Physics 481: Condensed Matter Physics - Homework 11

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due date: April 22, 2011

Problem 1: Microscopic Drude theory (Ashcroft-Mermin 1.1, 20 points + 5 BONUS)

A microscopic picture of the Drude theory can be built from the following assumptions: (i) While moving through the metal, the electrons suffer random collisions with the ions, between the collisions, there are no interactions between the electrons and ions or among the electrons. (ii) Collisions are instantaneous events that abruptly change the velocity of the electrons. (iii) Collisions occur with a rate (probability per time) of $1/\tau$. (iv) The collisions maintain local thermal equilibrium, i.e., after a collision the electron emerges in a random direction with a speed appropriate to the local temperature at the place of the collision.

a) Show that an electron picked at random had no collision during the preceding time interval t with probability $\mathrm{e}^{-t/\tau}$. Show that it will have no collisions during the following time interval t with the same probability.

b) Show that the probability that the time interval between two successive collisions is in the range $[t, t + dt]$ is $(dt/\tau)\mathrm{e}^{-t/\tau}$

c) Show (as a consequence of a)) that at any moment the mean time back to the last collision or up to the next collision is τ.

d) Show (as a consequence of b)) that the mean time between successive collisions is τ.

e) (5 BONUS points) Part c) implies that at any moment the time T between the last and next collision averaged over all electrons is 2τ. Why is this not inconsistent with the result of (b)? To explain, you may want to derive a probability distribution of T. This factor 2 problem is responsible for Drude's mistake in the Wiedeman-Franz law.

Problem 2: Joule heating (Ashcroft-Mermin 1.2, 15 points)

Consider a metal at uniform temperature in a static uniform electric field $\mathbf{E}$. An electron experiences a collision, and then, after a time t, a second collision. In the Drude model, energy is not conserved in a collision, for the mean speed of an electron emerging from a collision does not depend on the energy that the electron acquired from the field since the time of the preceding collision.

a) Show that the average energy lost to the ions in the second of two collisions separated by a time t is $(eEt)^2/2m$. (The average is over all directions in which the electron emerged from the first collision.)

b) Show, using the result of Problem 1b), that the average energy loss to the irons per electron per collision is $(eEt)^2/m$, and hence that the average energy loss per volume and time is σE^2. Deduce that the power loss in a wire of length L and cross section A is I^2R, where I is the current flowing and R is the resistance of the wire.

Problem 3: Semiclassical dynamics in the tight binding model (Marder 16.4, 15 points)

Consider a tight-binding Hamiltonian on an (infinite) square lattice:

$$H = U \sum_{\mathbf{r}} |\mathbf{r}\rangle\langle\mathbf{r}| + t \sum_{\langle\mathbf{r},\mathbf{r}'\rangle} \left(|\mathbf{r}\rangle\langle\mathbf{r}'| + |\mathbf{r}'\rangle\langle\mathbf{r}|\right)$$

where the second sum is over nearest-neighbor pairs.

a) Find the energy eigenvalues of this Hamiltonian (the band structure)

b) Calculate the effective mass tensor in one of the eigenstates. How does it change with t? Why?

c) Repeat the calculation for a cubic lattice.

Problem 4: Bloch oscillations (Marder 16.1, 10 points)

Consider whether it should be possible to observe Bloch oscillations.

a) Take the relaxation time τ in copper to be approximately 2×10^{-13}s. How strong an electric field would be needed in order to have one Bloch oscillation in less than a relaxation time?

b) Assuming a characteristic band gap of 2eV, how large is this field compared to one that could induce Zener tunneling?

c) Consider next GaAs where the low-temperature relaxation times can rise to 3×10^{-10}s, and where it is possible to build artificial structures for which the lattice constant is on the order of $a = 10$nm. How large an electric field would be needed to see Bloch oscillations in this case?