Physics 481: Condensed Matter Physics - Homework 4

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due date: Friday, Feb 11, 2011

Problem 1: Nematic order parameter (15 points)

Consider a three-dimensional nematic liquid crystal consisting of N rod-like molecules. The nematic order parameter (i.e., a quantity characterizing the degree of nematic order) can be defined as

$$S = \sum_{j=1}^{N} \left\langle \cos^2 \Theta_j - \frac{1}{3} \right\rangle$$

where Θ_j is the angle between the axis of molecule j and the director $\hat{n}$. $\langle \ldots \rangle$ indicates the thermodynamic average.

a) Show that S is indeed a nematic order parameter. To this end, show that it vanishes if the rods point in random directions. Also calculate S for a state with perfect nematic order.

b) In a magnet, an order parameter can be defined as $M = \sum_j \langle \cos \Theta_j \rangle$ where Θ_j is the angle between the spin j and the magnetization direction $\hat{n}$. Is this an order parameter for the nematic, too? Explain your answer.

c) Consider hypothetical molecules shaped like a + sign (symmetric under 90° rotations). Suggest an order parameter characterizing the degree of order in their relative orientations. Hint: Construct the order parameter analogous to S, but take the different symmetry into account!

Problem 2: Nematic-to-isotropic transition (25 points)

A liquid crystalline material can undergo a transition from an isotropic state ($S = 0$) to a nematic state ($S \neq 0$) upon lowering the temperature. The behavior close to the transition can be understood in terms of the free energy density f as a function of S (this is a so-called Landau free energy):

$$f = \frac{1}{2} r S^2 - w S^3 + u S^4 \ .$$

Here, w and u are positive constants, and r is a measure for the temperature. The physical value of the nematic order parameter S can be found by minimizing f.

a) Sketch the Landau free energy for various values of r (positive and negative) while treating w and u as fixed constants.

b) Find S as a function of r by minimizing the free energy.

c) The nematic phase transition occurs when the nematic phase ($S \neq 0$) has a lower free energy then the isotropic phase ($S = 0$). Find the value r_c of the temperature parameter at which the transition occurs. Find the value S_c of the order parameter at the transition.

d) Determine the limits of metastability, i.e., the r-range for which several solutions coexist.