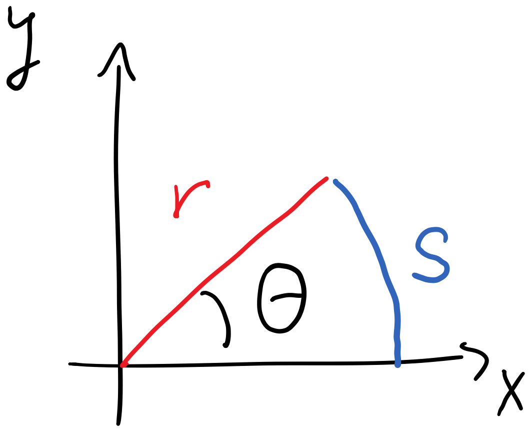
Lecture 20:
Rotational kinematics and energetics

- Angular quantities
- Rolling without slipping
- Rotational kinetic energy
- Moment of inertia
- Energy problems

Angle measurement in radians



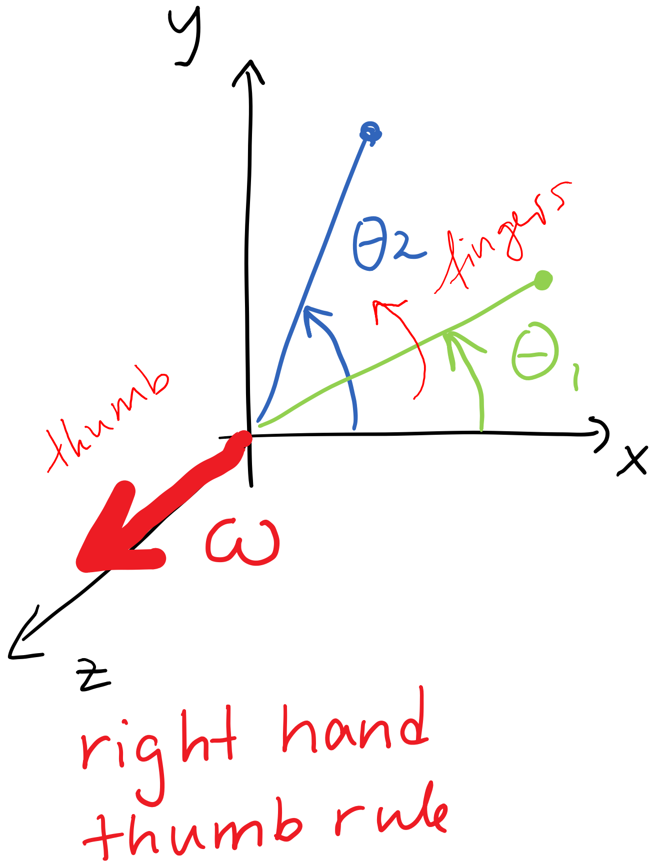
θ(in radians) **=** s/r

s is an arc of a circle of radius r

Complete circle:

$s = 2\pi r$, θ=2π

Angular Kinematic Vectors



Position: θ

Angular displacement:

$$\Delta\theta = \theta_2 - \theta_1$$

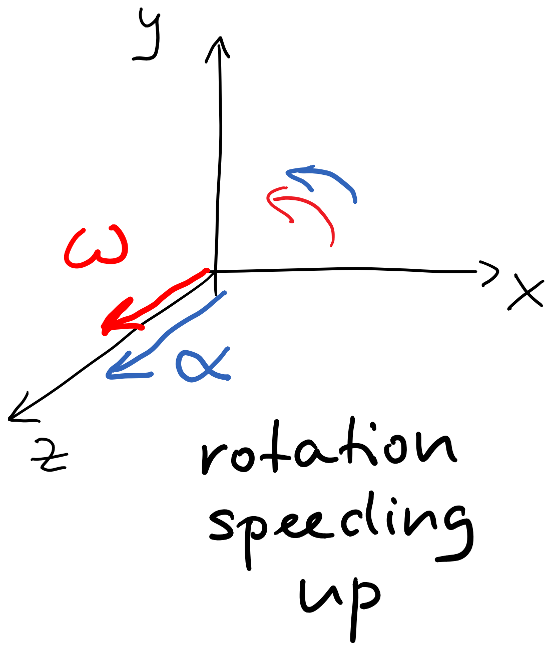
Angular velocity:

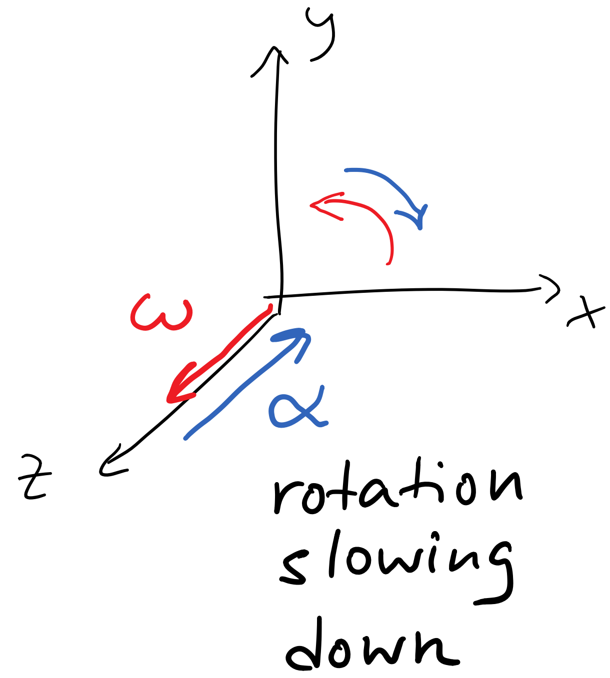
$$\omega_z = \frac{d\theta}{dt}$$

Angular velocity vector perpendicular to the plane of rotation

Angular acceleration

$$\alpha_z = \frac{d\omega_z}{dt}$$





Angular kinematics

For constant angular acceleration:

Compare constant a_x:

$$\theta = \theta_0 + \omega_{0z}t + \frac{1}{2}\alpha_z t^2$$

$$x = x_0 + v_{0x}t + ½a_x t^2$$

$$v_x = v_{0x} + a_x t$$

$$\omega_z = \omega_{0z} + \alpha_z t$$

$$v_x^2 = v_{0x}^2 + 2a_x(x - x_0)$$

$$\omega_z{}^2 = \omega_{0z}{}^2 + 2\,\alpha_z(\theta - \theta_0)$$

$\theta \leftrightarrow x$

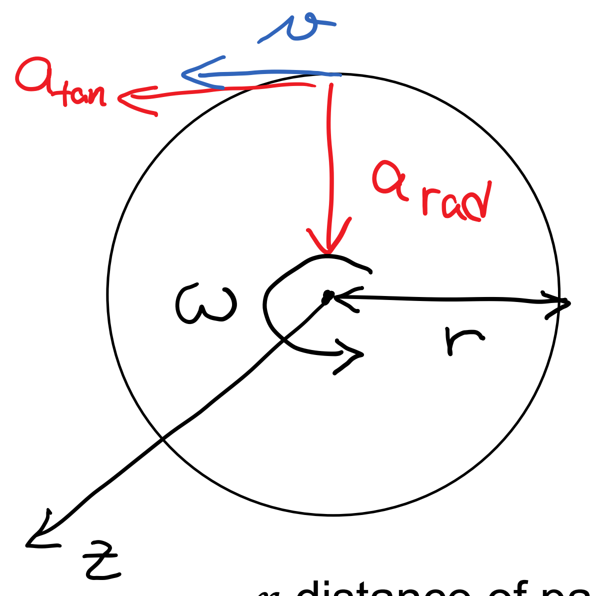
$\omega_z \leftrightarrow v_x$

$\alpha_z \leftrightarrow a_x$

Relationship between angular and linear motion

linear velocity tangent to circular path:

$$v = \frac{ds}{dt} = \frac{d}{dt}(r\theta) = r\frac{d\theta}{dt}$$



$$v = \omega r$$

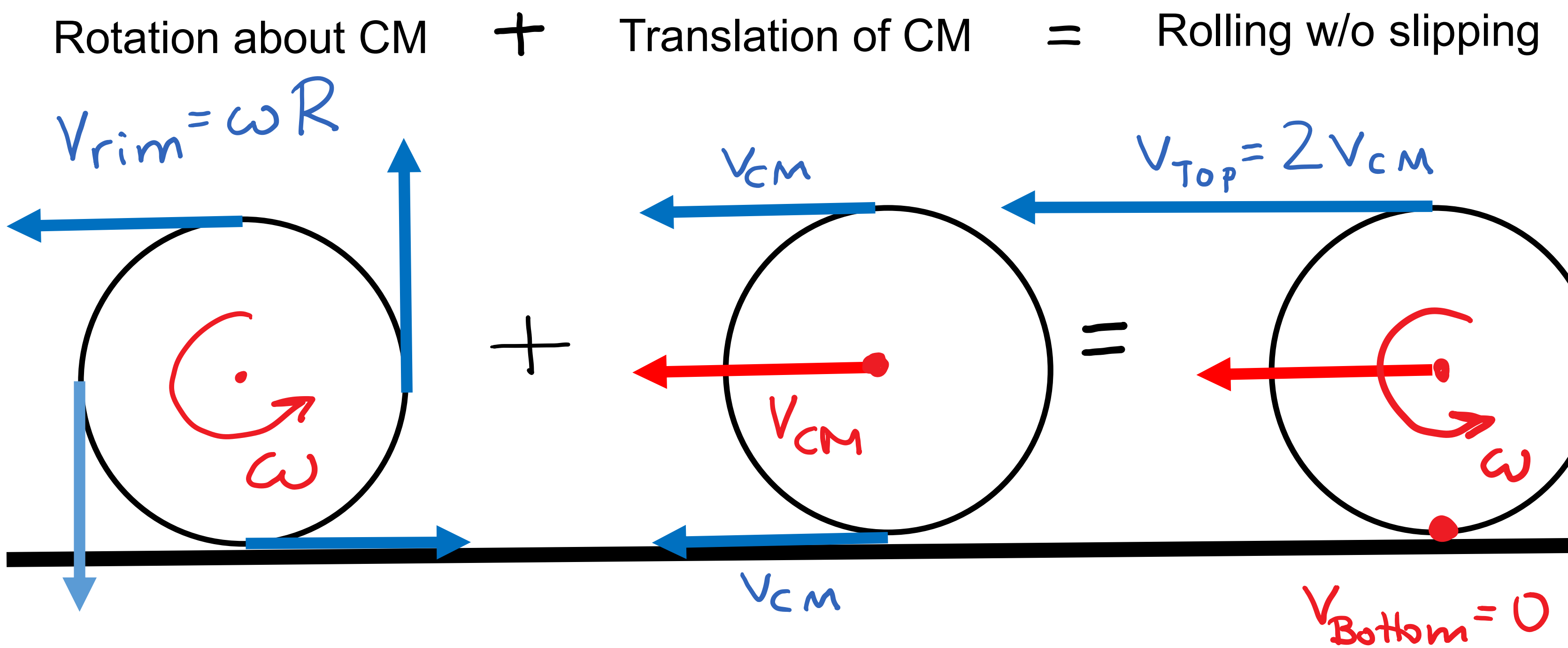
$$a_{tan} = \alpha r$$

$$a_{rad} = \frac{v^2}{r} = \omega^2 r$$

r distance of particle from rotational axis

Angular speed ω is the same for all points of a rigid rotating body!

Rolling without slipping



If $v_{rim} = v_{CM}$: $v_{bottom} = 0$ no slipping

$v_{CM} = \omega R$

$a_{CM} = \alpha R$

Rotational kinetic energy

$$K_{rotation} = \sum ½ m_n v_n^2 = \sum ½ m_n (\omega_n r_n)^2$$

$$= \sum ½ m_n (\omega\, r_n)^2 = ½ \left[\sum m_n r_n^2 \right] \omega^2$$

$$K_{rot} = ½\, I\, \omega^2$$

with $$I = \sum_n m_n r_n{}^2$$ Moment of inertia

Moment of inertia

$$I = \sum_n m_n {r_n}^2$$

r_n perpendicular distance from axis

Continuous objects:

$$I = \sum_n m_n {r_n}^2 \longrightarrow \int r^2 dm$$

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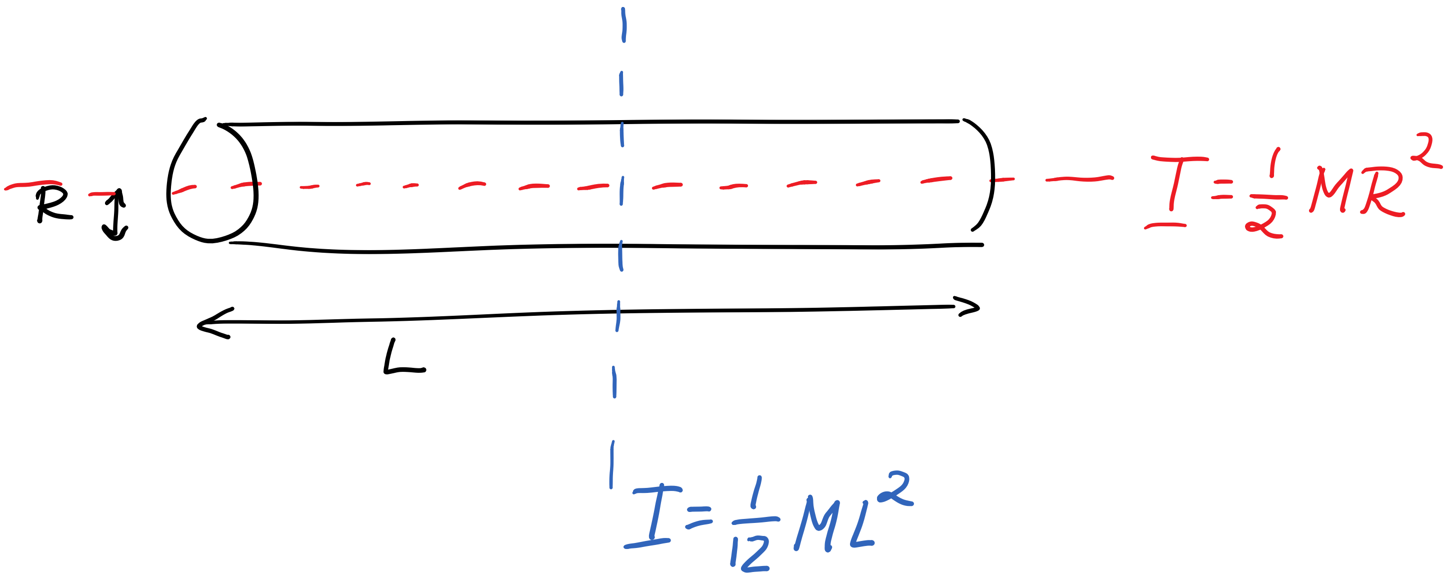
* No calculation of moments of inertia by integration in this course.

Properties of the moment of inertia

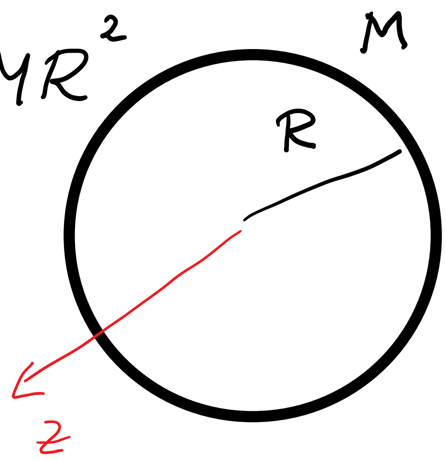
1. Moment of inertia I depends upon the axis of rotation. Different I for different axes of same object:



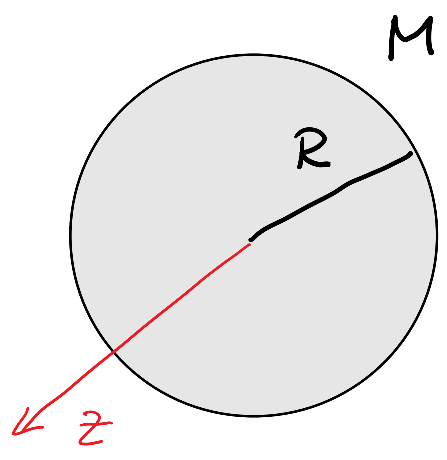
Properties of the moment of inertia

2. The more mass is farther from the axis of rotation, the greater the moment of inertia.

Example:
Hoop and solid disk of the same radius R and mass M.



$$I_z = MR^2$$



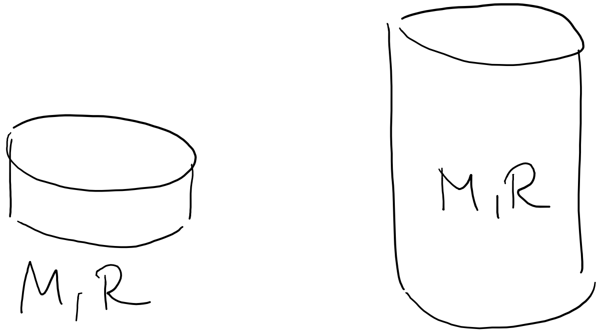
$$I_z = \frac{1}{2} MR^2$$

Properties of the moment of inertia

3. It does not matter where mass is along the rotation axis, only radial distance $r_{\boldsymbol{n}}$ from the axis counts.

Example:

I for cylinder of mass M and radius R: $I = ½MR^2$
same for long cylinders and short disks

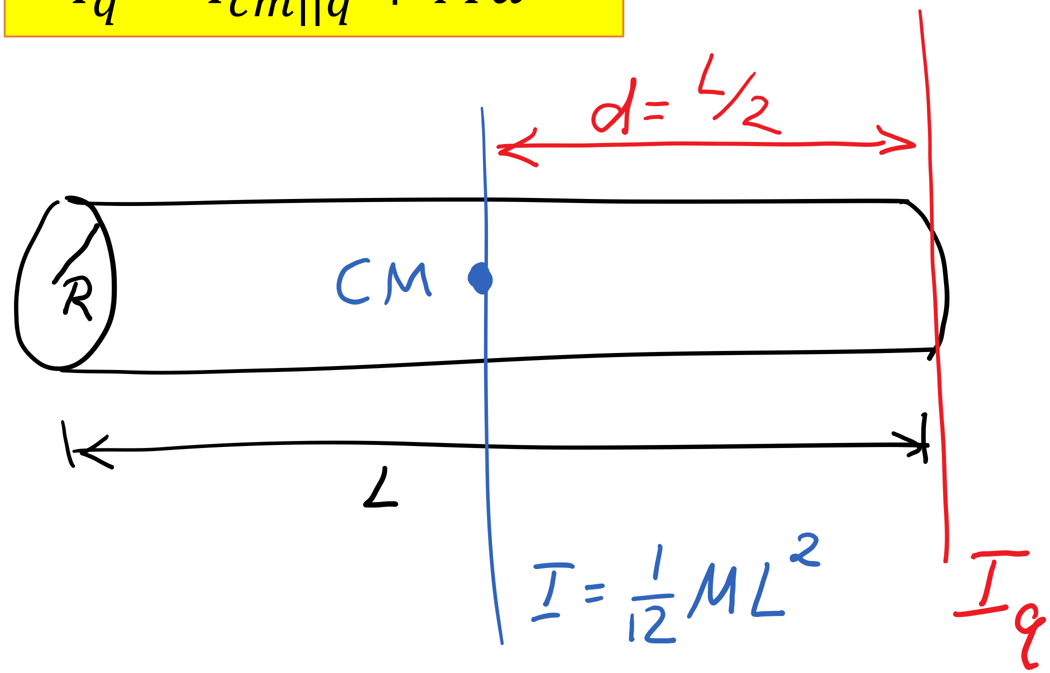


Parallel axis theorem

$$I_q = I_{cm||q} + Md^2$$

distance CM ↔ q

Example:



$$I_q = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{1}{12}ML^2 + \frac{1}{4}ML^2 = \frac{1}{3}ML^2$$

$\frac{1+3}{12} ML^2$

Rotation and translation

$$K = K_{trans} + K_{rot} = ½ M v_{CM}^2 + ½\, I\, \omega^2$$

with $$v_{CM} = \omega R$$

Hoop-disk-race

Demo: Race of hoop and disk down incline

Example: An object of mass M, radius R and moment of inertia I is released from rest and is rolling down incline that makes an angle θ with the horizontal. What is the speed when the object has descended a vertical distance H?