For questions on this page, write the letter which you believe to be the best answer in the underlined space provided to the left of the question number. For problems on subsequent pages: your solution to a question with OSE in front of it must begin with an Official Starting Equation. The expression for the final result must be in system parameters and simplified as far as possible. Draw a box around your answer to each question. Neglect air resistance. Calculators and notes cannot be used during the test. If you have any questions, ask the proctor.

1) (10 pts) An object has density \( \frac{1}{3} \) the density of water. You push the object under water so that it is fully submerged and then you let go. At this instant, the object’s acceleration equals

A) \( \frac{1}{3} g \)  
B) \( \frac{2}{3} g \)  
C) \( 2 g \)  
D) \( 3 g \)

2) (10 pts) The figure shows four situations in which two different liquids, depicted in black and grey, respectively, are in a U-tube that is open to the atmosphere. In which situation is it impossible for the liquids to be in static equilibrium?

A)  
B)  
C)  
D)  

3) (10 pts) A mass is oscillating at the end of a massless spring. The amplitude of the oscillation is \( D \). At what distance from the equilibrium position does the mass have one half its maximum speed?

A) \( \frac{1}{2} D \)  
B) \( \frac{3}{4} D \)  
C) \( \frac{\sqrt{3}}{3} D \)  
D) \( \frac{\sqrt{3}}{4} D \)

4) (10pts) You have two oscillators: a simple pendulum and a mass at the end of a spring. Which one will oscillate with the same period on Moon and on Earth?

A) Both  
B) Neither  
C) Simple Pendulum  
D) Mass on spring

5) (10pts) A uniform rod of mass \( M \) and length \( L \) is pivoted at its end and undergoes harmonic oscillations. Its period for small oscillations is

A) \( 2\pi \sqrt{\frac{L}{g}} \)  
B) \( 2\pi \sqrt{\frac{L}{6g}} \)  
C) \( 2\pi \sqrt{\frac{L}{12g}} \)  
D) \( 2\pi \sqrt{\frac{2L}{3g}} \)
6. An S&T student of mass $M$ has designed a special bicycle which has a very light (massless) frame plus two hooplike wheels, each having mass $m$ and diameter $D$. The rider is moving with speed $V_0$ up a slope of angle $\theta$ with respect to the horizontal when he stops pedaling and begins to coast.

a)(5) Add all information to the diagram that you use to solve part b).

b)(45) OSE: Using energy methods, determine the distance $L$ the bicycle and rider can travel up the hill by coasting before coming to a complete stop. Assume that the mass of each wheel is distributed uniformly around its circumference and that the rider maintains a fixed position on the bicycle. The bicycle wheels roll without slipping.

\[ E_f - E_i = W_{\text{other}} \]
\[ E_i = E_f \]
\[ \frac{1}{2} (M+2m) V_0^2 + \frac{1}{2} I \omega_0^2 + (M+2m) g y_f = 0 \]
\[ \frac{1}{2} (M+2m) V_f^2 + \frac{1}{2} I \omega_f^2 + (M+2m) g y_f = 0 \]
\[ \frac{1}{2} (M+2m) V_0^2 + \frac{1}{2} \left[ 2 \cdot m \left( \frac{D}{2} \right)^2 \right] \omega_0^2 = (M+2m) g L \sin \theta \]
\[ \omega_0 = \frac{V_0}{D/2} \]
\[ \frac{1}{2} (M+2m) V_0^2 + m \left( \frac{D}{2} \right)^2 \frac{V_0^2}{(D/2)^2} = (M+2m) g L \sin \theta \]
\[ V_0^2 \left[ \frac{1}{2} (M+2m) + m \right] = (M+2m) g L \sin \theta \]

\[ L = \frac{V_0^2 \left[ \frac{1}{2} M + 2m \right]}{(M+2m) g \sin \theta} = \frac{V_0^2 (M+4m)}{(2M+4m) g \sin \theta} = \frac{V_0^2 (M+4m)}{2g (M+2m) \sin \theta} \]
7. A uniform horizontal beam of length \( L \) and weight \( W \) is attached to a wall at its base by pivot \( P \). The other end of the beam is supported by a cable that makes an angle \( \theta \) with the vertical wall, as shown. A superman of weight \( 2W \) stands on the beam at a distance \( \frac{3}{4} L \) from the wall.

(a) (5 points) Complete the diagram on the right with all information necessary to solve parts b) and c) below.

(OSE) (b) (30 points) Derive an expression for the tension the cable in terms of relevant system parameters by taking the torques about the pivot \( P \).

\[ \sum T_x = T_{s_2} + T_{w_2} + T_{w_{s_2}} + T_{s_2} = T_{s_2} = 0 \]

\[ W \frac{L}{2} + 2W \frac{3}{4}L - TL \sin (90^\circ - \theta) = 0 \]

\[ W \left( \frac{1}{2}L + \frac{3}{4}L \right) = TL \cos \theta \]

\[ T = \frac{2W}{\cos \theta} \]

(OSE) (c) (15 points) Derive expressions for the horizontal and vertical components of the support force that the pivot exerts on the beam in terms of relevant system parameters. You may treat the tension \( T \) in the cable as a system parameter for this part.

\[ \sum F_x = S_x + W_x + W_{s_2} + T_x = M_x = 0 \]

\[ S_x - T \sin \theta = 0 \]

\[ \sum F_y = S_y + W_y + W_{s_2} + T_y = M_y = 0 \]

\[ S_y - W - 2W + T \cos \theta = 0 \]

\[ S_y = 3W - T \cos \theta = W \]

\[ S_x = T \sin \theta = 2W \tan \theta \]
8. A rod of mass $3M$ and length $2L$ is free to rotate about a frictionless pivot through its center. The rod is initially at rest. Two bullets, each of mass $\frac{1}{2}M$, simultaneously strike the rod perpendicularly at each of its ends. The bullet on the right has initial speed $V$, while the one on the left is only moving with a speed of $\frac{1}{4}V$ before impact. The bullet on the left is made of lead and imbeds itself in the rod after impact. The bullet on the right is made of rubber and rebounds from the rod with a speed of $\frac{1}{2}V$ as shown.

\begin{align*}
\text{initial} & \quad \text{final} \\
3/4V & \quad \omega_f \\
\frac{1}{2}M & \quad \frac{1}{2}M \\
\frac{1}{2}M & \quad \frac{1}{2}M \\
3M & \quad \omega_f \\
V & \quad \frac{1}{2}V
\end{align*}

\begin{align*}
a) \quad & \text{(10 points) In terms of system parameters, derive an expression for the moment of inertia of the rod with the lead bullet embedded.} \\
I & = I_{rod} + I_{p,m} = \frac{1}{12} (3M)(2L)^2 + \frac{1}{2} M L^2 \\
& = ML^2 + \frac{1}{2} M L^2 = \frac{3}{2} M L^2
\end{align*}

\begin{align*}
b) \quad & \text{(OSE) (40 points) In terms of system parameters, derive an expression for the angular speed } \omega_f \text{ of the rod with the imbedded bullet just after impact.} \\
I_i & = I_f \quad \text{if } \sum F_{ext} = 0 \\
-\frac{1}{2} M \left(\frac{3}{4} V L\right) & + \frac{1}{2} M V L = \left(\frac{3}{2} M L^2\right) \omega_{f_2} - \frac{1}{2} M \left(\frac{1}{2} V\right) L \\
-\frac{3}{8} M V L + \frac{1}{2} M V L & = \frac{1}{4} M V L = \frac{3}{2} M L^2 \omega_{f_2} \\
\omega_{f_2} & = \frac{V}{4L}
\end{align*}