

Lecture 36: Static Fluids

- Pressure
- Pascal's Principle
- Buoyancy force

Pressure

An object submerged in a fluid will experience a force acting on the surface.

Pressure p = Force magnitude per Area

$$p = \frac{F}{A}$$

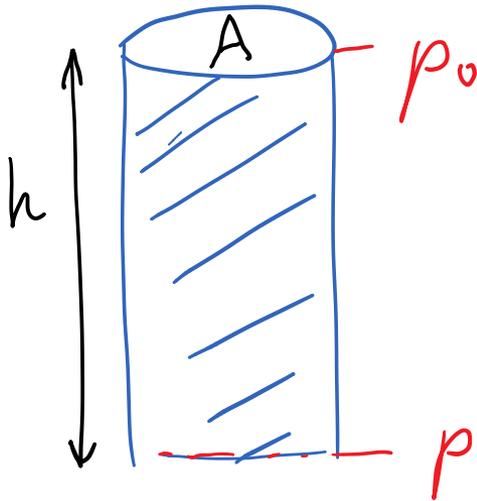
Unit: $\text{N/m}^2 = \text{Pa}$

Fluid at rest:

- at given depth, p is same in all directions.
- force due to pressure is perpendicular to all surfaces

Pressure increase with depth

Due to weight of column of fluid above



$$W = Mg = \rho Vg = \rho Ahg$$
$$\Delta p = \frac{W}{A} = \frac{\rho Ahg}{A} = \rho gh$$

$$p_{\text{below}} - p_{\text{above}} = \rho gh$$

Atmospheric pressure

$$p_{atm} = 100kPa = 10^5 N/m^2$$

On 1cmx1cm: $10N \approx 1kg * g$

Above head (10cmx10cm): weight of 100kg

Demo: Magdeburg hemispheres

Magdeburg hemispheres



Otto von Guericke, 1654. **30 horses.**

Magdeburg Hemispheres

$$D = 50 \text{ cm}$$

$$p = \frac{F}{A}$$

$$F = p \cdot A$$

$$A = \pi \frac{D^2}{4}$$

(cross section)

$$F \sim 10^5 \frac{\text{N}}{\text{m}^2} \cdot \pi \frac{(0.5 \text{ m})^2}{4} \sim 2 \times 10^4 \text{ N}$$

(\approx weight of a mass of 2000 kg)

Demo:

$$D = 10 \text{ cm}$$

$$F \approx 800 \text{ N} *$$

and not perfect vacuum inside

Pascal's Principle

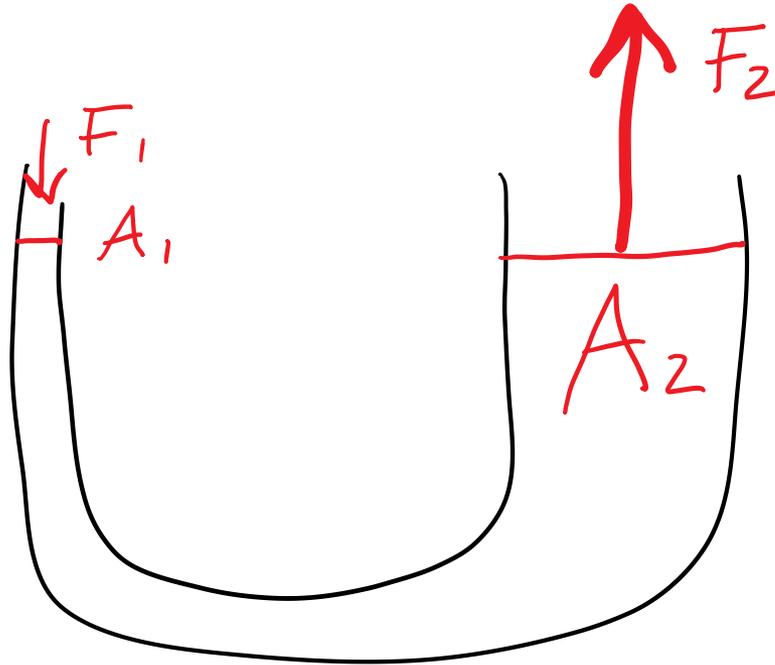
Pressure applied to a confined fluid increases the pressure throughout the fluid by the same amount.

All points at the same level in a **contiguous** fluid have the same pressure.

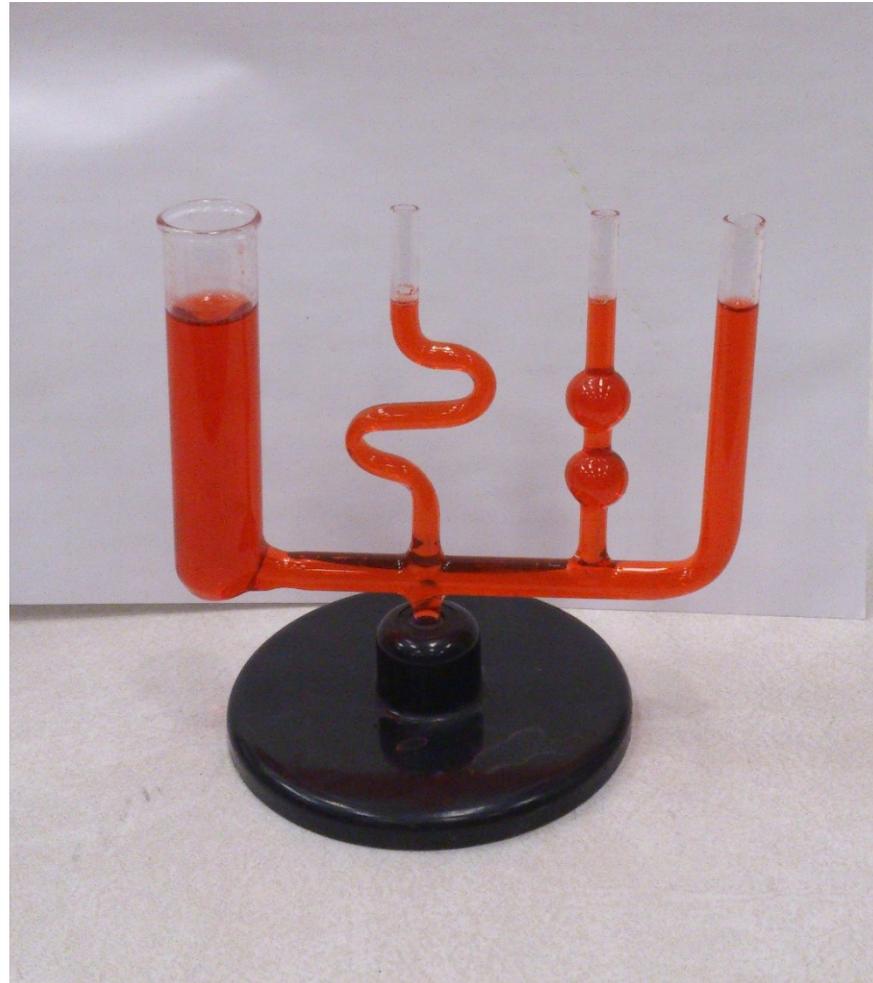
Applications of Pascal's Principle

Hydraulic lift

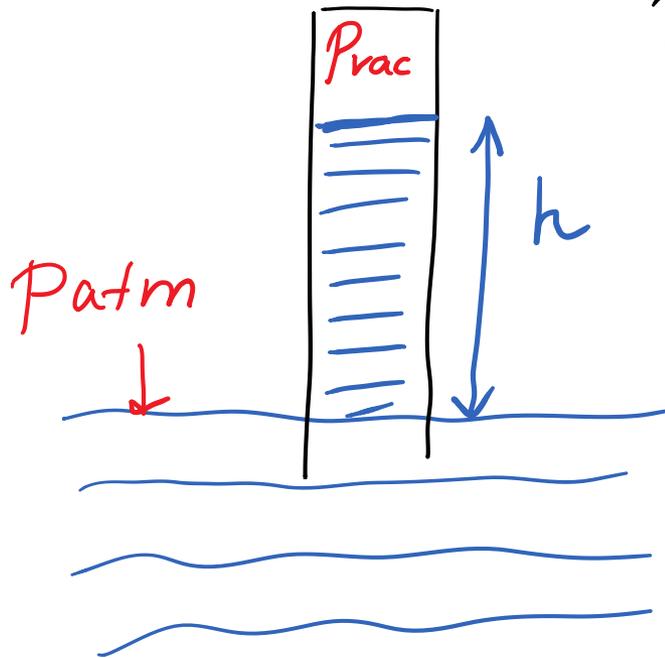
$$\Delta P = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$



Demo: same water level in connected tubes of different shapes and cross sections



The longest straw... or: How high can you pump water by suction?



$$P_{vac} \approx 0$$

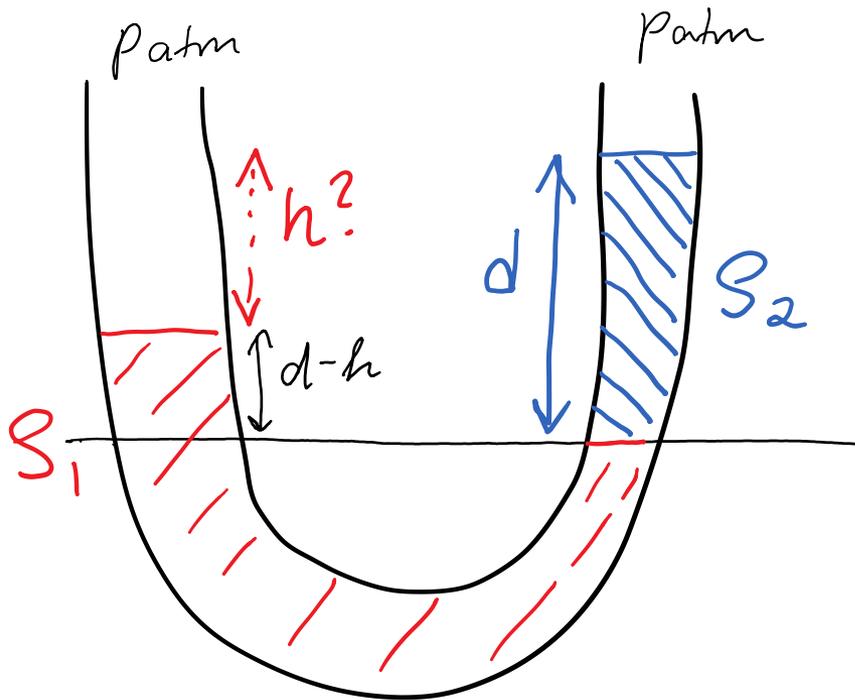
$$P_{below} - P_{above} = \rho g h$$

$$P_{atm} - 0 = \rho g h$$

$$h_{max} = \frac{P_{atm}}{\rho_{water} \cdot g}$$

$$h_{max} \approx 10 \text{ m}$$

Example 1



$$P_L = P_R$$

$$P_{atm} + \rho_1 g (d-h) = P_{atm} + \rho_2 g d$$

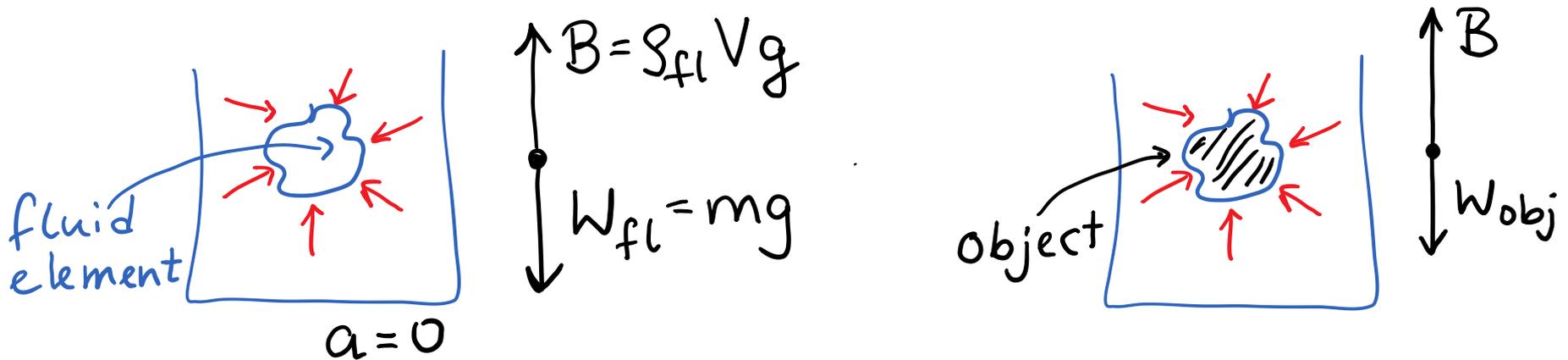
$$\rho_1 d - \rho_1 h = \rho_2 d$$

$$\rho_1 h = \rho_1 d - \rho_2 d$$

$$h = \frac{\rho_1 - \rho_2}{\rho_1} d$$

Buoyancy and Archimedes' Principle

An object fully or partially submerged in a fluid experiences an upward buoyancy force equal to the weight magnitude of the fluid displaced by the object.



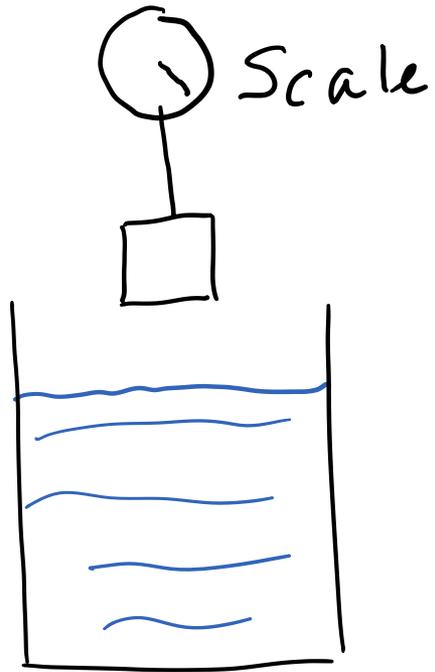
$$B = \rho_{fluid} V_{disp} g$$

Consequences of Archimedes' Principle

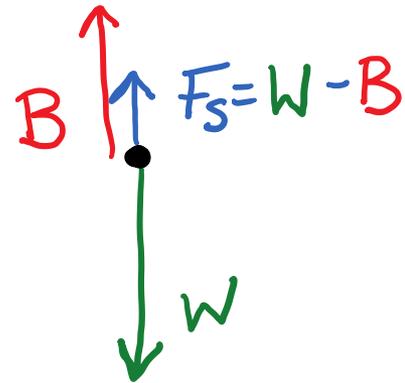
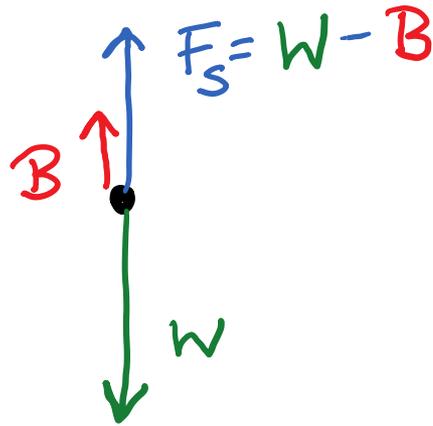
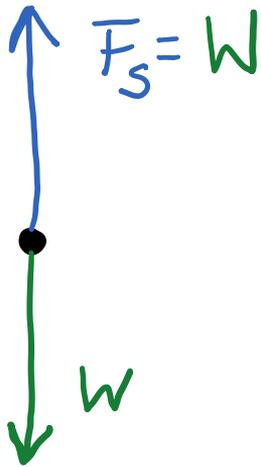
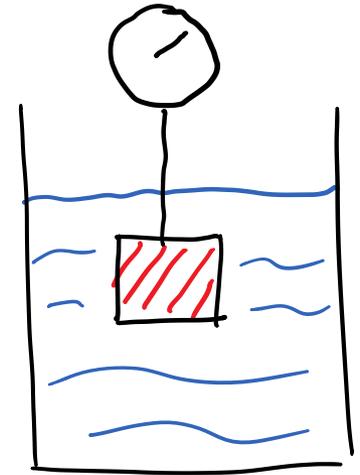
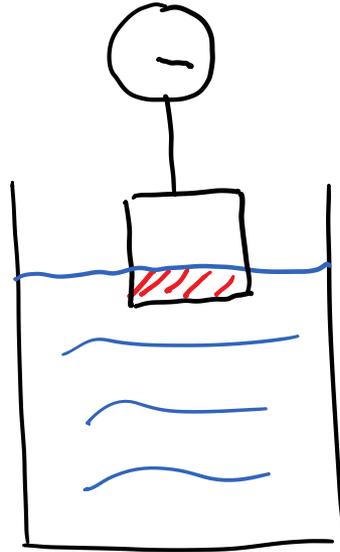
Density of object less than density of fluid:
Object floats

Density of object larger than density of fluid:
Object sinks

Demo: Buoyancy force

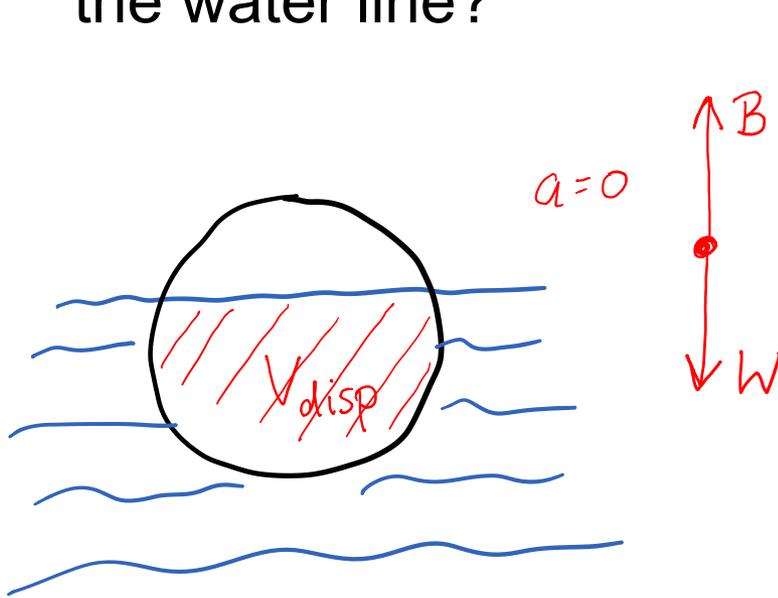


$$B = \rho_{fl} V_{disp} g$$



Example 2

A ball has a uniform mass density of $\frac{1}{3}$ the density of water. What fraction of the ball's volume is below the water line?



$$\sum F_y = B_y + W_y = m a_y^0$$

$$B - W = 0$$

$$B = W$$

$$\rho_w V_{disp} g = mg = \rho_{ball} V g$$

$$\rho_w V_{disp} = \frac{1}{3} \rho_w V$$

$$\frac{V_{disp}}{V} = \frac{\frac{1}{3} \rho_w}{\rho_w} = \frac{1}{3}$$

Example 3

A cube of side length L is placed in water and an object with twice the cube's weight is placed on top of it. Because the density of water is ρ and the cube has a uniform density of $\frac{1}{4}\rho$, a portion of the cube remains above the waterline. If the cube stays in a level orientation, what is the difference between the pressure at the cube's lower (submerged) surface and atmospheric pressure, i.e., what is the gauge pressure at the lower surface?

$$p - p_0 ?$$

