Angular Distribution of Transmission Eigenchannels

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Light in scattering media

• **Diffusive propagation**: chaotic, random-walk-like propagation through a medium

• Optically diffusive media contain **refractive index inhomogeneities**

• Examples: **paper, fog, biological tissue**
The transmission matrix $t$

- $t$ matrix determines how light scatters:
  $$t\hat{C}_{\text{in}} = \hat{C}_{\text{out}}$$

- Scattering is deterministic

- How can we construct $t$?

- Knowing $t$ allows us to manipulate transport
Extracting information with the SVD

\[ t = U \tau^{1/2} V^+ \]

- nth column of \( V = \) nth transmission eigenvector
- Transmission eigenvectors excite transmission eigenchannels
- nth element of \( \tau^{1/2} = \) sqrt of nth transmission eigenvalue
- Near-perfect transmission is usually possible
Perfect transmission without knowing $t$

- Often impossible to measure all elements of $t$
- Can we still excite transmission eigenchannels?
- What are the general characteristics of transmission eigenchannels?

What is the structure of $\langle |V|^2 \rangle$?
Numerical simulations in **kwant**

- Quantum transport package for Python
- Waveguide geometry with two semi-infinite leads
- \( L = 300, \ D = 1000 \)

(Rotter and Gigan, 2017)
Structure of $\langle |t|^2 \rangle$ matrix

- Structure of $\langle |t|^2 \rangle$ related to structure of $\langle |V|^2 \rangle$ through SVD
- $\langle |t|^2 \rangle = \rho \rho^T$
- $\rho$ = first column & first row of $\langle |t|^2 \rangle$
- Elements of $\rho$ distributed according to 1D Chandrasekhar equation:

$$\rho(x) = \frac{\pi}{4} + \frac{\sqrt{1 - x^2}}{\pi}$$

![Diagram showing incident angle vs. outgoing angle with color-coded data points]
Continuous $\langle |V|^2 \rangle$ matrix

- $\langle |V|^2 \rangle$ has same 1\textsuperscript{st} row & column, like $\langle |t|^2 \rangle$

\[
f(x, 0) = \frac{\pi}{4} + \frac{\sqrt{1-x^2}}{\frac{\pi}{2}}, \quad f(y, 0) = \frac{\pi}{4} + \frac{\sqrt{1-y^2}}{\frac{\pi}{2}}
\]

- “Magic square” normalization

\[
\int_0^1 f(x, y) dx = 1 \text{ for all } y \in [0, 1] \\
\int_0^1 f(x, y) dy = 1 \text{ for all } x \in [0, 1]
\]

- $\langle |V|^2 \rangle$ changes smoothly

\[
f(x, y + \Delta) = f(x, y) + \Delta [\mathcal{F}[f(x, y)] - \int_0^1 \mathcal{F}[f(x, y)] dx] \times \text{const}
\]
Results

\[ f(x, y) = 1 + (f_0(y) - 1)(f_0(x) - 1) \]

\[ |V|^2 \] from simulation

\[ |V|^2 \] from analytic expression

incident angle

outgoing angle

incident angle

outgoing angle
Conclusion

• Derived expression for the structure of $\langle |V|^2 \rangle$ matrix
• Can generalize results with 3D Chandrasekhar equation
• Transmission eigenchannels have predetermined, identifiable characteristics
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References
