Physics 1135: Homework for Recitation # 24: Harmonic motion

1. A toy figure of mass 2.0kg is at the end of a horizontal spring of spring constant 200N/m on a frictionless horizontal surface. The toy is pulled, stretching the spring a distance 6.0cm from equilibrium, and released from rest.
Find the angular frequency and the period of the oscillation. What is the maximum speed reached by the block?

2. A cork at the end of a spring oscillates with an angular frequency $\omega = 5.0 \text{ rad/s}$. At $t=0$, the cork is at position $x=2.0\text{cm}$ and is moving with speed 20cm/s in the negative $x$-direction. The position can be described through the equation $x=A \cos (\omega t+\phi)$. Find the amplitude $A$ and the phase angle $\phi$.

3. A mass at the end of a spring is undergoing simple harmonic oscillations with amplitude $A$.
   a) What fraction of the total mechanical energy is kinetic if the displacement is $\frac{1}{3}$ the amplitude?
   b) In terms of $A$, find the value of displacement $x$ at which the potential energy equals $\frac{1}{16}$ of the total mechanical energy.

4. Two pendula consist of a uniform ball of mass $M$ each, suspended from a massless string as shown in the figure. The ball of the left pendulum is very small. The ball of the right pendulum has radius $\frac{1}{2}L$.
   a) Find the period of each pendulum.
   b) How do the periods change when the mass of each pendulum is doubled?

5. A uniform rod of mass $M$ and length $L$ is pivoted at a distance $x$ from its center and undergoes harmonic oscillations. Derive an expression for the rod’s period $T$ for small oscillations about its pivot point, in terms of $M$, $L$ and $x$. In terms of $L$, find the value of $x$ for which the period is a minimum.