $8.9 \times 10^{22}$ kg. It is orbiting Jupiter at a distance $4.22 \times 10^{8}$ m and has an orbital period of 1.77 Earth days. Find the mass of Jupiter.

2. The center of a moon of mass $m$ is a distance $D$ from the center of a planet of mass $M$. At some distance $x$ from the center of the planet, along a line connecting the centers of planet and moon, the net force on an object will be zero.
a) Derive an expression for $x$ in terms of $m$, $M$, and $D$.
b) If the net force is zero a distance $\frac{2}{3}D$ from the planet, what is the ratio $R$ of the mass of the planet to the mass of the moon, $M/m$?

3. A geosynchronous satellite moves in a circular orbit around the Earth and completes one circle in the same time $T$ during which the Earth completes one revolution around its own axis. The satellite has mass $m$ and the Earth has mass $M$ and radius $R$. In order to be geosynchronous, the satellite must be at a certain height $h$ above the Earth’s surface. Derive an expression for $h$ in terms of $m$, $M$, $R$, $T$ and constants and find a numerical value. Earth’s mass and radius can be found in your textbook.

4. Binary Star. Two identical stars of mass $M$ orbit around the center of a circle with radius $R$. The stars are always on opposite sides of the circle as shown.

Derive expressions for the orbital speed and the orbital period of each star.