Physics 1135 Homework for recitation 14: Gravitational potential energy

1. A comet of mass $m$ is in an elliptical orbit around a star of mass $M$. At the closest point of its orbit, point P, the comet is a distance $D$ from the center of the star, and at the farthest point, point A, the distance is $3D$. You may disregard the presence of all other celestial bodies.
Derive an expression for the change in the comet’s kinetic energy as it moves from point P to point A, in terms of system parameters and constants.
Does the comet have its greatest speed at point A or point P?

2. A planet has mass $M$ and radius $2R$.
   a) Derive an expression for the escape speed from the planet.
   b) A projectile of mass $m$ is shot directly away from the surface of the planet at \( \frac{1}{3} \) of the escape speed from the planet. Derive an expression for the maximum distance from the center of the planet the projectile reaches, in terms of $R$. Simplify as far as possible. (Ignore the existence of all other celestial objects.)

3. A spaceship of mass $m$ has its engines switched off and is moving in a circular orbit at height $R$ above the surface of a planet of mass $M$ and radius $R$.
   a) Derive an expression for total mechanical energy $E$ of the orbiting spaceship, in terms of $G$, $m$, $M$ and $R$.
   b) Derive an expression for the minimum speed $V$ the spaceship would need to escape from this orbit into deep space, in terms of system parameters. (The engines can’t fire for the whole trip; they can only give the spaceship one boost so it obtains this velocity. Ignore all other celestial objects.)
4. The kings of planet A (mass $4M$, radius $2R$) and planet B (mass $M$, radius $3R$) want to meet for negotiations. The planets are a distance $10R$ from one another, center to center. For absolute fairness, the kings (who possess no physics knowledge) decide that the meeting place $P$ is to be exactly halfway between the planets. A space capsule of mass $m$ is launched from point $X$ on the surface of planet A by means of a giant cannon, which gives it a launch speed $V_L$. It travels directly along the line that connects the centers of both planets. Ignore the orbital motion of the planets.

a) Derive an expression for the speed $V$ with which the capsule arrives at the meeting place $P$, in terms of relevant system parameters.

b) Derive an expression for the net force (magnitude and direction) experienced by the capsule when it is at point $P$.

c) At what distance from planet A is the net gravitational force zero?