Physics 481: Condensed Matter Physics - Test prep homework

Problem 1: Tightly bound electrons in 1D (10 points)

Consider a one-dimensional electron system with lattice constant a in tight binding approximation. The energy-momentum relation reads

$$\epsilon(k) = -2t\cos(ka) \ .$$

a) Calculate the electronic density of states $D(\epsilon)$.

b) Does it have van-Hove singularities? If so, discuss their character!

c) Calculate the Fermi energy for 0.5, 1, and 2 electrons per unit cell.

d) For one electron per unit cell, calculate the low-temperature specific heat (per cell)!

Problem 3: Two-dimensional band structure (10 points)

Consider a two-dimensional crystal with a rectangular unit cell of length $a = 5$Å and width $b = 3$Å.

a) Determine the 1st Brillouin zone.

b) For free electrons, calculate the Fermi wavevector for 1, 2, 3 electrons per unit cell. Sketch the projected Fermi surfaces in the extended and reduced zone schemes.

c) Now assume a weak periodic potential. Where will gaps open in k-space? Sketch the Fermi surfaces for *nearly* free electrons for 1, 2, 3 electrons per unit cell.