Physics 481: Condensed Matter Physics - Homework 5

---

due date: Friday, Feb 18, 2011

Problem 1: Fibonacci chain (8 points)

Determine the ratio between the numbers of A and B elements in Fibonacci chains of generation 2,3,4,5. Calculate the limiting value in an infinite chain. Use the inflation rule!

Problem 2: Linear ionic crystal (12 points)

Consider a one-dimensional chain of $2N$ ions of alternating charge $\pm q$ ($N \gg 1$). In addition to the Coulomb interaction, there is a repulsive potential A/R^n between nearest neighbors only. (R is the distance between nearest neighbor ions.)

a) Determine the equilibrium distance R_0.

b) Determine the cohesive energy E_0 for this distance and show that it can be written as

$$E_0 = -N\,2\ln 2\left(1 - \frac{1}{n}\right)\frac{q^2}{R_0}\ .$$

c) Determine the work necessary to compress the crystal such that $R = R_0(1-\delta)$ to leading order in the small parameter $\delta \ll 1$

Problem 3: Polymer stiffness (Marder, problem 5.6, 20 points)

Consider a polymer composed of a sequence of N rigid rods of length a. The polymer is confined to two dimensions, and the rods are connected by springs. If the angle between rod l and rod $l+1$ is Θ_l then the energy of this joint is $\kappa\Theta_l^2$. (assume low temperatures such that $\kappa/k_BT \gg 1$). Show that long enough polymers behave as a random walks. To this end:

a) Write down the probability of having a particular set of angles $\Theta_1, \ldots, \Theta_N$ at temperature T (use canonical ensemble, i.e., Boltzmann distribution)

b) Put one end of the polymer at the origin. Find the coordinates (x_N, y_N) of the other end as a function of the angles $\Theta_1, \ldots, \Theta_N$. (Hint: It helps to formulate the problem in the complex plane!)

c) Find the thermal average $\langle x_N^2 + y_N^2 \rangle$. (Assume a sufficiently long polymer such that $Nk_BT \gg \kappa$.)

d) The result has the same form as expected for an ideal random walk, but the segment length a has to be replaced by by an effective length $\tilde{a}$. What is $\tilde{a}$?