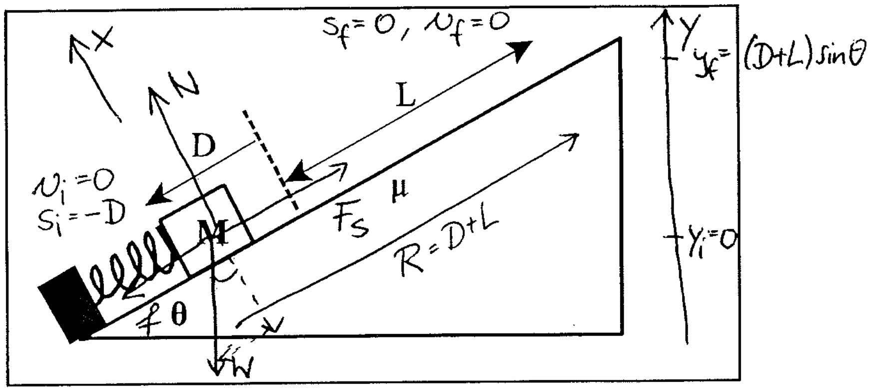
7) A box of mass M is on a rough incline that makes an angle θ with the horizontal. The coefficient of kinetic friction between the box and the incline is μ. The box is placed against a spring whose other end is secured to a wall at the lower end of the incline. The block is used to compress the spring a distance D and is then released from rest.



a)(10) Complete the diagram with all information necessary to solve Part b) below. Remember, any algebraic quantity that you use must appear in the diagram. You may want to add elements as you go along.

(OSE) b)(40) In terms of relevant system parameters, derive an expression for the minimum spring constant k necessary to ensure that the box reaches a distance L up the incline **from the equilibrium position of the spring**. (Treat the box as a point mass.)

$$E_f - E_i = W_{other}$$

$$\frac{M}{2}\cancel{v_f}^{2} + Mg\,y_f + \frac{1}{2}k\cancel{s_f}^{2} - \left(\frac{M}{2}\cancel{v_i}^{2} + Mg\cancel{y_i} + \frac{1}{2}k s_i^{2}\right) = \cancel{W_N} + W_{fr}$$

$$Mg(D+L)\sin\theta - \frac{1}{2}k(-D)^2 = \vec{f}\cdot\vec{R} = -(\mu N)(D+L)$$

to find N: $\sum F_x = N_x + W_x + \cancel{f_x} + \cancel{F_{s_x}} = M\cancel{a_x}$

$$N + (-Mg\cos\theta) = 0$$

$$N = Mg\cos\theta$$

$$Mg(D+L)\sin\theta - \frac{1}{2}kD^2 = -\mu Mg\cos\theta\,(D+L)$$

$$\frac{1}{2}kD^2 = Mg(D+L)\sin\theta + \mu Mg(D+L)\cos\theta$$

$$\boxed{k = \frac{2Mg(D+L)\left[\sin\theta + \mu\cos\theta\right]}{D^2}}$$

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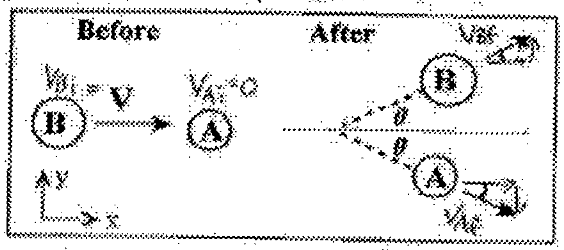
As shown in the diagram, object A of mass M is initially at rest on a flat, smooth frictionless surface. Object B, which has **twice** the mass of A, is traveling with speed V before it collides **elastically** with A. Immediately after the collision, both objects move off at angles θ with respect to the original direction of **B**.



(OSE) a)(50) Calculate the value of the angle θ. [Hint: Note that the collision is **elastic**.]

$$\vec{P}_f - \vec{P}_i = \cancel{\vec{J}_{ext}} = 0$$

$$P_{iy} = P_{fy}$$

$$2M \cancel{V_{Biy}}^{0} + M \cancel{V_{Aiy}}^{0} = 2M V_{Bfy} + M V_{Afy}$$

$$0 = 2M(+V_{Bf}\sin\theta) + M(-V_{Af}\sin\theta)$$

$$V_{Af} = 2V_{Bf}$$

$$P_{ix} = P_{fx}$$

$$2M V_{Bix} + M\cancel{V_{Aix}}^{0} = 2M V_{Bfx} + M V_{Afx}$$

$$2MV = 2M(+V_{Bf}\cos\theta) + M(+V_{Af}\cos\theta)$$

$$2V = 2V_{Bf}\cos\theta + (2V_{Bf})\cos\theta$$

$$V = 2V_{Bf}\cos\theta \longrightarrow V_{Bf} = \frac{V}{2\cos\theta}$$

$$V_{Af} = \frac{V}{\cos\theta}$$

Collision elastic: $E_f - E_i = \cancel{W_{other}}^{0} \Rightarrow K_f = K_i$

$$\frac{2M}{2}V_{Bf}^2 + \frac{M}{2}V_{Af}^2 = \frac{2M}{2}V_{Bi}^2 + \frac{M}{2}\cancel{V_{Ai}^2}^{0}$$

$$\left(\frac{V}{2\cos\theta}\right)^2 + \frac{1}{2}\left(\frac{V}{\cos\theta}\right)^2 = V^2$$

$$\frac{1}{4\cos^2\theta} + \frac{1}{2\cos^2\theta} = 1$$

$$\cos^2\theta = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$\boxed{\cos\theta = \frac{1}{2}\sqrt{3}} \Rightarrow \boxed{\theta = 30^\circ}$$

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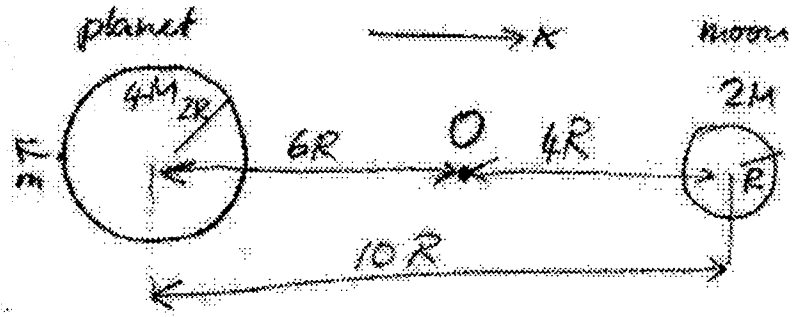
2.



a) $E_f - E_i = \cancel{W_{others}}^{\,0}$

$$K_f + U_{P_f} + U_{M_f} - (K_i + U_{P_i} + U_{M_i}) = 0$$

$$\Delta K = K_f - K_i = U_{P_i} + U_{M_i} - U_{P_f} - U_{M_f}$$

$$= -\frac{G(4M)m}{2R} - \frac{G(2M)m}{12R} - \left(-\frac{G(4M)m}{6R}\right) - \left(-\frac{G(2M)m}{4R}\right)$$

$$= \frac{GMm}{R}\left[-\frac{4}{2} - \frac{2}{12} + \frac{4}{6} + \frac{2}{4}\right] = \frac{GMm}{R}\left[\frac{-12-1+4+3}{6}\right]$$

$$\boxed{\Delta K = -\frac{GMm}{R}}$$

b) $\sum F_x = F_{P_x} + F_{M_x}$

$$= -\frac{G(4M)m}{(6R)^2} + \frac{G(2M)m}{(4R)^2}$$

$$= \frac{GMm}{R^2}\left[-\frac{4}{36} + \frac{2}{16}\right] = \frac{GMm}{R^2}\left[-\frac{1}{9} + \frac{1}{8}\right]$$

$$\boxed{F_{net_x} = \frac{1}{72}\frac{GMm}{R^2}}$$