Physics 481: Condensed Matter Physics - Homework 1

---

due date: Jan 21, 2011

Problem 1: Honeycomb lattice (10 points, Marder - Problem 1.1)

The honeycomb lattice can be constructed by starting from the hexagonal Bravais lattice with primitive vectors $\vec{a}_1 = a(\sqrt{3}/2, 1/2)$ and $\vec{a}_2 = a(\sqrt{3}/2, -1/2)$ where a is the lattice constant. Each lattice point is then decorated with basis particles at relative positions $\vec{v}_1 = a(\sqrt{3}/6, 0)$ and $\vec{v}_2 = a(-\sqrt{3}/6, 0)$

a) Verify that this construction leads to a regular honeycomb lattice (the distances between all neighboring points are identical, and all internal angles are 120°).

b) Sketch the neighborhoods of two particles in the honeycomb lattice which are not equivalent, and describe the rotation that would be needed to make them identical.

Problem 2: General reflection matrix in two dimensions (15 points)
Consider the reflection of a lattice point $\vec{R} = (x, y)$ about an axis through the origin that forms an angle ϕ with the x-axis. Show that the matrix form of this operation is

$$\begin{pmatrix} \cos(2\phi) & \sin(2\phi) \\ \sin(2\phi) & -\cos(2\phi) \end{pmatrix}$$

Hint: An efficient way of finding the matrix consists in rotating the lattice such that the reflection axis coincides with the x-axis, performing the reflection, and rotating back!

Problem 3: Allowed rotation axes (15 points, Marder - Problem 1.4)

Prove that the only allowed rotation axis in a two-dimensional Bravais lattice are twofold, threefold, fourfold, and sixfold!

To this end, consider the images of the lattice point $(a, 0)$ under rotations around the origin by angles ϕ and $-\phi$. Both must be in the Bravais lattice! From these conditions derive a simple expression that implicitly specifies all possible ϕ.